Murray McCutcheon 1

P. 523 Assignment 1

96/100

(1a) Choose BBO, as it has high nonlinear susceptibility, and is transparent to below 350 nm, as required.
(a1) └ good.

5/5

Referring to Table I (p. 263) of "Handbook of Nonlinear Optics" by Sutherland, → ref. [1], for BBO:

$$d_{11} = 1.6 \text{ pm/V} \quad [2]$$
$$d_{22} = 2.2 \text{ pm/V}$$
and $$d_{31} = 0.16 \text{ pm/V}$$

Note that these are not unambiguous values. Ref. [2] (D. Eimerl et al, J.A.P. 62, 1968 (1987).), from which d_{11} was extracted for [1], gives $\frac{d_{22}}{d_{11}} < 0.05$, which clearly disagrees with Sutherland.

Moreover, Sutherland gives d_{eff} for Type I (ooe) phase matching in a 3m crystal as

or d_{31} (depending on symmetry ... I will take $d_{31} = d_{15}$).

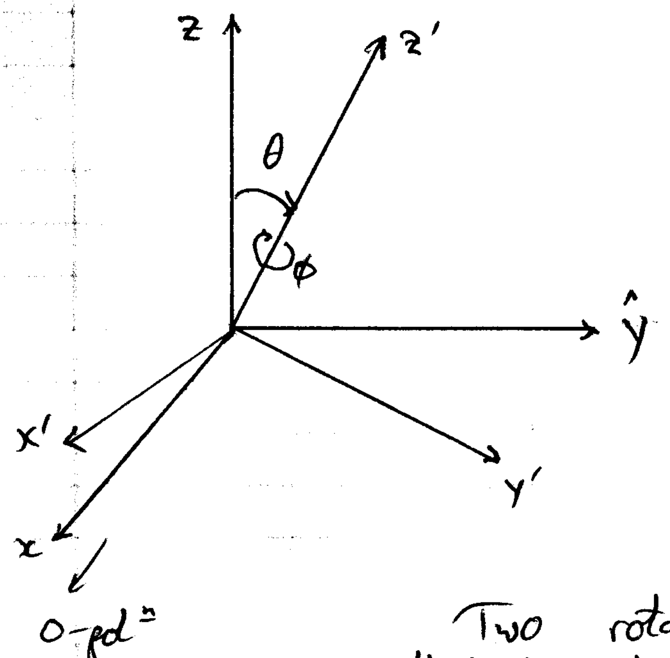
$$d_{eff} = (d_{15}) \sin\theta - \underline{d_{22}} \cos\theta \underline{\sin 3\phi}$$

whereas Eimerl et al. derive

$$d_{eff} = d_{31} \sin\theta - \underline{d_{11}} \cos\theta \underline{\cos 3\phi}$$

for seemingly similar definitions of angles etc.

I will try to derive the Sutherland result, based on the $LiIO_3$ phasematching solutions posted on the web.



First rotate around $\hat{x}$ by $-\theta$, then around z' by $-\phi$.

$T_x(-\theta)$, $T_{z'}(-\phi)$

Two rotation matrices, which, when multiplied together, give the transformation:

Crystal frame:
$$\begin{pmatrix} E'_x \\ E'_{y'} \\ E'_z \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi\cos\theta & \sin\phi\sin\theta \\ \sin\phi & \cos\phi\cos\theta & -\cos\phi\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}$$

$$\vec{E}(1\omega) = \begin{pmatrix} E_0 \\ 0 \\ 0 \end{pmatrix}$$ for ordinary pol input,

so
$$\begin{pmatrix} E'_x \\ E'_y \\ E'_z \end{pmatrix} = \begin{pmatrix} E_0\cos\phi \\ E_0\sin\phi \\ 0 \end{pmatrix}.$$

Use the d-matrix for 3m crystals from Sutherland, p. 18; and noting that with Kleinman symmetry, $d_{15} = d_{31}$,

$$\begin{pmatrix} 0 & 0 & 0 & 0 & d_{31} & -d_{22} \\ -d_{22} & d_{22} & 0 & d_{31} & 0 & 0 \\ d_{31} & d_{31} & d_{33} & 0 & 0 & 0 \end{pmatrix}$$

Nonlinear polarization: $\quad (E_{z'} = 0)$

$$\begin{pmatrix} P_x' \\ P_y' \\ P_z' \end{pmatrix} = \epsilon_0 \begin{pmatrix} 0 & 0 & 0 & 0 & d_{31} & -d_{22} \\ -d_{22} & d_{22} & 0 & d_{31} & 0 & 0 \\ d_{31} & d_{31} & d_{33} & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} E_{x'}^2 \\ E_{y'}^2 \\ E_{z'}^2 \\ 2E_{y'}E_{z'} \\ 2E_{x'}E_{z'} \\ 2E_{x'}E_{y'} \end{pmatrix}$$

$$= \epsilon_0 \begin{pmatrix} -2d_{22}E_{x'}E_{y'} \\ -d_{22}E_{x'}^2 + d_{22}E_{y'}^2 \\ d_{31}E_{x'}^2 + d_{31}E_{y'}^2 \end{pmatrix}$$

$$\begin{pmatrix} P_{x'} \\ P_{y'} \\ P_{z'} \end{pmatrix} = \epsilon_0 \begin{pmatrix} -2d_{22}E_0^2 \cos\phi \sin\phi \\ d_{22}E_0^2(\sin^2\phi - \cos^2\phi) \\ d_{31}E_0^2 \end{pmatrix}$$

Transforming back to the lab frame, using

$$\vec{P} = T^{-1}\vec{P}' = T^T\vec{P}'$$

$$\begin{pmatrix} P_x \\ P_y \\ P_z \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi\cos\theta & \cos\phi\cos\theta & \sin\theta \\ \sin\phi\sin\theta & -\cos\phi\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} P_{x'} \\ P_{y'} \\ P_{z'} \end{pmatrix}$$

Only P_z is phasematched:

$$P_z = \sin\phi\sin\theta(-2d_{22}E_0^2\cos\phi\sin\phi) - \cos\phi\sin\theta\, d_{22}E_0^2 \underbrace{(\sin^2\phi - \cos^2\phi)}_{-\cos 2\phi}$$

$$+ d_{31}E_0^2\cos\theta$$

so $\quad d_{eff} = d_{31}\cos\theta - 2d_{22}\sin^2\phi\cos\phi\sin\theta + d_{22}\cos\phi\cos 2\phi\sin\theta$

$$2\sin\phi\cos\phi = \sin 2\phi$$

$$= d_{31}\cos\theta - d_{22}\sin\theta\big(\underbrace{\sin\phi\sin 2\phi - \cos\phi\cos 2\phi}_{-\cos 3\phi}\big).$$

$$= d_{31}\cos\theta + d_{22}\sin\theta\cos 3\phi$$

but $\theta = \left(\frac{\pi}{2} - \theta_p\right)$, and $\cos\theta = \sin\theta_p$, $\sin\theta = \cos\theta_p$.

$$\boxed{d_{eff} = d_{31}\sin\theta_p - d_{22}\cos\theta_p\cos 3\phi}$$

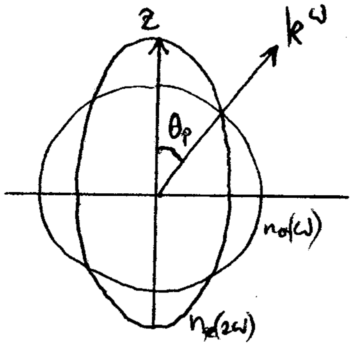
where did the minus sign come from?

