

The Quantum Model of the Hydrogen Atom

In this experiment you will use a spectrometer to determine the wavelengths of the visible lines of atomic hydrogen. The goal of this experiment is to verify Bohr's theory for the hydrogen atom. The apparatus is more complex than you have used so far and the physics comes from quantum mechanics. Nevertheless, you will follow the familiar steps of gathering data, developing a linearized plot, fitting, and interpreting the results of the fit.

Hydrogen and other atoms emit light at particular wavelengths, rather than the continuous spectrum that we see coming from hot objects such as the filament of an incandescent bulb, or from the sun. The wavelength of a photon (a single quantum of light) is related to its energy by

$$E = h\nu = \frac{hc}{\lambda} \quad (1)$$

where Planck's constant h is

$$\begin{aligned} h &= 6.63 \times 10^{-27} \text{ erg-sec} \\ &= 4.14 \times 10^{-15} \text{ eV-sec} \\ &= 6.63 \times 10^{-34} \text{ J-sec} . \end{aligned}$$

Thus, the observation of emission at particular wavelengths means that the light coming from a hydrogen atom is emitted only at particular energies. The reason for this is illustrated in Figure 1. In the quantum mechanical description of the hydrogen atom, the electron is only permitted to occupy a set particular energies in the attractive potential created by the proton.

The light emitted by a hydrogen atom occurs when an electron in one of the excited states drops down to a lower energy level. The energy of the photon that is emitted is the difference between the energies of the atom in its higher (n_1) and lower (n_2) states. That is, the emitted photon energy is

