96/100

Scott Webster

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Phys 523 Assignment 1

Jan 30 '05

1\. a) We want to frequency double 784 nm light to produce 100 mW at 392 nm with a ~100 MHz linewidth (very narrow).

Need to choose a crystal. I'll choose BBO (β-BaB_2O_4 – Barium Borate) because it allows phase matching for these wavelengths, has a relative large nonlinear response and is uniaxial, making it "easy" to deal with. okay.

a1 5/5

Sellmeier equations: $n^2 = A + \dfrac{B}{\lambda^2 + C} + D\lambda^2$

Get A, B, C, D from Eimerl et al. J. Appl. Phys. 62 p. 1968

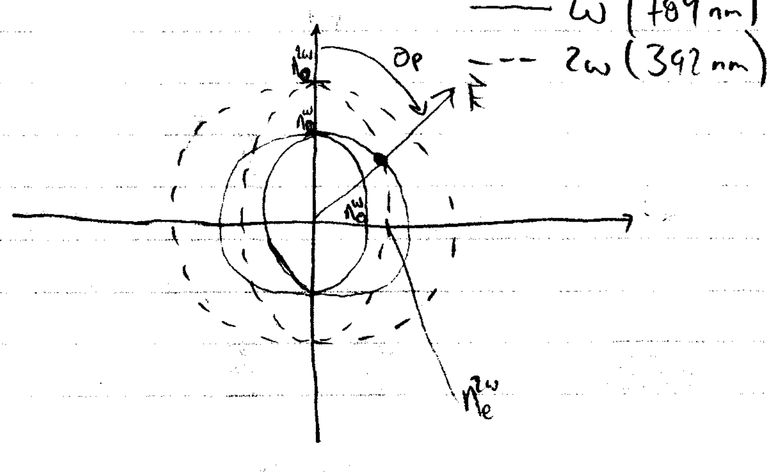
	A	B (μm^2)	C (μm^{-2})	D (μm^{-2})
n_e	2.3730	0.0128	−0.0156	−0.0044
n_o	2.7405	0.0187	−0.0179	−0.0155

Solve for our λ:

	784 nm	392 nm
n_e	1.54650	1.57004
n_o	1.66187	1.69518

Conditions for 90° phase matching are not met. I won't consider temperature tuning here, but will proceed with angle tuned phasematching.

K-surface plot:



This is just a sketch. We can tell the relative dimensions of the circles and ellipses because we calculated $n_o^{\omega}, n_e^{\omega}, n_o^{2\omega}, n_e^{2\omega}$ above.

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We need $n_o(\omega) = n_e(2\omega, \theta)$. This is satisfied when $\theta = \theta_p$, the phase matching angle.

Solve for θ_p as in class and the example problem to get:

$$\sin\theta_p = \left[\frac{n_o^{-2}(\omega) - n_o^{-2}(2\omega)}{n_e^{-2}(2\omega) - n_o^{-2}(2\omega)}\right]^{\frac{1}{2}}$$

This expression makes sense because it includes the relevant indices, leaving out $n_e(\omega)$, which clearly does not affect the solution.

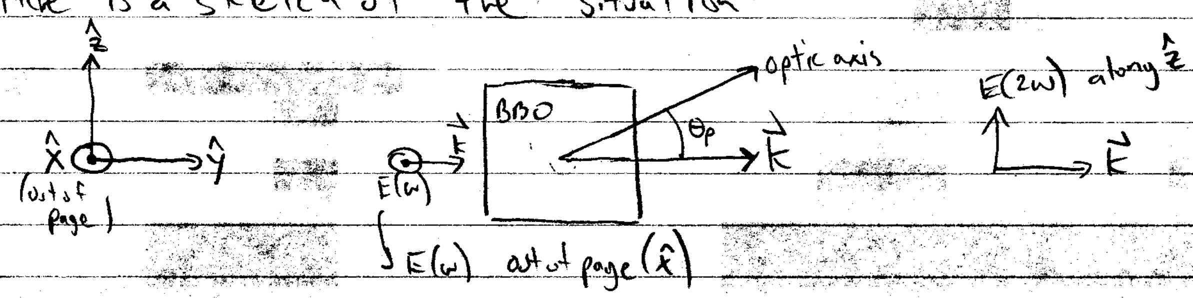
(a2) 5/5

$$\theta_p = \arcsin\left(\left[\frac{(1.66187)^{-2} - (1.69518)^{-2}}{(1.57004)^{-2} - (1.69518)^{-2}}\right]^{\frac{1}{2}}\right) = 29.6^\circ$$

good

I checked the SNLO software and it seems to come up with $\theta_p = 29.9^\circ$. I will use my calculated value.

Here is a sketch of the situation.



So we cut the crystal at an angle θ_p from the optic axis as shown above.

To eliminate reflections (of the fundamental beam) one can use Brewster windows.

Here $\tan\theta_B = \frac{n_2}{n_1} = \frac{n_o(\omega)}{n_{air}} = 1.66187 \Rightarrow \theta_B = 58.96^\circ$

Hilroy

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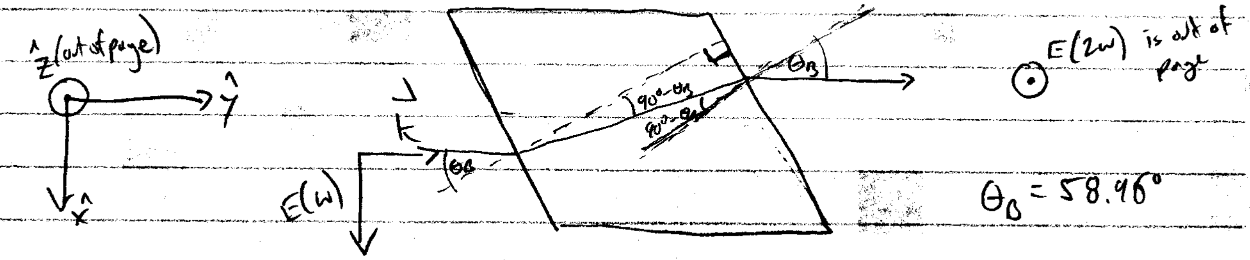
We now adjust the crystal cut as follows: From a new viewpoint, looking "down from above":



$\theta_B = 58.96°$

(b3) 5/5

The optic axis of the crystal is now pointing up out of the page on an angle. yes, good

Now we need to calculate how the second harmonic is generated, We have: (gaussian units)

generally: $\nabla^2 E(r,\omega) + \frac{\omega^2}{c^2}\varepsilon(\omega)\cdot E(r,\omega) = -\frac{4\pi\omega^2}{c^2}P^{(2)}_{NL}(r,\omega)$

Let $\omega_2 = 2\omega$

$$\Rightarrow \left[\frac{d}{dz^2} + \frac{\omega_2^2}{c^2}\varepsilon^{(1)}(\omega_2)\right]E(z,\omega_2) = -4\pi\frac{\omega_2^2}{c^2}P^{(2)}_{NL}(z,\omega_2) \quad (1)$$

following Mills here more or less..

