1 Introduction

The purpose of this experiment is to measure the ratio of two fundamental physical constants e/k_B using a simple transistor circuit. The quantity e is the magnitude of the electron charge and k_B is Boltzmann's constant which relates the microscopic energy scale with temperature. Using the known value of the electron charge ($e = 1.602 \times 10^{-19}$ C) Boltzmann's constant can be extracted from your measurements. You will also verify the Boltzmann distribution law $I \propto \exp(-eV/k_BT)$ at several temperatures T. Here I denotes current and V is voltage.

This experiment will require that you measure currents spanning five orders of magnitude, from tens of nano-amps to milli-amps. These measurements will be made using a simple 741 op-amp circuit, a device with which you became familiar in the first term of this course.

This experiment will also introduce you to vacuum systems and the cryogenic fluid liquid nitrogen which boils at 77 K. Using liquid N_2 will allow you to perform your measurements at any temperature between 77 K and room temperature. Using a water bath and a hot plate you could extend your temperature range up to 100°C. The temperature will be measured using a simple diode circuit.

2 Pre-lab Questions

Before attempting these pre-lab questions first read the entire lab manual, you may find some useful information that will be helpful when answering some of these questions.

1. Rearrange Eq. 2 given in the text below and show how Boltzmann's constant can be obtained from the slope of a straight line. Justify any approximations you make.

2. As mentioned above, you will use an op-amp circuit to measure current. Show that for the circuit pictured in Fig. 1 the current is given by $i_{in} = -V_{out}/R_f$.

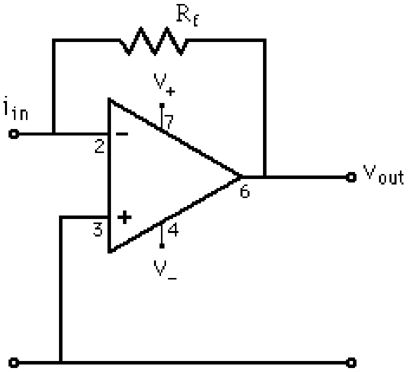

Figure 1: Op-amp circuit: current to voltage converter.

3. In this experiment, liquid N_2 is used to reach a base temperature of 77 K. How could lower base temperatures be reached? Can you think of any technical problems we might encounter if we tried to cool the probe used in this experiment to very low temperatures, say between 1-10 K?

3 Semiconductor devices - Diodes and Transistors

The information presented in this section is based on a useful article entitled *Measurement of Boltzmann's constant* by D.E. Evans [1] which you are encouraged to read. You can pick up a copy of this article at the bench in the lab.

Diodes are devices made by "joining" a p-type semiconductor with an n-type semiconductor with an abrupt junction. An n-type semiconductor is made by doping a material (*eg.* silicon) with *donor* atoms that have one extra valence electron. Thus for each donor atom there will be one loosely bound electron introduced. Conversely, p-type semiconductors are doped with *acceptor* atoms that have one too few valence electrons and thus each acceptor atom introduces one *hole* that can propagate freely through the crystal. Abrupt junctions can be made by switching from donor atoms to acceptor atoms during the semiconductor crystal growth.

With no externally applied voltage across the diode the net current flowing through the diode is zero. Near the diode junction a *depletion region* with a *contact potential* $V_\circ$ is created because electrons (holes) from the n-type (p-type) region diffuse across the junction and combine with holes (electrons) in the p-type (n-type) region (see Fig. 2). Once the depletion region is created, electrons in the n-type region can diffuse into the p-type region only if they have enough energy to overcome the contact potential energy $eV_\circ$. The number of electrons with sufficient energy is determined by the *Boltzmann distribution*, which says that at temperature T, the probability of an electron having energy $eV_\circ$ is given by: $\exp(-eV_\circ/k_BT)$. Thus, a *forward* electron current I from the n- to the p-type material is created and is given by:

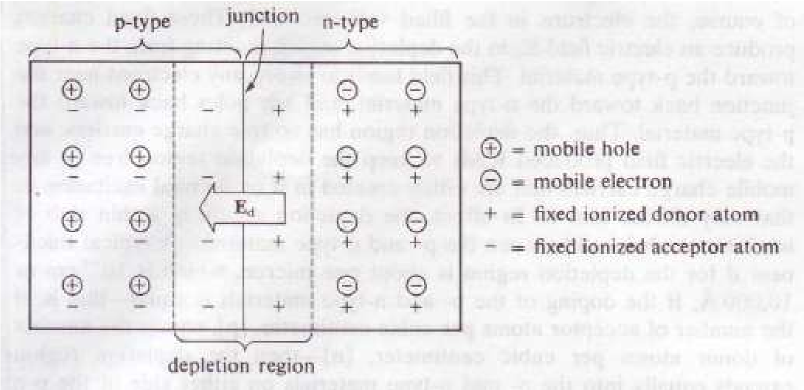

Figure 2: Depletion region of a diode. Figure taken from reference [2].