If I can orient the crystal to have $\boxed{\phi = 60^\circ}$, then $\cos 3\phi = -1$, and

okay

$$d_{eff} = d_{31}\sin\theta_p + d_{22}\cos\theta_p.$$

To determine θ_p, we use the usual Type I construction:



$$\frac{1}{n_e^2(\theta)} = \frac{\cos^2\theta}{n_o^2} + \frac{\sin^2\theta}{n_e^2}$$

$$\Rightarrow \sin^2\theta_m = \frac{(n_o^{\omega})^{-2} - (n_o^{2\omega})^{-2}}{(n_e^{2\omega})^{-2} - (n_o^{2\omega})^{-2}}$$

Sellmeier equations for BBO (from newlightphotonics.com); also Eimerl et al. JAP, 62, 1968 (1987).

$$n_o^2 = 2.7359 + \frac{0.01878}{\lambda^2 - 0.01822} - 0.01354\lambda^2$$

$$n_e^2 = 2.3753 + \frac{0.01224}{\lambda^2 - 0.01667} - 0.01516\lambda^2$$

$$\Rightarrow (n_o^{\omega})^2 = 2.7582$$
$$(n_o^{2\omega})^2 = 2.8792$$

$$(n_e^{2\omega})^2 = 2.4667$$

which gives $\sin^2\theta_m = \dfrac{\frac{1}{2.7582} - \frac{1}{2.8792}}{\frac{1}{2.4667} - \frac{1}{2.8792}} = 0.2622$

$$\Rightarrow \boxed{\theta_m = 30.8^\circ}.$$ I got 29.9°

so finally $d_{eff} = 0.16\sin(30.8) + 2.2\cos(30.8)$ a2

$$\boxed{d_{eff} = 1.97 \text{ pm/V}}$$ a6 5/5 5/5

In the undepleted pump approximation, in SI units, for perfect phase-matching,

$$\eta_{2\omega}^{0} = \frac{8\pi^2 d_{eff}^2 L^2 I_\omega}{\varepsilon_0 (n_\omega)^3 c \lambda_\omega^2}$$ (p. 36 Sutherland).

where $P_{2\omega} = \eta_{2\omega} P_\omega$ in Watts.

let $I_\omega = \frac{P_\omega}{w_0^2}$ where w_0 is our beam waist.

so $$P_\omega = \sqrt{\frac{\varepsilon_0 (n_\omega)^3 c \lambda_\omega^2 w_0^2 P_{2\omega}}{8\pi^2 d_{eff}^2 L^2}}$$

Suppose we use a 1 mm crystal; note that the aperture length, as defined by Ashkin et al. (IEEE J. of Q. E), is

$$l_a = \frac{\sqrt{\pi}\, w_0}{\rho}$$. Calculate: $\tan\rho = \frac{\left(\frac{n_o^2}{n_e^2} - 1\right)\tan\theta_m}{\frac{n_o^2}{n_e^2}\tan^2\theta + 1}$ (from web notes)

$\rho = 3.95^\circ$ for BBO $\Rightarrow$ 69 mrad

Suppose $w_0 = 20 \times 10^{-6} = 20\ \mu m$,

then $l_a = \frac{\sqrt{\pi}\cdot 20.10^{-6}}{0.069} = \underline{0.5\ mm}$.

a3 8/10

Effective interaction of the beams is only maintained up to l_a, so suppose we have a $l = 0.5$ mm BBO crystal.

what is the diffraction length? How does l_f compare to l_a? Later (p.10) you motivate your choice of crystal length using the Boyd + Kleinman result which accounts for these competing effects... – the goal!!

$$\begin{cases} \lambda_\omega = 784\ nm \\ \varepsilon_0 = 8.85 \times 10^{-12}\ F/m \\ P_{2\omega} = 100\ mW \end{cases}$$

$$P_\omega = \sqrt{\frac{(8.85 \times 10^{-12})(1.6608)^3 \cdot 3\times 10^8\,(784 \times 10^{-9})^2 \cdot (20.10^{-6})^2\ 0.1}{8\pi^2\,(1.97\times 10^{-12})^2\,(0.0005)^2}}$$

this will depend on l_a and l_f...

$\cong$ 62.(48) ~ 63 W — this is probably right... W, which is clearly unfeasible.

a4 5/5

Depleted pump approximation

From Sutherland, we have (or Mills, (4.32a)).

$\frac{P_{2\omega}}{P_\omega} = \eta_{2\omega} = \tanh^2(L/L_{NL})$

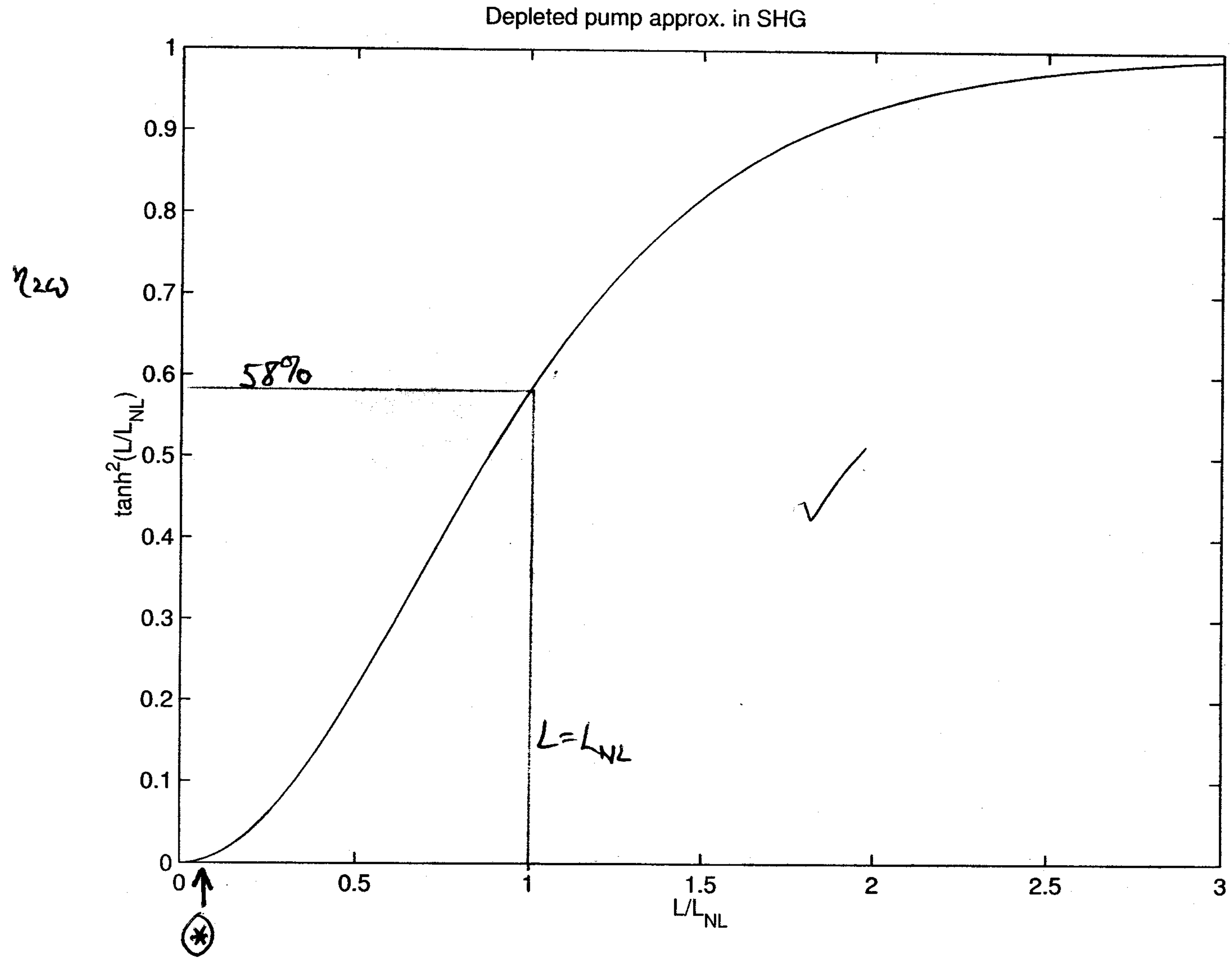
$$L_{NL} = \frac{1}{4\pi d_{eff}}\sqrt{\frac{2\epsilon_0 n^3 c \lambda_\omega^2}{I_\omega(0)}} \qquad \text{take } I_\omega = \frac{P_\omega}{w_0^2}.$$