Use slowly varying envelope approx: $E(z,\omega_2) \simeq E(z,\omega_2)e^{ik_2z}$

Where the variation of e^{ik_2z} is assumed large compared to $E(z,\omega_2)$

Now, $\frac{\partial^2 E}{\partial z^2} = -\left(k_2^2E - 2ik_2\frac{\partial E}{\partial z} - \frac{\partial^2 E}{\partial z^2}\right)e^{ik_2z}$

with $\frac{\partial^2 E}{\partial z^2} \approx 0 \Rightarrow \frac{\partial^2 E}{\partial z^2} = -\left(k_2^2E - 2ik_2\frac{\partial E}{\partial z}\right)e^{ik_2z}$

with $k_2^2 = \frac{\omega_2^2}{c^2}\varepsilon(\omega_2)$. Now sub into (1), k^2E terms cancel.

get: $2ik_2\frac{\partial E}{\partial z} = -4\pi\frac{\omega_2^2}{c^2}P^{(2)}_{NL}(z,\omega_2)e^{-ik_2z}$

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So, if $P^{(2)}_{NL}(z,\omega_2) = E^2(\omega_1)\chi^{(2)} e^{i2k_1 z}$

$$\frac{dE}{dz} = \frac{2\pi i \omega_2^2}{c^2 k_2} E^2(\omega_1)\chi^{(2)} e^{-ik_2 z + i2k_1 z}$$

Now integrate over z from $z=0$ to $z=z$ (bad notation)

let $\Delta k = 2k_1 - k_2$

$$E(\omega_2, z) = \frac{2\pi i \omega_2^2}{c^2 k_2} E^2(\omega_1)\chi^{(2)}\left(\frac{-i}{\Delta k}\left(e^{i\Delta k z} - 1\right)\right) \qquad (E(\omega_2,0)=0)$$

Now, $e^{i\Delta k z} - 1 = e^{i\frac{\Delta k z}{2}}\left(e^{i\frac{\Delta k z}{2}} - e^{-i\frac{\Delta k z}{2}}\right) = 2i e^{i\frac{\Delta k z}{2}} \sin\frac{\Delta k z}{2}$

So $$E(\omega_2, z) = \frac{4\pi i \omega_2^2 E^2(\omega_1)\chi^{(2)}}{c^2 k_2 \Delta k} e^{i\frac{\Delta k z}{2}} \sin\frac{\Delta k z}{2}$$

Now $$S = \frac{c^2 k_2 |E(\omega_2, z)|^2}{2\pi\omega_2}$$

So $$S = \frac{\cancel{c^2}\cancel{k_2}}{\cancel{2\pi\omega_2}}\left(\frac{\overset{8}{\cancel{16}}\pi^{\cancel{2}} \omega_2^{\overset{3}{\cancel{4}}} \chi^{(2)^2} |E(\omega_1)|^4}{c^{\cancel{4}2} k_2^{\cancel{2}}} \frac{\sin^2\frac{\Delta k z}{2}}{(\Delta k)^2}\right)$$

$$S = \frac{8\pi\omega_2^3 |\chi^{(2)}|^2 |E(\omega_1)|^4}{c^2 k_2} \frac{\sin^2\frac{\Delta k z}{2}}{(\Delta k)^2}$$

$\Delta k \to 0$ for phase matching, so $\frac{\sin^2\frac{\Delta k z}{2}}{(\Delta k)^2} \to \left(\frac{z}{2}\right)^2$

$$S = \frac{2\pi\omega_2^3 |\chi^{(2)}|^2 |E(\omega_1)|^4 z^2}{c^2 k_2}$$

Mills somehow manages to have c^3 in the denominator, which I believe is correct, but I can't see how.

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Also, I am having difficulty converting from power to electric field... a great deal of difficulty. This is compounded by the confusion between CGS and MKS units.

So, I guess I'll give up on that and use an expression from Ashkin et al (IEEE J. Quant. Elec. QE-2 p 109)

$$P_2 = \frac{128\pi^2\omega_1^2}{(n_1 c)^3} d_{eff}^2 \frac{P_1^2 z^2}{w_0^2}$$ w_0 is the waist radius

$$d_{eff} = \frac{1}{2}\chi^{(2)}_{eff}$$? ✓

I guess now I'll find deff.

Following the solution given in the supplemental notes:

$$\begin{bmatrix} E_x' \\ E_y' \\ E_z' \end{bmatrix} = \underbrace{\begin{bmatrix} \cos\phi & -\sin\phi\cos\theta & \sin\phi\sin\theta \\ \sin\phi & \cos\phi\cos\theta & -\cos\phi\sin\theta \\ 0 & \sin\theta & \cos\phi \end{bmatrix}}_{T} \begin{bmatrix} E_0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} E_0\cos\phi \\ E_0\sin\phi \\ 0 \end{bmatrix}$$

This is with $E(\omega)$ along $\hat{x}$ as before. The transformation is done by rotating about $\hat{x}$ by θ, then about $\hat{z}'$ by ϕ. Now $\theta = 90° - \theta_p$ for consistency.

The Eimerl paper I referenced above has:

$d_{11} = 1.6$ pm/V

$\left|\frac{d_{22}}{d_{11}}\right| < 0.05$

$\left|\frac{d_{31}}{d_{11}}\right| < 0.05$

This seems inconsistent with other references.