$$I = A\exp(-eV_\circ/k_BT), \tag{1}$$

where A is a constant of proportionality. There is also a *reverse* electron current from the p- to n-type region. Electrons are created when bonds in the p-type region are broken due to thermal energy. If these created electrons make it into the depletion region they are swept into the n-type material causing current to flow. In equilibrium the two currents are equal and opposite to each other [1].

A diode is said to be forward biased if a voltage V is applied across the diode such that the p-type region is made positive with respect to the n-type region. In this case the electric field produced by the battery $\mathbf{E}_B$ opposes the natural electric field $\mathbf{E}_d$ due to the depletion region. See Fig. 3. The applied voltage alters the height of the potential barrier across the depletion region so that the forward electron current is

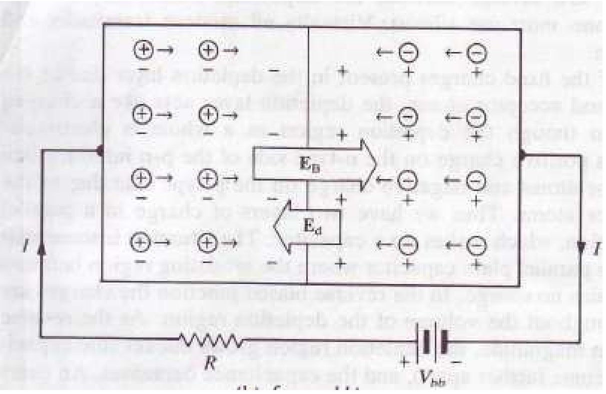

Figure 3: Forward biased diode. Figure taken from reference [2].

proportional to $\exp[-e(V_\circ - V)/k_BT]$. The reverse electron current is unchanged so that the net forward current is given by:

$$\begin{aligned} I &= A\mathrm{e}^{-e(V_\circ - V)/k_BT} - A\mathrm{e}^{-eV_\circ/k_BT} \\ &= A\mathrm{e}^{-eV_\circ/k_BT}\left(\mathrm{e}^{eV/k_BT} - 1\right) \\ &= I_\circ\left(\mathrm{e}^{eV/k_BT} - 1\right). \end{aligned} \tag{2}$$

Unfortunately there are additional contributions to the diode current that we have neglected. When these additional currents are accounted for, the above relationship is modified such that:

$$I = I_\circ\left(\mathrm{e}^{eV/mk_BT} - 1\right), \tag{3}$$

where m is a parameter that varies from transistor to transistor and is typically between 1 and 2.5. Because of these additional currents, diodes cannot be used to determine k_B [1].

Fortunately Eq. 2 can be applied to a transistor circuit. An npn transistor is a p-type region sandwiched in between two n-type regions. The regions are termed *emitter*, *base*, and *collector*. The base is the central p-type region and the emitter and collector are the n-doped regions on either end. If we forward bias the base-emitter junction, then a forward current will flow. If the collector is maintained at the same potential as the base, then it is found that the collector current is described very accurately by Eq. 2 and the extra diode currents that cause the modification of Eq. 3 are drained through the base and do not interfere with the collector current.

4 Experimental Setup

The transistor used in this experiment is a TIP31C npn power transistor. Because the emitter is n-type and the base is p-type, the base-emitter voltage $V_{BE} = V_B - V_E$ must be positive to forward bias the base-emitter junction. Because the base is held at 0, V_E must be negative. The circuit used in this experiment is shown in Fig. 4. The transistor is already wired and housed in a low-temperature probe, but you will need to build the op-amp current detection circuit. You will also need to supply a variable negative voltage to the emitter of the transistor. One possible method using a variable resistor is shown in the figure. Typical emitter voltages that you will use are ≈ -0.5 V.