Solving for P_ω using fsolve in Maple gives $P_\omega = 62.51\,W$, for $P_{2\omega} = 0.1\,W$. This is negligibly larger than in the undepleted case. L_{NL} sets a nonlinear length scale which, with the current parameters, and $P_\omega \sim 63\,W$, is $L_{NL} \sim 12$ ~~cm~~ mm. good.
With $L = 0.5$ mm, $\eta_{2\omega} \sim 0.0017$ which is still very small, so there is essentially no difference with the undepleted approx.

a5

5/5

See figure next page.

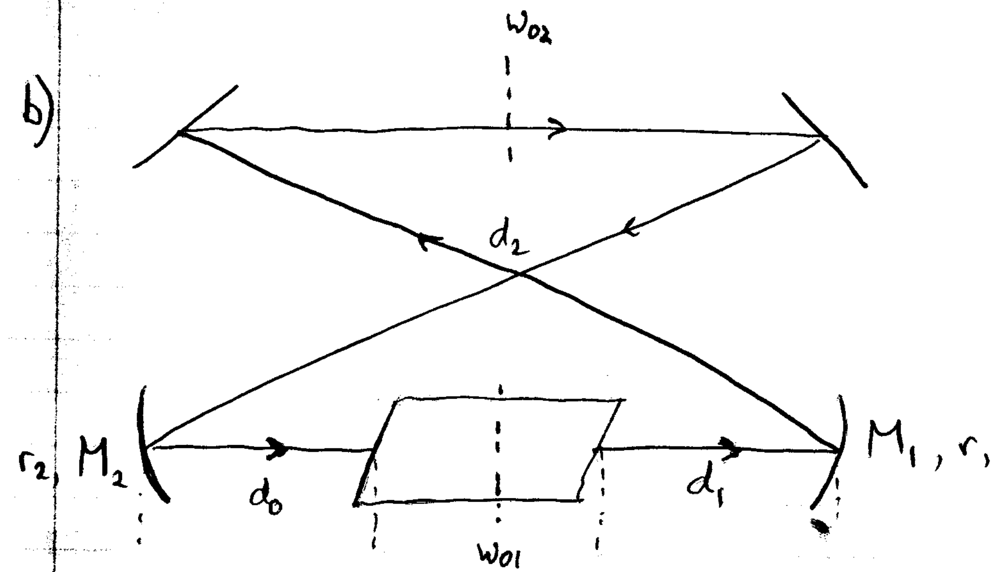


$L_{NL} \sim 12$ mm

and $L = 0.5$ mm, so we are at (*) on the graph, where $\eta_{2\omega}$ is small, and so is pump depletion.

nice.

b)



d_2 - path from M_1 to M_2 via long arm.

Good reference for Gaussian beam optics:

- Kogelnik and Li, Applied Optics, 1966, p. 1550. ✓
- Abitan and Skettrup, J of Optics A 7 (2005) 7-20 ✓
  - bow tie resonators.

To simplify matters, assume $r_1 = r_2$. okay

Use the ABCD matrix method to solve this system. But first, we need to decide on a crystal length. This is far from trivial. We know the aperture length

$$l_a = \frac{\sqrt{\pi}\, w_0}{\rho} \sim 0.6 \text{ mm for a } 20\ \mu\text{m}$$

waist, which, as I will show, is quite reasonable for this resonator design.

However, if we refer to the seminal Boyd and Kleinman, JAP, 1968, p. 3597 paper, "Parametric Interaction of Focused Gaussian Light Beams", there is a quantitative numerical calc. of SHG power fully accounting for diffraction and walk-off. ✓

Boyd & Kleinman (BK) give the following 2 plots:

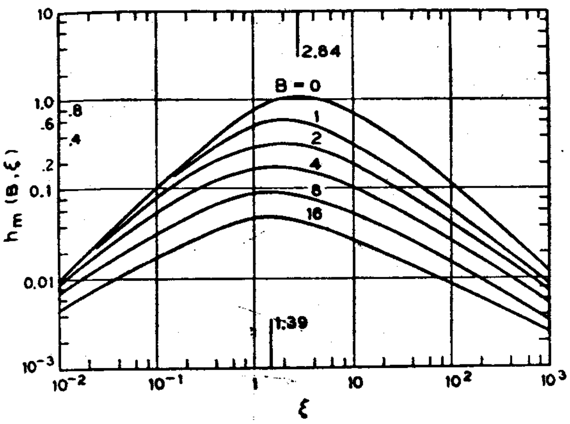


FIG. 2. SHG power (2.22) represented by the function $h_m(B, \xi)$ (2.29) for optimum phase matching as a function of focusing parameter $\xi = l/b$ for several values of double-refraction parameter $B = \rho(lk_1)^{1/2}/2$. Vertical lines indicate optimum focusing in the limits of small and large B.

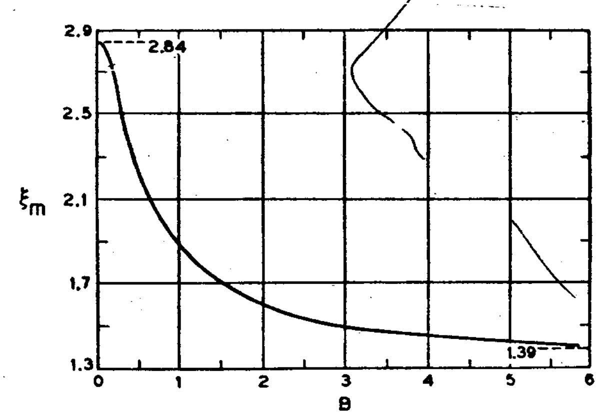


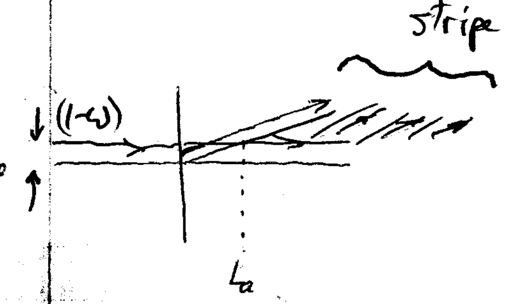
FIG. 4. Optimum focusing parameter $\xi_m(B)$ defined by the maxima of the curves in Fig. 2.