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"Handbook of Nonlinear Optics" by Sutterland has

BBO 3m symmetry

$d_{11} = 1.6$ pm/V
$d_{22} = 2.2$ pm/V
$d_{31} = 0.16$ pm/V

$$\bar{d} = \begin{bmatrix} 0 & 0 & 0 & 0 & d_{31} & -d_{22} \\ -d_{22} & d_{22} & 0 & d_{31} & 0 & 0 \\ d_{31} & d_{31} & d_{33} & 0 & 0 & 0 \end{bmatrix}$$

this is also from sutterland for 3m crystals.
Note: no d_{11} appears. Very odd.

$$\begin{bmatrix} P_x' \\ P_y' \\ P_z' \end{bmatrix} = \epsilon_0 \bar{d} \begin{bmatrix} E_x'^2 \\ E_y'^2 \\ E_z'^2 \\ 2E_y'E_z' \\ 2E_x'E_z' \\ 2E_x'E_y' \end{bmatrix}$$

$$P' = \begin{bmatrix} d_{31}\cancel{2E_x'E_z'} - 2d_{22}E_x'E_y' \\ -d_{22}E_x'^2 + d_{22}E_y'^2 + \cancel{2d_{31}E_y'E_z'} \\ d_{31}E_x'^2 + d_{31}E_y'^2 + \cancel{d_{33}E_z'^2} \end{bmatrix} \qquad E_z' = 0$$

$$P = T^T P'$$

$$P = \epsilon_0 \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi\cos\theta & \cos\phi\cos\theta & \sin\theta \\ \sin\phi\sin\theta & -\cos\phi\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} -2d_{22}E_0^2\cos\phi\sin\phi \\ -d_{22}E_0^2(\cos^2\phi - \sin^2\phi) \\ d_{31}E_0^2(\underbrace{\cos^2\phi + \sin^2\phi}_{1}) \end{bmatrix} \cos 2\phi$$

$$P = \epsilon_0 \begin{bmatrix} \text{—— not phase matched} \\ \text{—— not phase matched} \\ -2d_{22}\sin^2\phi\cos\phi\sin\theta E_0^2 + E_0^2 d_{22}\cos 2\phi\cos\phi\sin\theta + d_{31}E_0^2\cos\theta \end{bmatrix}$$

$$P_Z = \epsilon_0 E_0^2 \left(-2d_{22} \sin^2\phi \cos\phi \sin\theta + d_{22} \cos 2\phi \cos\phi \sin\theta + d_{31} \cos\theta \right)$$

$$= \epsilon_0 E_0^2 \left(d_{31} \cos\theta + d_{22} \sin\theta \left(\cos 2\phi \cos\phi \; 2\sin^2\phi \cos\phi \right) \right)$$

$$\cos 2\phi \cos\phi - 2\sin^2\phi \cos\phi = \cos 2\phi \cos\phi - \sin 2\phi \cos\phi = \cos 3\phi$$

So $P_Z = \epsilon_0 E_0^2 \left(d_{31} \cos\theta + d_{22} \sin\theta \cos 3\phi \right)$

But my $\theta_p = 90 - \theta$

So $P_Z = \epsilon_0 E_0^2 \left(\overbrace{d_{31} \sin\theta + d_{22} \cos\theta \cos 3\phi}^{d_{eff}} \right)$

Sutterland's result for d_{eff} has a negative sign... error in T matrix? — your T matrix looks correct to me...

If we take the −ve sign then to maximize d_{eff} we set $\phi = 60°$. Now we adjust our crystal cut to attain this.

Numerically then,

$$d_{eff} = 0.16 \text{ pm/V} \sin 29.6° + 2.2 \text{ pm/V} \cos 29.6°$$

$$\underline{d_{eff} = 1.99 \text{ pm/V}}$$

a6 5/5

Convert d_{eff} to CGS units

$$d_{eff}^{CGS} = 1.99 \frac{\text{pm}}{\text{V}} \times \frac{1 \text{ m}}{10^{12} \text{ pm}} \; \frac{3 \times 10^4}{4\pi}$$

$$= 4.75 \times 10^{-9} \frac{\text{cm}}{\text{statvolt}}$$

Crystal length: Use method of Boyd + Kleinman ✓
J. Appl. Phys. v39 p3597 (1968)

Define focusing parameter $\xi = \dfrac{l}{b}$

b is confocal parameter $b = w_0^2 k_1 = \dfrac{w_0^2 n_1 \omega_1}{c}$ (since $k_1 = \dfrac{n_1 \omega_1}{c}$)

Define Double refraction (walkoff) param. $B = \dfrac{\rho\,(l k_1)^{\frac{1}{2}}}{2}$

$$\tan\rho = \frac{\left(\frac{n_o^2}{n_e^2} - 1\right)\tan\theta_p}{\frac{n_o^2}{n_e^2}\tan\theta_p + 1} = \frac{\left(\frac{1.69519^2}{1.57057^2} - 1\right)\tan 29.6^\circ}{\frac{1.69519^2}{1.57057^2}\tan 29.6^\circ + 1}$$

$\Rightarrow \rho = 0.068$ rad $= 3.9^\circ$ walkoff

Find B. Guess 1mm = 0.1cm crystal first.

$$B = \frac{\rho}{2}\left(\frac{l\, n_1 \omega_1}{c}\right)^{\frac{1}{2}} = \frac{\rho}{2}\left(\frac{l\, n_1}{c}\frac{2\pi c}{\lambda}\right)^{\frac{1}{2}} = \frac{0.068}{2}\left(\frac{0.1 \cdot 1.662 \cdot 2 \cdot \pi}{784\times10^{-7}}\right)^{\frac{1}{2}}$$

$= 3.92 \approx 4$ ✓

Now Fig. 2 from Boyd + Kleinman:

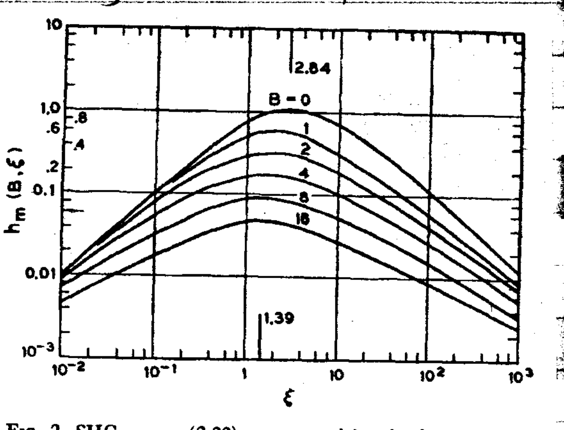

FIG. 2. SHG power (2.22) represented by the function $h_m(B, \xi)$ (2.29) for optimum phase matching as a function of focusing parameter $\xi = l/b$ for several values of double-refraction parameter $B = \rho(lk_1)^{1/2}/2$. Vertical lines indicate optimum focusing in the limits of small and large B.