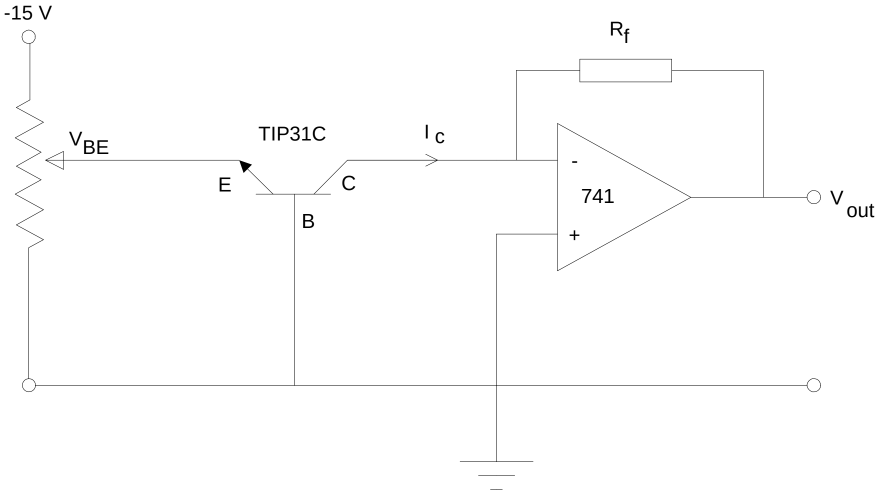

Figure 4: Circuit used to determine Boltzmann's constant.

Temperature Measurement

When supplied with a constant current, a diode can be used as a thermometer. When a diode is forward biased there is a voltage drop V_f across the diode that depends linearly on temperature from room temperature to ≈ 30 K. The constant current source used in this experiment is a UBC built 0-10 mA DC current supply which is powered by ±15V DC.

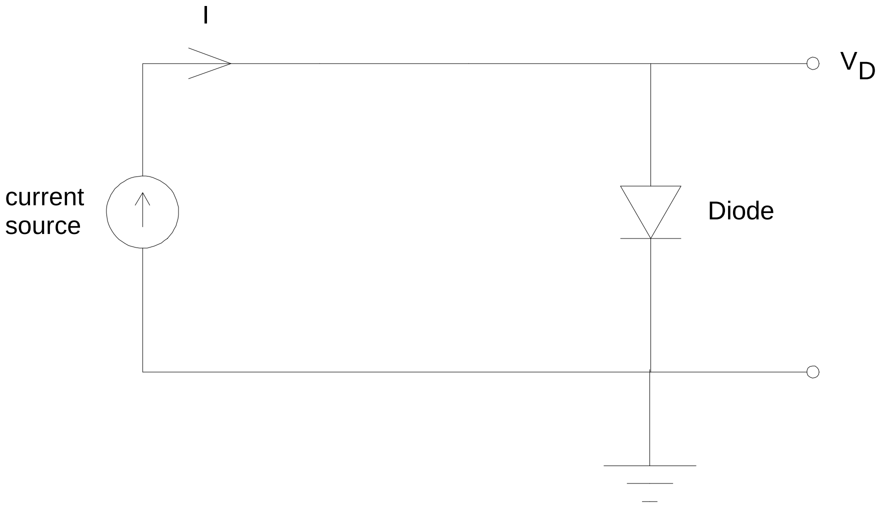

Figure 5: Diode circuit used to measure temperature.

4.1 The Low-Temperature Probe

In order to allow you to perform measurements over a broad range of temperatures, a custom low-temperature probe has been built. A vacuum environment is required to perform measurements at 77 K, otherwise moisture and oxygen from the air would condense onto the electrical components potentially damaging them. A cross-section of the probe is pictured in Fig. 6. A seal is formed between the top