These are very important and relevant, and are referenced by Sutherland and Kozlovsky, independently.

Note that $\xi = \frac{l}{b} \geq 1.39$ for all $B = \frac{\rho\sqrt{lk}}{2}$, which is a walk-off parameter.

For $w_0 = 20\,\mu m$, the confocal parameter $b = 5.3$ mm, much in excess of $L_a = 0.6$ mm.

What is happening here is that although the original SHG beam has completely separated from 1-ω beam after L_a, a continuous stripe of SHG is generated as 1-ω propagates through the crystal. ✓ yes!



Whilst a lot of power is generated beyond L_a, the beam (2ω) becomes highly astigmatic and elliptic.

Assumption : I'm going to optimize for power, and worry about correcting the 2ω beam profile outside the resonator (co-op student ?).

okay

This is a fair assumption. The Kaler paper on the reading list uses an $f = 300$ mm lens and a tilted concave mirror to compensate for beam shape distortion.

I will show that $w_0 \sim 20\ \mu m$ waist for my resonator design; so

insert here

$$B = \frac{0.070}{2}\sqrt{0.005 \times 2\pi / 784 \cdot 10^{-9}} = 9.0$$

ℓ from below

$$b = w_0^2 \cdot k = (20 \cdot 10^{-6})^2 \cdot \left(\frac{1.697 \cdot 2\pi}{784 \times 10^{-9}}\right) = 5.3 \text{ mm}$$

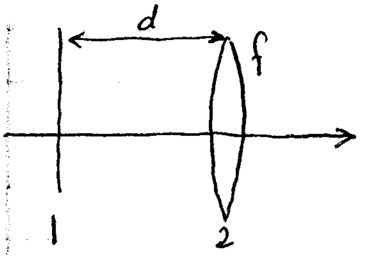
so at the very least $\ell = (1.39)(5.3) = 7.4$ mm. However, from BK's Fig. 2 (on prev. page), we can see that $h(\xi)$ is not very sensitive to ξ near its peak. For the purposes of this design problem, there will be negligible difference in taking $\xi = 1$, or

$$\ell = b \cong 5 \text{ mm}$$ (nice round number for ease of calc ^).

With $B = 9$, $h(\xi) \sim 0.09$, which we will use later in the calculation.

Now, back to the resonator design, with $l = 5\,mm$

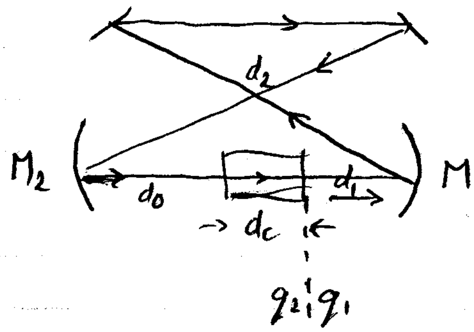
The ray transfer matrix for a spherical mirror is the same as for a lens:



$$M = \begin{pmatrix} 1 & d \\ -\frac{1}{f} & 1-\frac{d}{f} \end{pmatrix} = \begin{pmatrix} 1 & d \\ -\frac{2}{r} & 1-\frac{2d}{r} \end{pmatrix}$$

where $\frac{r}{2} = f$.

We have a matrix for mirrors M_1, M_2, path d_0, and d_c inside the crystal.



We wish to propagate the Gaussian beam from q_1 to q_2 with the matrix

$$M = M_c M_0 M_2 M_1 = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

(multiply from left)

The ABCD law is

$$q_2 = \frac{A q_1 + B}{C q_1 + D}$$

and then we postulate self-consistency, $q_1 = q_2 = q$.

where q is the Gaussian beam parameter, such that

$$\frac{1}{q} = \underset{\text{rad. of curvature}}{\frac{1}{R}} - j\underset{\text{beam width}}{\frac{\lambda}{\pi w^2}}$$

Note once we know ABCD matrix, all is determined.

$$q = \frac{Aq + B}{Cq + D}$$ for a resonant mode.

let $\tilde{q} = \frac{1}{q}$

$$\frac{1}{\tilde{q}} = \left(\frac{A}{\tilde{q}} + B\right) / \left(C/\tilde{q} + D\right)$$

$$\frac{C}{\tilde{q}^2} + \frac{D}{\tilde{q}} = \frac{A}{\tilde{q}} + B$$

or, $$B\tilde{q}^2 + (A-D)\tilde{q} - C = 0$$

which has solutions

$$\frac{1}{q} = \frac{D-A}{2B} \mp \frac{\sqrt{(A-D)^2 + 4BC}}{2B}$$

But in general, $AD - BC = 1$
so $4BC = 4AD - 4$

which gives $$\frac{1}{q} = \frac{D-A}{2B} \mp \frac{i\sqrt{4-(A+D)^2}}{2|B|}$$ ✓

Clearly $R = \dfrac{2B}{D-A}$ (Δ)

and $w^2 = \dfrac{2\lambda|B|}{\pi} \dfrac{1}{\sqrt{4-(A+D)^2}}$ (✱)

which gives the stability criterion

$-1 < \sqrt{4-(A+D)^2} < 1$

We have $M_1 = \begin{pmatrix} 1 & d_1 \\ -\frac{2}{r_1} & 1-\frac{2d_1}{r_1} \end{pmatrix}$ $M_0 = \begin{pmatrix} 1 & d_0 \\ 0 & 1 \end{pmatrix}$

$M_2 = \begin{pmatrix} 1 & d_2 \\ -\frac{2}{r_2} & 1-\frac{2d_2}{r_2} \end{pmatrix}$ $M_c = \begin{pmatrix} 1 & \frac{d_c}{n} \\ 0 & 1 \end{pmatrix}$

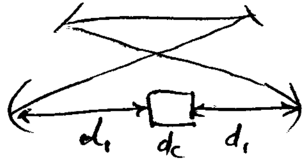
Use Maple to calculate M (big, nasty...), and get w^2 from (✱), then use

→ either $w_0^2 = \dfrac{\lambda^2 z R}{\pi^2 w^2} = \dfrac{\lambda^2 z}{\pi^2 w^2} \left(\underbrace{\dfrac{2B}{D-A}}_{\Delta} \right)$

(and z is the position of the waist)

→ or $w_0^2 = \dfrac{w^2}{1+\left(\dfrac{\pi w^2}{\lambda R}\right)^2}$.

As a starting point, use the parameters from Hemmerich et al., Optics Letters 1990, p.372, which solves a similar problem.



Resonator Design...

In the following Maple output, I calculate w_0 as a function of

$d_2, d_c, d_1\ (= d_0), r_1, r_2, \lambda, n$.