So we can read off the optimum ξ for $B = 4$.

$\xi \approx 1.5$

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So, solve for l in ξ equation.

$$\xi = \frac{l}{b} = \frac{l c}{w_0^2 n_1 \omega_1} = \frac{l c \lambda_1}{w_0^2 n_1 2\pi c}$$

$$\Rightarrow l = \frac{\xi w_0^2 n_1 2\pi}{\lambda}$$

Choose $w_0 = 20 \times 10^{-4}$ cm as a reasonable value... similar to in the Henrerich paper

$$l = \frac{1.5 (20\times10^{-4})^2 \, 1.662 \cdot 2\pi}{784 \times 10^{-7}}$$

$= 0.80$ cm much larger than 1mm guess

$\Rightarrow$ Try again w/ 0.8cm crystal.

$$B = \frac{0.068}{2} \left(\frac{0.8 \; 1.662 \; 2\pi}{784\times10^{-7}} \right)^{\frac{1}{2}} = 11$$

Read off Figure again $\Rightarrow$ $\xi \approx 1.4$ for optimal $h_m(B, \xi)$

So not much change!

Resolve for $l = \frac{1.4 (20\times10^{-4})^2 \, 1.662 \; 2\pi}{784\times10^{-7}} = 0.75$ cm

Could iterate again but will just choose 0.75cm crystal length. okay. that's your optimal length in this case...

a3 10/10

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So now we can use this deff in the $P_2 = \eta P_1$ formula.

$$P_2 = \frac{128\pi^2 \omega_1^2}{(n_1 c)^3} d_{eff}^2 \frac{P_1^2 z^2}{W_0^2}$$

Use $z = 0.75cm$, $W_0 = 0.02 mm = 0.002 cm$

Need $P_2 = 100 mW = 0.1 \times 10^7 erg/s = 10^6 erg/s$

$$P_1 = \sqrt{\frac{P_2 (n_1 c)^3 W_0^2}{128\pi^2 \omega_1^2 d_{eff}^2 z^2}} = \sqrt{\frac{10^6 (1.66 \cdot 3\times10^{10})^3 \, 0.002^2}{128 \cdot 3.14^2 \left(\frac{3\times10^{10} 2\pi}{784\times10^{-7}}\right)^2 (4.75\times10^{-9})^2 \cdot 0.75^2}}$$

$$P_1 = 7.30\times10^{-7} \frac{erg}{s} \times \frac{1W}{10^7 erg/s} = \boxed{7.3 W}$$ which is it?

a4 4/5

$\boxed{54.8 W}$ is probably a bit much, hence the need for a resonator.

Depleted pump (still single pass)

From Sutherland this time:

$$\eta = \tanh^2 (z/L_c) \qquad L_c = \frac{1}{16\pi^2 d_{eff}} \sqrt{\frac{n_\omega^3 c \lambda_\omega^2}{2\pi I_\omega(0)}}$$

$$P_2 = \eta P_1 \qquad L_c = \frac{1}{16\pi^2 4.75\times10^{-9}} \sqrt{\frac{1.66^3 \, 3\times10^{10} (784\times10^{-7})^2}{2\pi \underbrace{\frac{10^6 erg/s}{\pi 0.002^2}}}}$$

a5 3/5

$= 54.8.$

Here you assume $P_\omega = .1 W$ yet this is what you are ~~trying to find~~

$$P_1 = \frac{P_2}{\eta} = \frac{10^6 erg/s}{\tanh^2(0.75/54.8)} = 5.34\times10^9 \frac{erg}{s} \times \frac{1W}{10^7 erg/s} = 534 W$$

Hilroy

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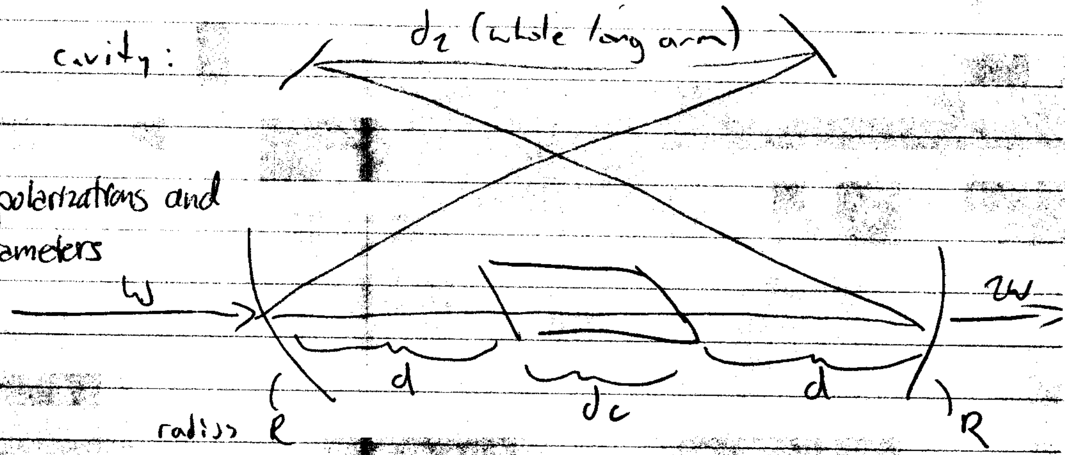
b) These required pump values are very large so now I'll design a resonator for the fundamental.

Bow tie cavity:

(b1)

10/10

your cavity polarizations and crystal cut parameters are done in b3 on page 3.



Start at left spherical mirror. Have 4 "elements".