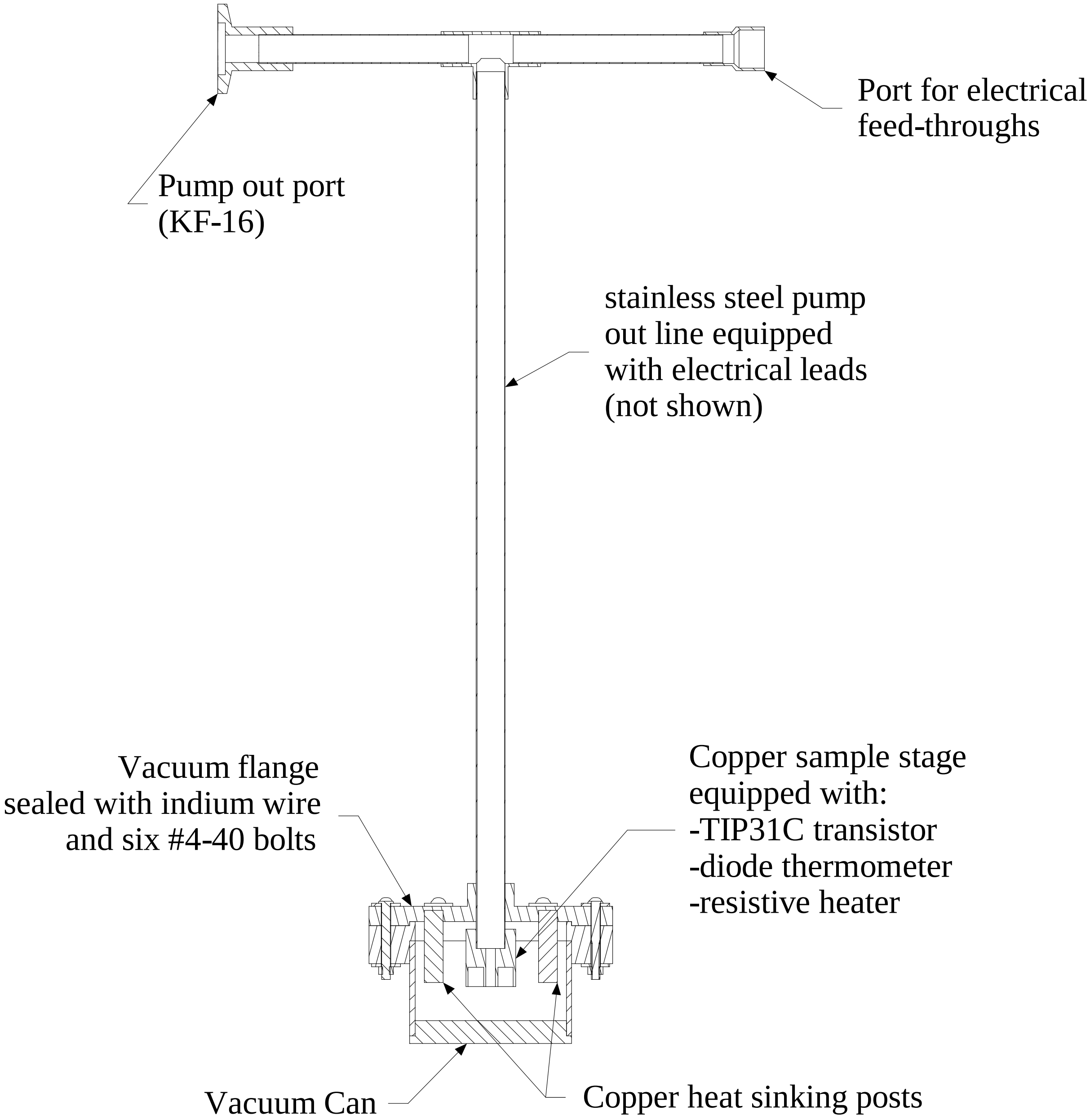

Figure 6: Cross-sectional view of the low-temperature probe. The sample stage is thermally isolated from the base temperature by a short section of stainless steel and can be independently controlled using a heater (not shown). The sample stage is equipped with a TIP31C npn transistor (not shown) and a diode thermometer (also not shown). The probe is also equipped with a low-pressure blow-off as a safety device. If the Kapton seal should happen to leak, the probe could fill with liquid N_2, which would evaporate and build up a high pressure when the probe was warmed. The blow-off allows this gas to escape.

vacuum flange and the vacuum can by clamping down on a Kapton o-ring using a series of bolts. The o-ring should first be given a light coating of vacuum grease.

Notice that the electrical leads coming room temperature have been wrapped and glued to copper posts to the top flange before making contacts to the transistor, diode, or heater. This step was taken to ensure that heat conducted through the wires from room temperature was removed before reaching the cold sample stage. Are there any other sources of heat that one may need to consider?

The time required to cool the sample stage is determined by the ratio of the thermal conductance of the thin stainless steel tube used to isolate the stage from the liquid N_2 bath and the heat capacity of the copper used to build the stage. The sample stage has been isolated from the liquid N_2 bath so that it can be independently controlled using a heater. The heater was made by wrapping 250 Ω of manganin wire around the sample stage and holding it in place with epoxy. By supplying power to the heater, the sample stage temperature can be raised.