Optimization: I plot w_0 vs. d_1 (½ length of short arm of resonator)

as I vary d_2, (the long arm length).

I construct 3 such families of curves, one for each $r_1 = r_2 = 40, 50, 60$ mm.

Conclusion: Clearly I can design a resonator with w_0 as small as 10 μm, but the stability region in d_1 is less than 1 mm.

b2 15/15

Given the complexity of this problem, and the number of independent parameters, I'm going to err on the side of caution and stability and chose a fairly modest spotsize of 20 μm.

good...

b1

nice justification for crystal length choice

For $r_1 = r_2 = 50$, $23.5 < d_1 < 26.8$ is the region of stability, with $d_2 = 450$ mm.

10/10

So choosing $d_1 = 25.1$ mm, to bisect the stability region, gives $w_0 = 20$ μm. If this proves feasible, one can always lengthen d_2 to reduce the small arm waist to 15 μm or smaller, with no change in mirror curvatures or crystal parameters.

```
> restart;
> M1:=linalg[matrix](2,2,[1, d1, -2/r1, 1-2*d1/r1]);
```

$$M1 := \begin{bmatrix} 1 & d1 \\ -2\frac{1}{r1} & 1-\frac{2\,d1}{r1} \end{bmatrix}$$

M_1

```
> with(linalg):with(plots):
> M2:=linalg[matrix](2,2,[1, d2, -2/r2, 1-2*d2/r2]);
```

$$M2 := \begin{bmatrix} 1 & d2 \\ -2\frac{1}{r2} & 1-\frac{2\,d2}{r2} \end{bmatrix}$$

M_2 ✓

```
> multiply(M2,M1);
```

$$\begin{bmatrix} 1-\frac{2\,d2}{r1} & d1+d2\left(1-\frac{2\,d1}{r1}\right) \\ -2\frac{1}{r2}-\frac{2\left(1-\frac{2\,d2}{r2}\right)}{r1} & -2\frac{d1}{r2}+\left(1-\frac{2\,d2}{r2}\right)\left(1-\frac{2\,d1}{r1}\right) \end{bmatrix}$$

✓ Sanity check – agrees with Kogelnik + Lee

```
> D0:=linalg[matrix](2,2,[1, d1, 0, 1]);
```

$$D0 := \begin{bmatrix} 1 & d1 \\ 0 & 1 \end{bmatrix}$$

M_0

```
> D1:= linalg[matrix](2,2,[1, dc/n, 0, 1]);
```

$$D1 := \begin{bmatrix} 1 & \frac{dc}{n} \\ 0 & 1 \end{bmatrix}$$

M_1

```
> M:=multiply(D1,D0,M2,M1);
```

$$M := \begin{bmatrix} 1-\frac{2\left(d1+\frac{dc}{n}\right)}{r2}-\frac{2\left(d2+\left(d1+\frac{dc}{n}\right)\left(1-\frac{2\,d2}{r2}\right)\right)}{r1} & \left(1-\frac{2\left(d1+\frac{dc}{n}\right)}{r2}\right)d1+\left(d2+\left(d1+\frac{dc}{n}\right)\left(1-\frac{2\,d2}{r2}\right)\right)\left(1-\frac{2\,d1}{r1}\right) \\ -2\frac{1}{r2}-\frac{2\left(1-\frac{2\,d2}{r2}\right)}{r1} & -2\frac{d1}{r2}+\left(1-\frac{2\,d2}{r2}\right)\left(1-\frac{2\,d1}{r1}\right) \end{bmatrix}$$

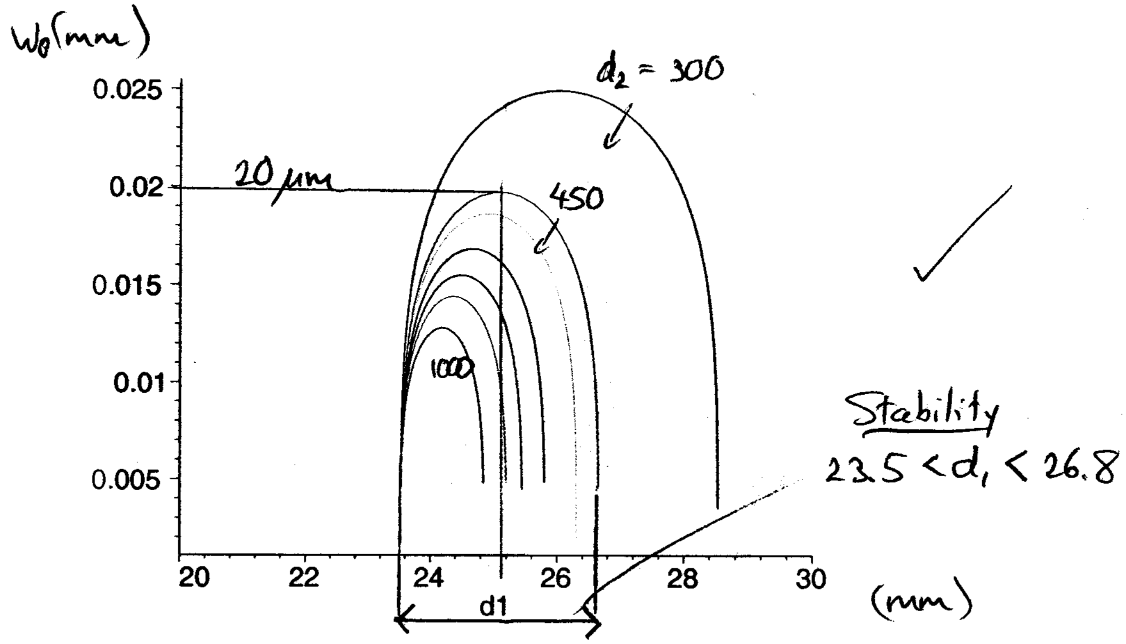
```
> w0:= sqrt((lambda/(2*Pi*M[2,1]))*sqrt(4-(M[1,1]+M[2,2])^2));
```

$$w0 := \frac{1}{2}\sqrt{2}\sqrt{\frac{\lambda\sqrt{4-\left(1-\frac{2\left(d1+\frac{dc}{n}\right)}{r2}-\frac{2\left(d2+\left(d1+\frac{dc}{n}\right)\left(1-\frac{2\,d2}{r2}\right)\right)}{r1}-\frac{2\,d1}{r2}+\left(1-\frac{2\,d2}{r2}\right)\left(1-\frac{2\,d1}{r1}\right)\right)^2}}{\pi\left(-2\frac{1}{r2}-\frac{2\left(1-\frac{2\,d2}{r2}\right)}{r1}\right)}}$$