1. Free space length d
2. Crystal length d_c
3. Free space length d + spherical mirror radius R
4. Free space length d_2 + spherical mirror radius R

Corresponding to matrices:

$$(3) = \begin{bmatrix} 1 & d \\ -\frac{2}{R} & 1-\frac{2d}{R} \end{bmatrix} \qquad (4) = \begin{bmatrix} 1 & d_2 \\ -\frac{2}{R} & 1-\frac{2d_2}{R} \end{bmatrix}$$

$$(1) = \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} \qquad (2) = \begin{bmatrix} 1 & d_c/n \\ 0 & 1 \end{bmatrix}$$

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So the ABCD matrix (call it M) is:

$$M = (4)(3)(2)(1) = \begin{bmatrix} 1 & d_2 \\ -\frac{2}{R} & 1-\frac{2d_2}{R} \end{bmatrix} \begin{bmatrix} 1 & d \\ -\frac{2}{R} & 1-\frac{2d}{R} \end{bmatrix} \begin{bmatrix} 1 & d_c/n \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix}$$

Solve w/ maple:

$$M = \begin{bmatrix} \frac{R-2d_2}{R} & \frac{2dRn - 4dnd_2 + d_cR - 2d_cd_2 + nd_2R}{Rn} \\ -4\left(\frac{R-d_2}{R^2}\right) & \frac{-8dRn + 8dnd_2 - 4d_cR + 4d_cd_2 + R^2n - 2nd_2R}{R^2n} \end{bmatrix}$$

Now analyze. (sort of following Kogelnik + Li Applied Optics v5 p1550 (1966))

Stable when $-1 < \frac{1}{2}(A+D) < 1$ ✓

Radius of curvature of beam $\mathcal{R} = \frac{2B}{D-A}$ (note: $\mathcal{R} \neq R$)

Beam radius $w = \frac{2\lambda|B|}{\pi} \Big/ \sqrt{4-(A+D)^2}$

waist $w_0^2 = w^2 \Big/ \left[1 + \left(\frac{\pi w^2}{\lambda \mathcal{R}}\right)^2\right]$

These formula's come from knowing that the q parameter: $\frac{1}{q} = \frac{1}{R} - \frac{i\lambda}{\pi w^2}$ is modified by the ABCD matrix as

$$q_2 = \frac{Aq_1 + B}{Cq_1 + D}$$

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and the requirement that $q_2 = q_1$

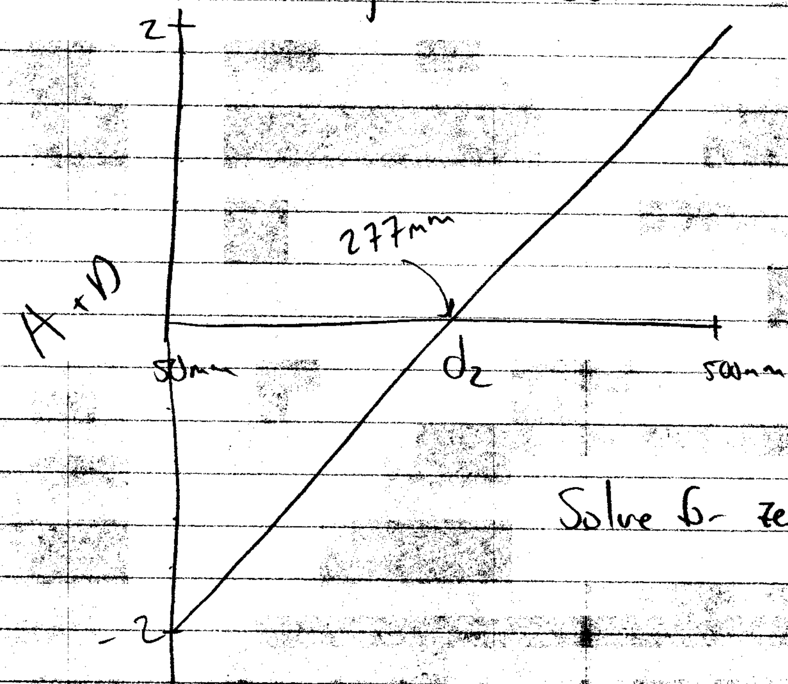
Knowing this we can solve the quadratic for $q = q_1 = q_2$ and arrive at the above formulas.

Start with some values similar to the Hemmerich paper.

$d = 25.5\,mm$ $R = 50\,mm$ and use $d_c = 7.5\,mm$ from before.

Plot variation of stability condition with d_2 (long arm length)

Use $S = A + D$, find zero:



Solve for zero, get $d_2 = 277\,mm$

Use maple to find waist of beam for this d_2.

Find $W_0 = 26\,\mu m$. Close to $20\,\mu m$ "guess".

Now analyze variation of W_0 with changes to d and R.

Maple sheet is attached with some comments. *Hilary*

All distances in metres.

```
> with(linalg):
Warning, the protected names norm and trace have been redefined and unprotected
> one:=matrix(2,2,[1,d,0,1]);
```

$$one := \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix}$$

```
> two:=matrix(2,2,[1,dc/n,0,1]);
```

$$two := \begin{bmatrix} 1 & \frac{dc}{n} \\ 0 & 1 \end{bmatrix}$$

```
> three:=matrix(2,2,[1,d,-2/R,1-2*d/R]);
```

$$three := \begin{bmatrix} 1 & d \\ -2\frac{1}{R} & 1-\frac{2d}{R} \end{bmatrix}$$

```
> four:=matrix(2,2,[1,d2,-2/R,1-2*d2/R]);
```

$$four := \begin{bmatrix} 1 & d2 \\ -2\frac{1}{R} & 1-\frac{2d2}{R} \end{bmatrix}$$

Optical elements

```
> M:=multiply(four,three,two,one);
```

$$M := \begin{bmatrix} 1-\frac{2d2}{R} & \left(1-\frac{2d2}{R}\right)d+\frac{\left(1-\frac{2d2}{R}\right)dc}{n}+d+d2\left(1-\frac{2d}{R}\right) \\ -2\frac{1}{R}-\frac{2\left(1-\frac{2d2}{R}\right)}{R} & \left(-2\frac{1}{R}-\frac{2\left(1-\frac{2d2}{R}\right)}{R}\right)d+\frac{\left(-2\frac{1}{R}-\frac{2\left(1-\frac{2d2}{R}\right)}{R}\right)dc}{n}-\frac{2d}{R}+\left(1-\frac{2d2}{R}\right)\left(1-\frac{2d}{R}\right) \end{bmatrix}$$