5 Procedure

You should first obtain and analyze a set of room temperature data to familiarize yourself with the experiment and analysis before proceeding to get lower temperature data using liquid N_2. For the room temperature data it is not necessary to seal or evacuate the vacuum can. Use a standard mercury or alcohol thermometer to measure the temperature of the room. You should build your circuits on the removable circuit board so that you can save them in the drawer behind the bench for the next session.

1. Begin by constructing the circuit shown in Fig. 4. There is a cable designed to be used with the probe. The ends of the wires have been labeled, "Base," "Emitter," and so on. Ensure that all power supplies are turned off when making electrical connections to the op-amp circuit. Don't forget that the op-amp needs ±15 V supplies!

2. Before using your circuit to collect data, you will need to measure the offset voltage at the output terminal of the op-amp. Start with a R_f=1 MΩ feedback resistor (which will allow you to measure currents from 100's of nano-amps up to $\approx$ 10.5 μA). After carefully ensuring that the circuit is connected correctly, set $V_E = 0$ V and turn on the ±15 V power to the op-amp. Now measure V_{out}. This value is the offset voltage. When making measurements of V_{out} with nonzero V_E you will need to subtract the offset voltage from your measurement. For example, if you have an offset voltage of 0.209 V and then turn on V_E and measure 2.090 V at the output of the op-amp, the true $V_{out} = (2.090 - 0.209)$ V $= 1.881$ V (make sure to account for propagation of errors each time you do this subtraction).

3. Now measure the collector current as a function of V_E. You must measure V_E very accurately so use the best digital voltmeter you can find. Start with small values of V_E and increase the voltage very slowly.

4. Once you have saturated the op-amp output (15 V) turn off all the power supplies and switch to a 1 MΩ feedback resistor. You will have to remeasure the op-amp offset voltage → repeat steps (2) and (3) above.

5. Collect data sets for R_f = 1 MΩ, 100 kΩ, 10 kΩ, and 1 kΩ. Using this set of feedback resistors will allow you to measure currents from tens of nano-amps to 10 mA. Two to three data points for each feedback resistor will be sufficient.

6. Analyze your data and extract a value for k_B.

Once you are satisfied with your room temperature results you may proceed with low temperature measurements.

1. First you will need to set up the temperature measurement circuit of Fig. 5. While still at room temperature, measure the diode voltage with a diode current of $\approx 100\ \mu$A. The exact value of this current is not very important, but it must be the same for all of your measurements.
2. Next seal the vacuum can. The can is sealed with a Kapton o-ring which when compressed will fill the gaps between the vacuum can and the top flange. The Kapton must first be given a fine coating of vacuum grease. After putting the o-ring in place, seal the vacuum can by applying uniform pressure with the bolts.
3. Then pump the can out using the mechanical pump. Let it pump for about 10 minutes to get out as much air as possible, then close the valve to the vacuum can. To shut down the pump properly, you first open the vent valve and then shut it off. This prevents oil from inside the pump from being drawn up to the probe.
4. Using a retort stand, position the vacuum can an inch or two above the bottom of the liquid N_2 dewar. When handling liquid N_2, you must wear gloves and eye protection - direct contact with the liquid N_2 will cause severe frostbite. Now fill the dewar with liquid N_2. Wait 45-60 minutes until the temperature of the sample stage becomes stable (i.e. when the diode voltage is stable). Then record the diode voltage and calibrate your diode thermometer (note that liquid N_2 boils at 77 K).
5. Now collect a set of I_C vs. V_E data as you did at room temperature.
6. If you have time, adjust the temperature of the sample stage to some temperature in between room temperature and 77 K. The temperature is controlled by running a current through the heater. The voltage across the heater should not exceed 25 V, and a voltage of 22 V should bring the temperature to about 200 K. Again, you should expect to wait $\approx 45-60$ minutes before the temperature becomes stable.

The final value of Boltzmann's constant you report should be a weighted average (with an associated weighted error) of all of your measurements at different temperatures. Your report should also include a plot of $\ln I_C$ versus V_{BE} with linear fits. You should plot all of your data sets on a single graph.

References

[1] D. E. Evans. Measurement of Boltzmann's constant. *Phys. Educ.*, 21:296, 1986.

[2] R. E. Simpson. *Introductory Electronics for Scientists and Engineers 2nd Ed.* Prentice Hall, Inc., New Jersey, 1987.