$\boxed{\text{Waist}}$

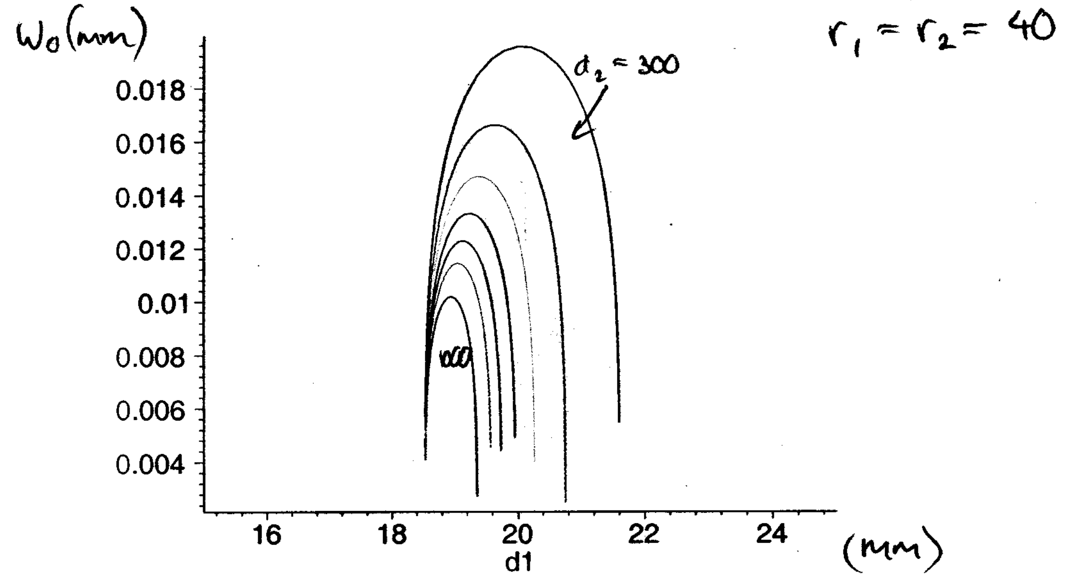
```
> w0a:=subs(dc=5,d2=300,r1=50,r2=50,lambda=784e-6,n=1.6968,evalf(w0)):
> w0b:=subs(dc=5,d2=450,r1=50,r2=50,lambda=784e-6,n=1.6968,evalf(w0)):
> w0c:=subs(dc=5,d2=500,r1=50,r2=50,lambda=784e-6,n=1.6968,evalf(w0)):
> w0d:=subs(dc=5,d2=600,r1=50,r2=50,lambda=784e-6,n=1.6968,evalf(w0)):
> w0e:=subs(dc=5,d2=700,r1=50,r2=50,lambda=784e-6,n=1.6968,evalf(w0)):
> w0f:=subs(dc=5,d2=800,r1=50,r2=50,lambda=784e-6,n=1.6968,evalf(w0)):
> w0g:=subs(dc=5,d2=1000,r1=50,r2=50,lambda=784e-6,n=1.6968,evalf(w0)):
> plot([w0a(d1),w0b(d1),w0c(d1),w0d(d1),w0e(d1),w0f(d1),w0g(d1)],d1=20..30);
```

$r_1 = r_2 = 50$



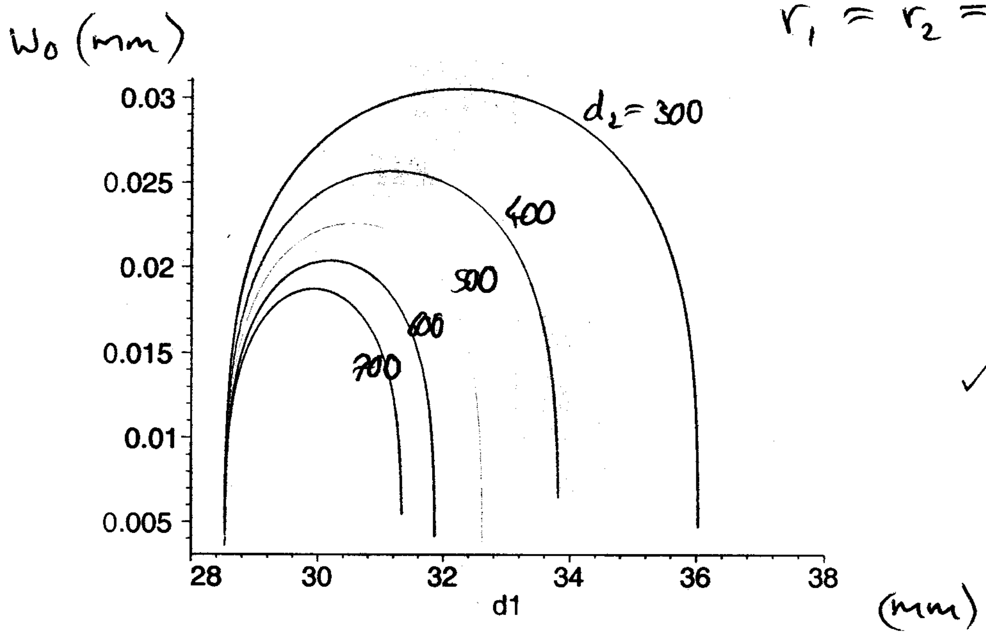

```
> w0a:=subs(dc=5,d2=300,r1=40,r2=40,lambda=784e-6,n=1.6968,evalf(w0)):
  w0b:=subs(dc=5,d2=400,r1=40,r2=40,lambda=784e-6,n=1.6968,evalf(w0)):
  w0c:=subs(dc=5,d2=500,r1=40,r2=40,lambda=784e-6,n=1.6968,evalf(w0)):
  w0d:=subs(dc=5,d2=600,r1=40,r2=40,lambda=784e-6,n=1.6968,evalf(w0)):
  w0e:=subs(dc=5,d2=700,r1=40,r2=40,lambda=784e-6,n=1.6968,evalf(w0)):
  w0f:=subs(dc=5,d2=800,r1=40,r2=40,lambda=784e-6,n=1.6968,evalf(w0)):
  w0g:=subs(dc=5,d2=1000,r1=40,r2=40,lambda=784e-6,n=1.6968,evalf(w0)):
> plot([w0a(d1),w0b(d1),w0c(d1),w0d(d1),w0e(d1),w0f(d1),w0g(d1)],d1=15..25);
```

$r_1 = r_2 = 40$




```
> w0a2:=subs(dc=5,d2=300,r1=60,r2=60,lambda=784e-6,n=1.6968,evalf(w0)):
  w0b2:=subs(dc=5,d2=400,r1=60,r2=60,lambda=784e-6,n=1.6968,evalf(w0)):
  w0c2:=subs(dc=5,d2=500,r1=60,r2=60,lambda=784e-6,n=1.6968,evalf(w0)):
  w0d2:=subs(dc=5,d2=600,r1=60,r2=60,lambda=784e-6,n=1.6968,evalf(w0)):
  w0e2:=subs(dc=5,d2=700,r1=60,r2=60,lambda=784e-6,n=1.6968,evalf(w0)):
>
> plot([w0a2(d1),w0b2(d1),w0c2(d1),w0d2(d1),w0e2(d1)],d1=28..38);
>
```

$r_1 = r_2 = 60$


Wo (mm)
0.03
0.025
0.02
0.015
0.01
0.005
28
30
32
34
36
38
d1
(mm)
d_2 = 300
400
500
600
700

<u>Cavity analysis</u> : Follow the method of Kozlovsky.