← ABCD matrix

```
> M:=simplify(%);
```

$$M := \begin{bmatrix} \frac{R-2d2}{R} & \frac{2dRn-4dnd2+dcR-2dcd2+d2nR}{Rn} \\ -4\frac{R-d2}{R^2} & -\frac{8dRn-8dnd2+4dcR-4dcd2-R^2n+2d2nR}{R^2n} \end{bmatrix}$$

```
> AA:=M[1,1];BB:=M[1,2];CC:=M[2,1];DD:=M[2,2];
```

$$AA := \frac{R-2d2}{R}$$

$$BB := \frac{2dRn-4dnd2+dcR-2dcd2+d2nR}{Rn}$$

$$CC := -4\frac{R-d2}{R^2}$$

$$DD := -\frac{8dRn-8dnd2+4dcR-4dcd2-R^2n+2d2nR}{R^2n}$$

```
> r:=simplify(2*BB/(DD-AA));
```

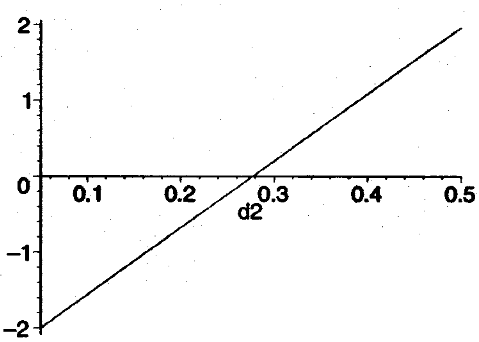
$$r := -\frac{1}{2}\frac{(2dRn-4dnd2+dcR-2dcd2+d2nR)R}{2dRn-2dnd2+dcR-dcd2}$$

→ R

```
> S:=AA+DD;
```

$$S := \frac{R-2d2}{R}-\frac{8dRn-8dnd2+4dcR-4dcd2-R^2n+2d2nR}{R^2n}$$

```
> plot(subs(R=50e-3,d=25.5e-3,dc=7.5e-3,n=1.662,S),d2=50e-3..500e-3);
```



stability plot

```
> solve(subs(R=50e-3,d=25.5e-3,dc=7.5e-3,n=1.662,S)=0,d2);
```

$$.2767518011$$

```
> w:=((2*lambda*abs(BB)/Pi)/(4-(AA+DD)^2)^(1/2))^(1/2);
```

$$w := \sqrt{2}\sqrt{\frac{\lambda\left|\frac{2dRn-4dnd2+dcR-2dcd2+d2nR}{Rn}\right|}{\pi\sqrt{4-\left(\frac{R-2d2}{R}-\frac{8dRn-8dnd2+4dcR-4dcd2-R^2n+2d2nR}{R^2n}\right)^2}}}$$

Beam radius w

```
> w0:=(w^2/(1+(Pi*w^2/lambda/r)^2))^(1/2);
```

$$w0 := \sqrt{2}\,\mathrm{sqrt}\left(\lambda\left|\frac{2dRn-4dnd2+dcR-2dcd2+d2nR}{Rn}\right| \Big/ \left(\pi\sqrt{4-\left(\frac{R-2d2}{R}-\frac{8dRn-8dnd2+4dcR-4dcd2-R^2n+2d2nR}{R^2n}\right)^2}\right.\right.$$

Beam waist W_0

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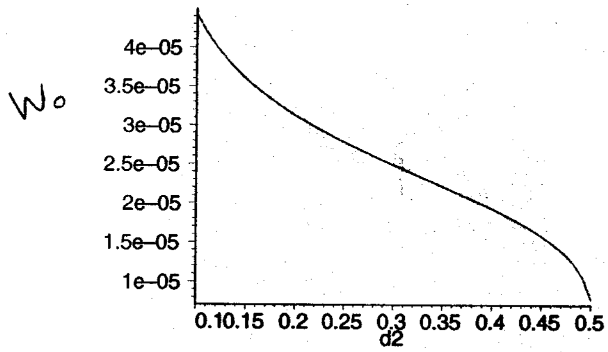
$$\left(1+\frac{16\left|\frac{2\,d\,R\,n-4\,d\,n\,d2+dc\,R-2\,dc\,d2+d2\,n\,R}{R\,n}\right|^2(2\,d\,R\,n-2\,d\,n\,d2+dc\,R-dc\,d2)^2}{\left(4-\left(\frac{R-2\,d2}{R}-\frac{8\,d\,R\,n-8\,d\,n\,d2+4\,dc\,R-4\,dc\,d2-R^2\,n+2\,d2\,n\,R}{R^2\,n}\right)^2\right)(2\,d\,R\,n-4\,d\,n\,d2+dc\,R-2\,dc\,d2+d2\,n\,R)^2\,R^2}\right)$$

```
> plot(subs(lambda=784e-9,R=50e-3,d=25.5e-3,dc=7.5e-3,n=1.662,w0),d2=100e-3..500e-3);
```



Variation of waist with long arm length d_2

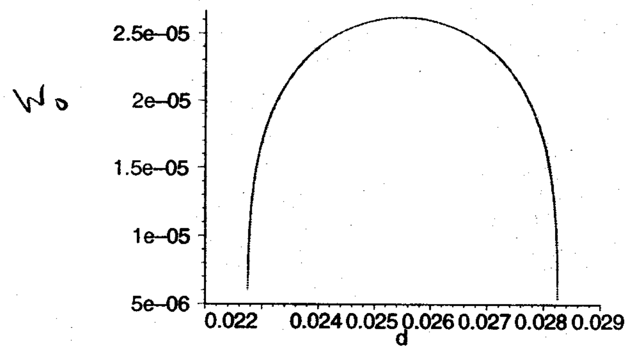
```
> evalf(subs(lambda=784e-9,R=50e-3,d=25.5e-3,dc=7.5e-3,n=1.662,d2=277e-3,w0));
```

.00002621259665

```
> %*1e6;
```

26.21259665

```
> plot(subs(lambda=784e-9,R=50e-3,dc=7.5e-3,n=1.662,d2=277e-3,w0),d=22e-3..29e-3);
```

