Convective and Radiative Cooling

Introduction:

There are three main mechanisms of heat transfer. These are: (1) Conduction, (2) Convection, and (3) Radiation. The theoretical basis and analysis of conduction and radiation are quite well established, but understanding convection is an extremely complex problem.

1) **Conduction**: The basic equation for conduction relates the flux of heat flow to the temperature gradient with a proportionality constant, which is called the thermal conductivity of the material. The numerical values of the conductivity vary over many orders of magnitude from one material to another. For conduction, the rate of heat flow is given by

$$\frac{dQ}{dt} = -kA\frac{dT}{dx}$$

where *k* is the thermal conductivity of the material and *A* is the cross-sectional area.

2) **Radiation**: The basic law governing radiative heat transfer is the Stefan-Boltzmann equation

$$\frac{dQ}{dt} = -Ae\sigma T^4$$

where T is the temperature of the object in Kelvin, *e* is the emissivity of the material and σ is the Stefan-Boltzmann constant. Here *A* is the surface area of the sample. The absorptivity of a material is equal to its emissivity, so materials with high absorption (low reflectance) have high emissivity and cool quickly by radiation. When calculating radiative heat transfer you also need to include the fact that a body absorbs radiation from the surroundings

$$\frac{dQ}{dt} = Ae\sigma T_0^4$$

where T_0 is the temperature of the surroundings. The sum of these two terms gives the rate of heat transfer.

3) **Convection**: A simple equation for convection is Newton's Law of Cooling, which relates the heat flow to a temperature difference. Unfortunately, the proportionality

$$\frac{dQ}{dt} = -hA(T - T_o)$$

'constant' h

is not a simple number, but is also dependent on the temperature difference, usually with some fractional exponent. The details depend on all of the complexity of fluid flow around a warm body and engineers use a mix of empirical results and sophisticated computer simulations to try to provide useful models of convection. A commonly accepted form for a cylinder is

$$h = 1.32 \frac{J}{m^{7/4} s K^{5/4}} \left(\frac{T - T_0}{D} \right)^{1/4}$$

where D is the diameter. Again A is the surface area of the sample.

These equations for heat flow can be related to the rate at which a body cools by using the heat capacity.

$$\frac{dQ}{dt} = MC\frac{dT}{dt} = \rho VC\frac{dT}{dt}$$

where M = mass, C = heat capacity, ρ = density and V = volume of the object.

Prelab Questions:

1) Set up a differential equation for heat loss by convection. Assuming there is no other heat loss by the cylinders, solve this equation to obtain an analytic solution. It may be necessary to neglect the ends of the cylinders. Use this solution to make a rough estimate of the time it takes for one of the aluminum rods in the lab to cool from 90 C to 30 C.
2) Make a similar estimate of the radiative cooling of a rod, assuming a maximum emissivity of 1.

Procedure:

You will investigate the cooling of 3 aluminum rods, one with a rough surface, one with a polished surface and one with a lacquered surface. The different surfaces have different emissivities. The rods can be heated in hot water, then placed in their stands with thermometers inserted into the top. Measure the temperature of the three aluminum cylinders as a function of time as they are cooling. You should take data at intervals of two minutes at the start of the experiment and increase the interval to five minutes later and then ten towards the end of your data collection.

Analysis

You should analyze data in terms of the mechanisms of heat transfer discussed in the Introduction.

1) Plot a graph of your data and compare them to the solution you obtained for the convective heat loss.

2) Set up a differential equation for heat loss by convection and radiation and simulate a solution by numerical integration for the temperature of the cylinder as a function of time. Try a simulation for emissivity = 1 as an extreme example and then see if you can find values that match your experimental data.