Firstly, calculate the nonlinear conversion factor γ_{SH} for a focused Gaussian beam:

$$\gamma_{SH} = \left(\frac{2\omega^2 d_{eff}^2 k_\omega}{\pi n^3 \epsilon_0 c^3}\right) L h(B, \xi)$$

Earlier, I showed $h(B, \xi) \sim 0.09$ for BBO $L = 5mm$

With the other parameters

$$\begin{aligned} \omega &= 2\pi c / 784 \cdot 10^{-9} . \\ d_{eff} &= 1.97 \cdot 10^{-12} m/V \\ k_\omega &= n\omega / c \\ n. &= 1.6608 \end{aligned}$$

$$\Rightarrow \quad \gamma_{SH} = 7.81 \times 10^{-5}$$

Treat the loss due to pump depletion phenomenologically (as a perturbation), so

$t_{SH} = (1 - \gamma_{SH} P_c)$ ✓ is the crystal transmission

I will assume there is no absorption of the fundamental by the crystal, or at least that this is negligible, which is justified by Kaler et al, who find their resonator finesse limited by losses in the mirror coatings.

The roundtrip cavity reflectance

$$r_m = t^2 t_{SH} r_2 ,$$ ✓

[redacted]

where t is the 1-way transmission, and r_2 is the reflectance of the output coupler.

From Ashkin et al. (IEEE J of QE, 1966 p.109),

Cavity/Input : $$\frac{P_c}{P_1} = \frac{t_1}{(1-\sqrt{r_1 r_m})^2 + 4\sqrt{r_1 r_m}\sin^2\phi/2}$$ ✓

Reflected/Input : $$\frac{P_r}{P_1} = \frac{(\sqrt{r_1}-\sqrt{r_m})^2 + 4\sqrt{r_1 r_m}\sin^2\phi/2}{(1-\sqrt{r_1 r_m})^2 + 4\sqrt{r_1 r_m}\sin^2\phi/2}$$

At resonance, $\phi = 0$. Modes should be spaced by $\Delta\left(\frac{\lambda}{2}\right) = 392$ nm in resonator length, so resonance should be easy to achieve in this respect.

Note that when $r_1 = r_m$, the impedance is matched and $P_r = 0$, so all P_1 is coupled into the cavity.

In this case,

since $r_1 + t_1 = 1$

r_1 okay

$$\frac{P_c}{P_1} = \frac{t_1}{(1-r_m)^2} = \frac{1-r_m}{(1-r_m)^2} = \frac{1}{1-r_m}$$ ✓

$$= \frac{1}{1-t^2 t_{SH} r_2} = \frac{1}{1-t^2(1-\gamma_{SH}P_c)r_2}.$$

$$\Rightarrow \boxed{P_c = \frac{-(1-t^2r_2) \pm \sqrt{(1-t^2r_2)^2 + 4t^2r_2\gamma_{SH}P_1}}{2t^2r_2\gamma_{SH}}}$$ ✓

This is a closed form result for the cavity power, P_c.

The enhancement of fundamental power is $\frac{P_c}{P_1}$.

For some realistic parameters for r_1, r_2, t^2, refer to Table I in Kozlovsky.

$$r_1 = 0.983 \qquad r_2 \approx 1.$$
$$t^2 \simeq 0.992$$

For $P_1 = 1\,W$ (which is feasible from a Ti-Sapph)

(b4) $P_c = 73\,W$.

15/15

I can plot $\left(\eta_{SH}^{T} = \gamma_{SH} P_c^2/P_1\right.$ i.e. conversion eff$\left.\right)$ vs. (input power)

total

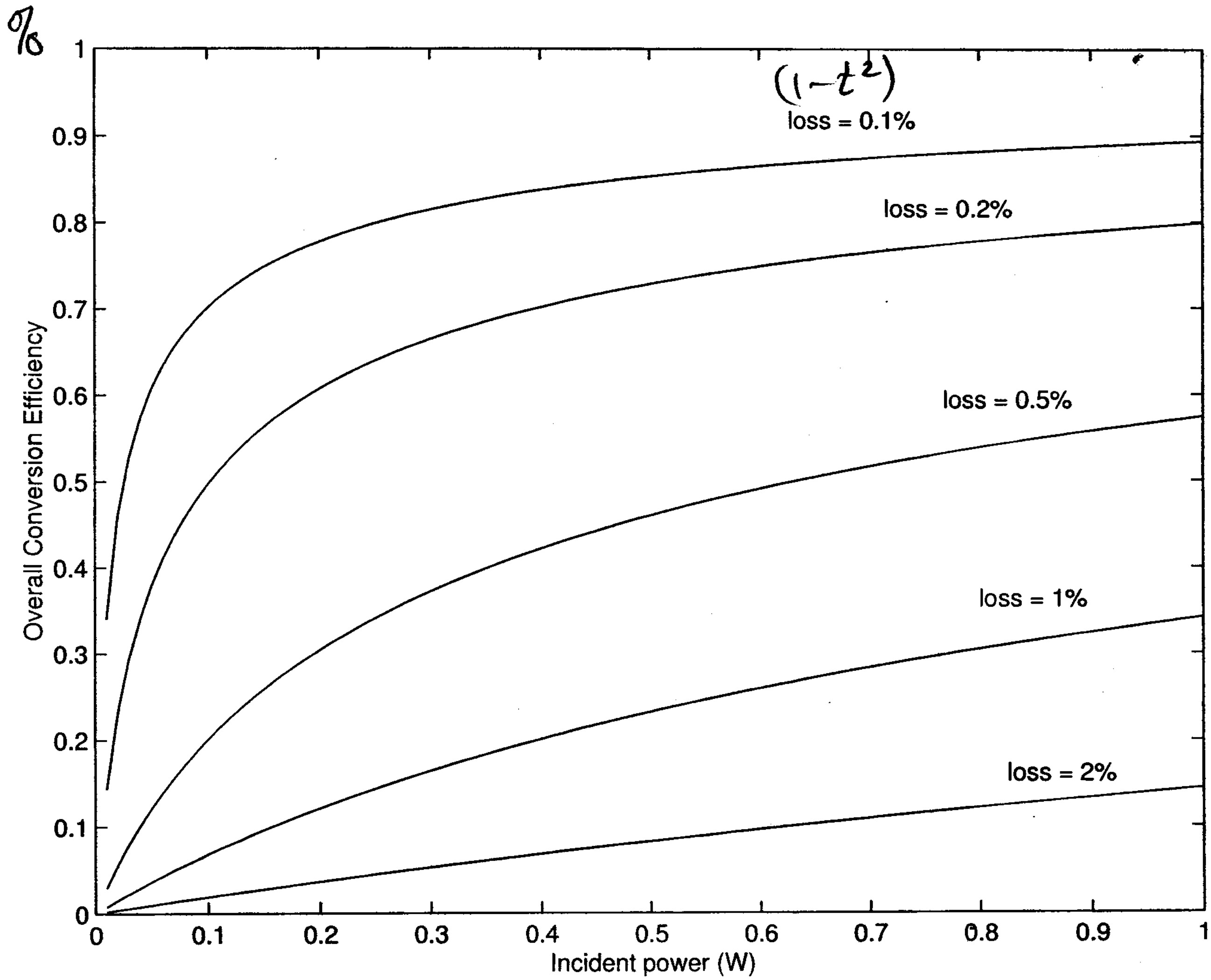
imbedded in this is the Finesse with depletion losses

for a range of cavity losses, $1-t^2$, from which it is very apparent that small losses have a dramatic effect on the overall efficiency.