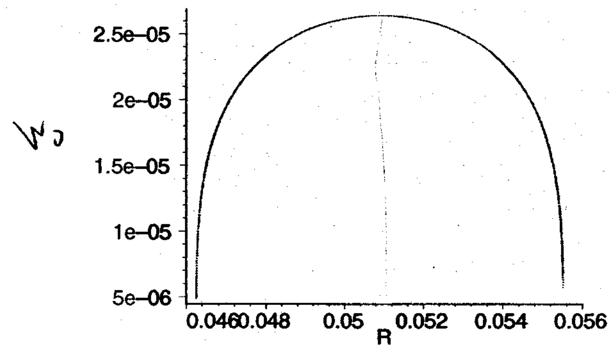


Variation of wo with d ($\frac{1}{2}$ short arm length)

```
> fsolve(diff(subs(lambda=784e-9,R=50e-3,dc=7.5e-3,n=1.662,d2=277e-3,w0),d)=0,d);
```

.02549698628

```
> plot(subs(lambda=784e-9,d=25.5e-3,dc=7.5e-3,n=1.662,d2=277e-3,w0),R=46e-3..56e-3);
```



Variation of wo with R (Radius of mirrors)

```
> fsolve(diff(subs(lambda=784e-9,d=25.5e-3,dc=7.5e-3,n=1.662,d2=277e-3,w0),R)=0,R);
```

.5030737667

```
> plot(subs(lambda=784e-9,d=25.5e-3,R=50e-3,n=1.662,d2=277e-3,w0),dc=1e-3..15e-3);
```

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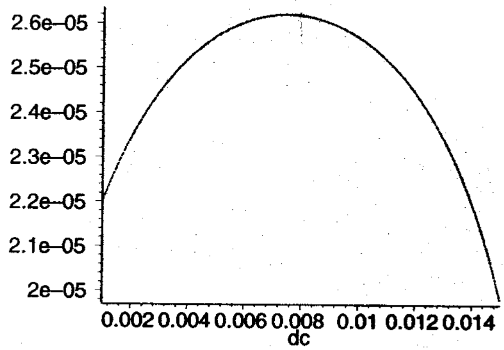

Variation of w_0 with d_c (crystal length)

```
> fsolve(diff(subs(lambda=784e-9,d=25.5e-3,R=50e-3,n=1.662,d2=277e-3,w0),dc)=0,dc);
                              .007489982380
>
```

As can be seen from the figures:

✓ The stability of w_0 with d is relatively good. Changing d by ± 1 mm does not greatly affect the waist.

R can similarly change without drastically changing w_0.

b2

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For d_c (which we set anyway) the waist does not change much at all with different lengths.

My waist value could be slightly off due to the crystal refractive index. I'm not sure if this calculation takes that into account, however the SNLO software gives a waist in the crystal of 29.13 μm, so I seem to be on the right track. ✓

To find the beam radius at the input (left spherical mirror)

We can use $$W^2(z) = W_0^2\left[1 + \left(\frac{\lambda z}{\pi W_0^2}\right)^2\right]$$

$$W^2(-28\,\text{mm}) = (26.2\times10^{-6})^2\left[1 + \left(\frac{794\times10^{-9}\cdot 28\times10^{-3}}{\pi\,(26.2\times10^{-6})^2}\right)^2\right]$$

$$\Rightarrow W(-28\,\text{mm}) = 268\ \mu\text{m}.$$

Also $$R(z) = z\left[1 + \left(\frac{\pi W_0^2}{\lambda z}\right)^2\right] = -0.0283\ \text{m}$$ ~~Radius of~~ curvature of beam at input

Now we can shape our input beam with lens(es) as necessary to match these parameters. true

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Now we need to find Q.

Follow Kozlovsky. Depletion in term $t_{SH} = (1 - \gamma_{SH} P_c)$

where $\gamma_{SH} = \left(\frac{2 \omega_1^2 d_{eff}^2 k_1}{\pi n^3 \epsilon_0 c^3} \right) L h(B, \xi)$

$$\gamma_{SH} = \left(\frac{2\, 2^2 \pi^2 c^2 d^2\, 2\pi}{\lambda^2 \pi n^3 \epsilon_0 c^3\, \lambda} \right) Lh = \left(\frac{16 \pi d^2}{\lambda^3 n^3 \epsilon_0 c} \right) Lh = 0.0339\, Lh(B, \xi)$$

Get $h(B,\xi)$ from plot on page 8

$$h(11, 1.4) \approx 0.06 \quad , \quad L = 0.0075\ m$$

$$\gamma_{SH} = 0.0339 \cdot 0.06 \cdot 0.0075 = 1.526 \times 10^{-5}$$

Let $r_m = t^2 t_{SH} r_2$ where t is the transmission through the air in the cavity, r_2 is the power reflection coefficient of the "right" spherical mirror.

From Kozlovsky: $\frac{P_{circ}}{P_{inp}} = \frac{t_1}{(1 - \sqrt{r_1 r_m})^2}$ — $t_1 = 1 - r_1$, r_1 is pow. reflection coeff for left spherical mirror.

To impedance match we want $r_1 = r_m$ — yes.

$t_{SH} = (1 - \gamma_{SH} P_c)$

then $\frac{P_{circ}}{P_{inp}} = \frac{1 - t^2(1 - \gamma_{SH} P_{circ}) r_2}{(1 - t^2 (1 - \gamma_{SH} P_{circ}) r_2)^2}$ — for impedance matching $r_1 = r_m$, $t_1 = 1 - r_m$

$$= \frac{1}{1 - t^2 (1 - \gamma_{SH} P_{circ}) r_2}$$

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Use quadratic formula, get:

$$P_c = \frac{-(1-t^2 r_2) \pm \sqrt{(1-t^2 r_2)^2 + 4\gamma_{SH} r_2 P_1}}{2\gamma_{SH} r_2}$$

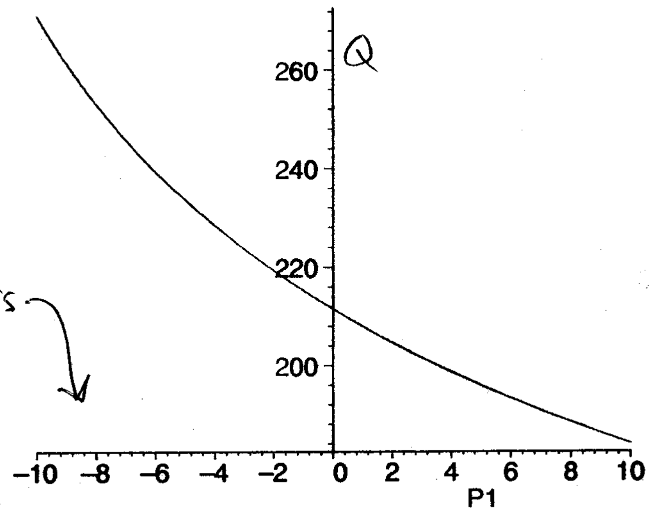
$Q = 2\pi \frac{P_{circ}}{P_{inp}}$ Can plot this.