(b5) 10/10

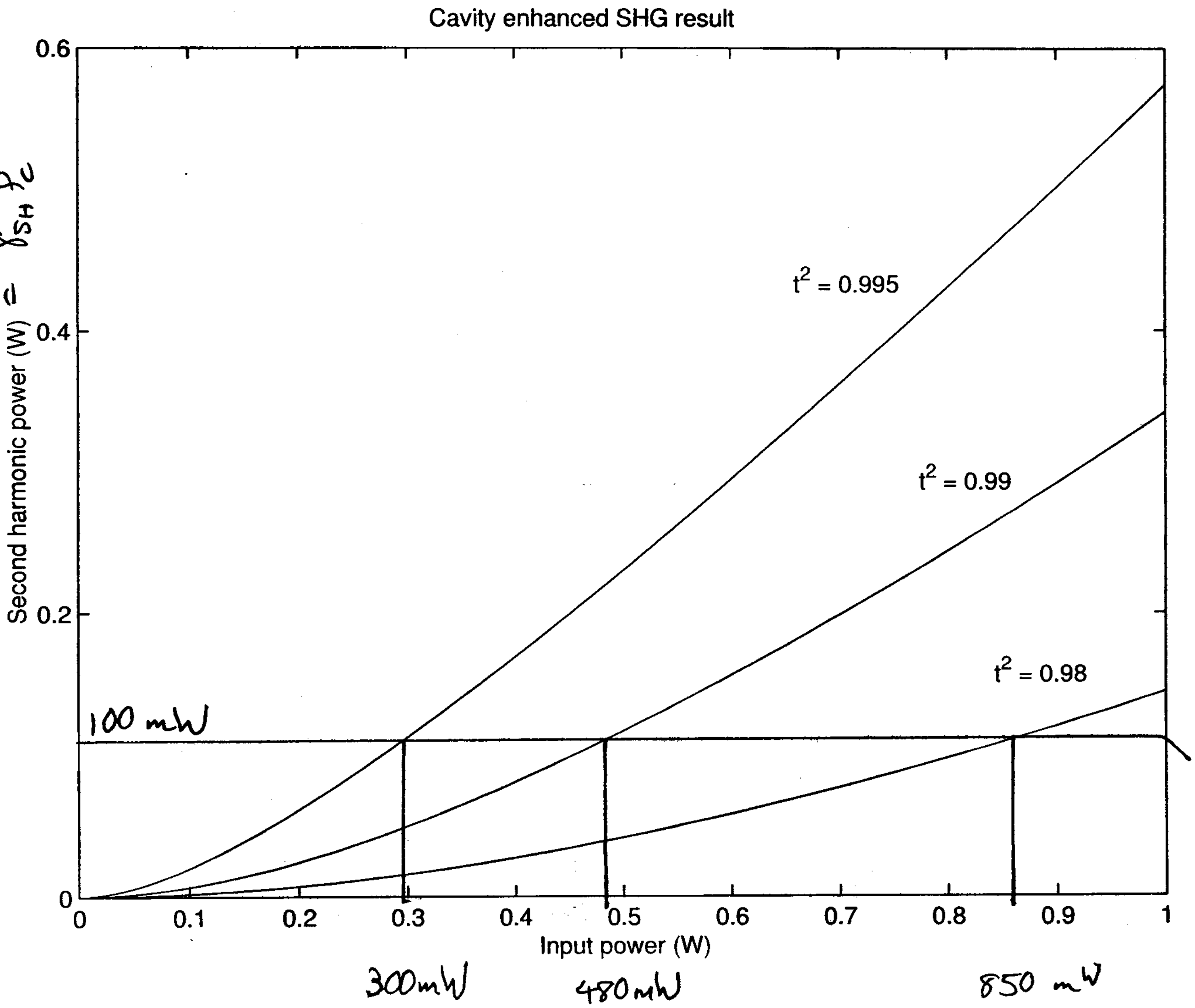
* The second plot is the main result, plotting P_{SH} vs. P_1. With ~~850~~ mW input, the desired 100 mW at 392 nm is generated if $t^2 = 0.98$. If $t^2 = 0.995$ similar to Kozlovsky, only 300 mW is required. But this is unrealistic for a non-monolithic cavity, given reflective losses at the crystal – see next section.

[redacted]



✓ very nice.

[redacted]

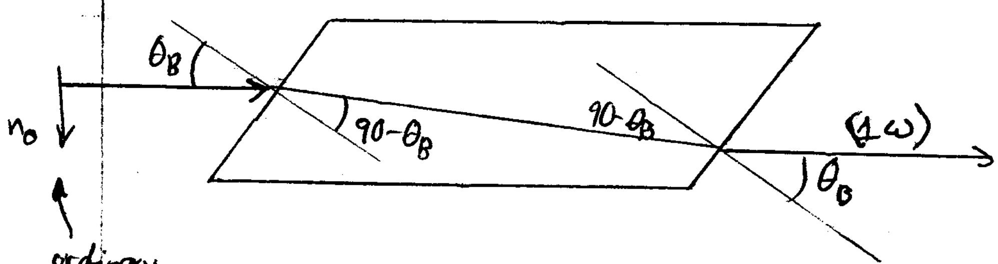


Cavity enhanced SHG result
Second harmonic power (W) = $\gamma_{SH} P_c^2$
Input power (W)
t^2 = 0.995
t^2 = 0.99
t^2 = 0.98
100 mW
300mW
480mW
850 mW
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0.2
0.4
0.6

Some major assumptions have been made along the way.

(1) Considering reflections at crystal interfaces, these will obviously lead to resonator loss, a reduction in Q, and reduced SHG.

To circumvent this, we cut the crystal at Brewster's angle. From the top, the crystal looks like:



ordinary polarization.

This leads to a slight displacement of the beam, but the resonator can be easily compensated for this. ✓

b3

5/5

$$\tan\theta_B = \frac{n_2}{n_1} = \frac{1.6608}{1}$$

$\theta_B = 59^\circ$ for BBO at 784 nm.

This ~~cut~~ Brewster angle of the crystal complicates the way ~~position determination of~~ the x'tal is cut for θ and ϕ for phasematching, and would need to be accounted for when ordering the x'tal.

yes!

(2) That the fundamental can be effectively mode-matched to couple with high efficiency into the resonator at the large waist in d_2. ✓ good...

Kaler et al. couple 75% incident light into the cavity. Using this value, I still get a fundamental power enhancement of

$\sim 0.75 (73\ W) \sim 55\times$.

Note that $\frac{55 W}{\pi(20 \cdot 10^{-4})^2} = 4\ MW/cm^2$, and the

damage threshold $> 1\ GW/cm^2$ (10 ns pulse), so we are well below the damage threshold! ✓

(3) Assuming roundtrip transmission $t^2 = \begin{pmatrix} 0.98 - \\ 0.995 \end{pmatrix}$,

but Kozbusky et al. achieved 0.9958, so this should be feasible, especially with some high quality Newport super-reflector mirrors.

$t^2 = 0.98$ represents a substantial 2% loss per roundtrip, and yet even then it is feasible to attain the desired 100 mW

8/10

Completeness

nice derivations + explanation of results

nice references provided...

almost everything here but sometimes a little hard to find immediately

also, your drawings could have been a little better and a bit more organization helpful.