Choose $t = 0.99$ and $r_2 = 0.99$ arbitrarily. Obviously this affect the result below. okay

```
> Pc:=(-(1-t^2*r2)+sqrt((1-t^2*r2)^2+4*gamma*r2*P1))/2/gamma/r2;
```

$$Pc := \frac{1}{2}\,\frac{-1 + t^2\, r2 + \sqrt{(1 - t^2\, r2)^2 + 4\,\gamma\, r2\, P1}}{\gamma\, r2}$$

```
> plot(2*Pi*subs(t=0.99,r2=0.99,gamma=1.526e-5,Pc)/P1,P1);
```



negative laser powers cool...

b4

15/15

So Q decreases with increasing pump power ($P_1 = P_{inp}$) because of depletion. ✓

Now, P_{SH}, the second harmonic power, is just

$$P_{SH} = \gamma_{SH} P_{circ}^2$$

Plot P_{SH} vs. P_1

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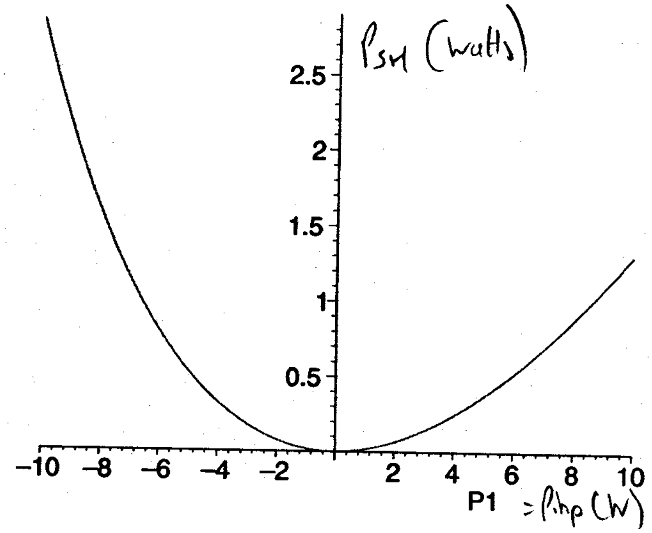
```
> Psh:=gamma*Pc^2;
```

$$Psh := \frac{1}{4}\frac{\left(-1+t^2 r2+\sqrt{(1-t^2 r2)^2+4\gamma r2\, P1}\right)^2}{\gamma r2^2}$$

```
> plot(subs(t=0.99,r2=0.99,gamma=1.526e-5,Psh),P1);
```



b5

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```
> solve(subs(t=0.99,r2=0.99,gamma=1.526e-5,Psh)=0.1,P1);
```

-2.305328502, 2.503328502 $= P_{inp}$ for 100 mW

```
> Q:=2*Pi*subs(t=0.99,r2=0.99,gamma=1.526e-5,Pc)/P1;
```

$$Q := 2\frac{\pi(-982.9950883+33096.36337\sqrt{.000882149401+.0000604296\,P1})}{P1}$$

```
> evalf(subs(P1=2.503,Q));
```

$Q =$ 203.1827940

```
>
```

So P_{SH} increases with increasing pump power.

We can solve for the pump power required to get 100 mW of P_{SH}, finding

$$P_{inp} = 2.50 \text{ W}$$

Solving for Q at this input power we get

$$Q = 203.$$ okay

Clearly a 2.5 W pump is easier to find than a > 500 W pump, so the resonator is useful. Well said!

The above assumes that all the harmonic escapes the cavity, but unfortunately some will be reflected inside the crystal and from the output mirror. true..

Hilroy

Scott Webster

I haven't discussed whether there is a mode of the cavity at 784 nm yet, but since the mode spacing will be

$$\delta_\nu = \frac{c}{2d} = \frac{c}{2\cdot(277mm + 56mm)}$$

$$= 450\ MHz$$

which is very small, there will be a mode of the cavity very near 784 nm. — yes.

Also, the input mirror power reflectivity needs to be chosen such that $r_1 = r_m$ for impedance matching.

For our 100 mW requirement, this gives

$$r_1 = r_m = t^2 t_{SH} r_2 = (0.99)^3 (1 - \gamma_{SH} P_c)$$

$$= 0.99^3 (1 - 1.526\times10^{-5} \cdot 80.8\ W)$$

$$= 0.969$$

This seems reasonable. Perhaps $0.99 = r_2 = t$ is a bit optimistic, but I'm assuming I have very fancy mirrors.

no, 99% is fine...
fancy is 99.999 or better

<u>Bonus</u>

I'll comment briefly on these points.

To stabilize the cavity we can mount one of the mirrors on a piezo stage and then feed that piezo a signal based on a comparison of the input beam and the cavity beam. The fundamental inside the cavity can be sampled by putting a detector outside the cavity, behind a mirror which will necessarily transmit a small amount of fundamental.

as described, cavity transmission alone does not provide a "which way" information for ~~the~~ proper feedback control

This feedback system allows, in principle, the cavity length to be stabilized. Bonus +1

Astigmatism should be able to be corrected by tuning the spherical mirrors, but I won't get into that here. yes

Well, I'm out of time, so I won't address any more of the (many) complications that would need to be dealt with in reality.
— you can work the rest of these out in the lab... —

Completeness

good job of including references
providing expressions + derivations where possible
nice discussions
your drawings could have been a bit more detailed making life easier for the final exam (4 months later) when you have to build your cavity based on these notes...

8/10

Hilary