$$h\nu = \frac{hc}{\lambda} = E_{n_1} - E_{n_2} \quad (2)$$

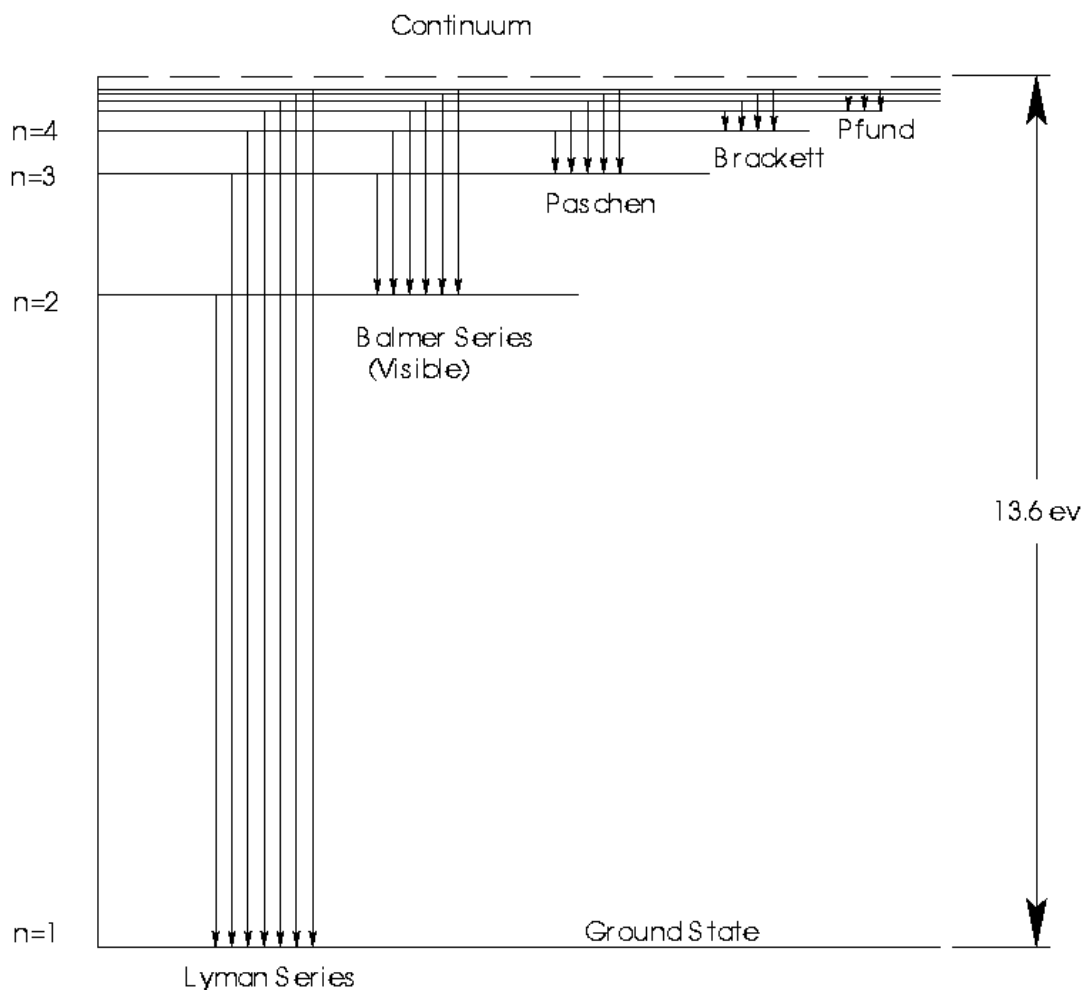


Figure 1: The Hydrogen Energy Level Diagram Showing Spectral Lines

The quantum model for this energy difference in the states of the hydrogen atom yields possible emitted wavelengths.

$$1/\lambda = R_H \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right) \quad (3)$$

The numbers n_1 and n_2 are integers corresponding to different energy levels shown in Fig. 1. Rydberg's constant is $R_H = 1.097 \times 10^7$ per meter. Visible wavelengths are from electrons dropping into the second lowest state $n_1 = 2$.

$$1/\lambda = R_H \left(\frac{1}{2^2} - \frac{1}{n_2^2} \right). \quad (4)$$

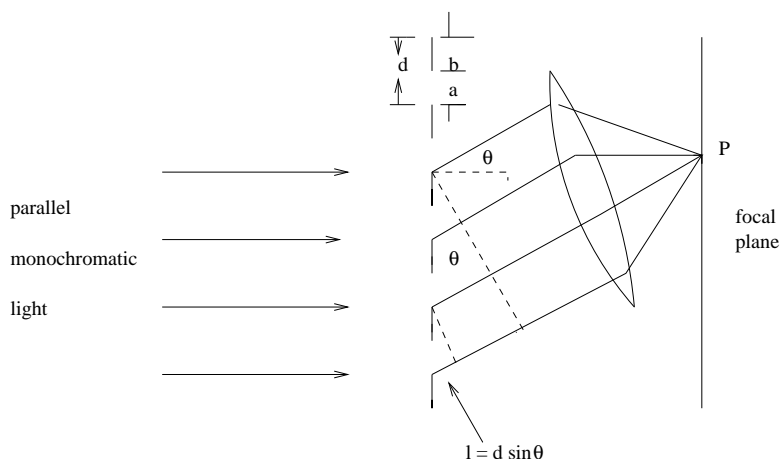


Figure 2: A multiple Slit Grating

1 Theory of the Diffraction Grating (Fraunhofer 1819):

You will be measuring wavelengths using a diffraction grating. This grating works by exploiting wave interference to create an intensity pattern. Diffraction gratings are made by ruling equally spaced parallel grooves into a glass or metal surface.

In Figure 2, a parallel beam of monochromatic light falls at right angles to a series of narrow transparent spaces - “a” - alternating with relatively opaque lines - “b”. The distance $d = a + b$ is called the grating spacing; $1/d$ gives the number of lines per unit length. The grating that you will be using has a grating spacing that is $d = 1755 \text{ nm}$.

Thousands of diffracted light beams are superimposed at the focal plane of a double convex lens. The light beams interfere, and a simple equation, $d \sin \theta = p\lambda$, where p is an integer, determines the angles at which constructive interference (bright slit images) occurs, for example at point P. in Fig. 2. The path length difference between successive paths from the input plane to the output plane is given by $d \sin \theta$. If the path difference is an integral multiple of the wavelength, an interference maximum will be seen in the output plane. The equation $d \sin \theta = p\lambda$ is a concise statement of this condition. With your apparatus you can observe:

$d \sin \theta = 0$ -zero order: undispersed light of all wavelengths
 $d \sin \theta = 1\lambda$ -first order maximum spectra at small angles to
either side of zero order.
and $d \sin \theta = 2\lambda$ -second order maxima.

2 Experiment

2.1 Setting up the Spectrometer

The principles and practice of properly setting up and using the spectrometer are typical of many optical instruments such as the theodolite, used in surveying. Let's start with two general pieces of advice for handling optics. **Keep your eyes relaxed.** If you squint while focusing your telescope you will need to squint to see through it. **Look through the optics** while adjusting it. This way you can see what you are doing.

2.2 Telescope Rotation

Examine the spectrometer. Make sure you understand how the table and telescope rotate - and how the clamp and fine adjustment work. Be sure that the clamp is tight when using the fine adjustment. If you have difficulties, ask your TA for assistance.

2.3 Adjustment of Eyepiece for Relaxed Viewing of the Crosswires

Look through the telescope at a piece of white cardboard placed about two meters in front. Keeping both eyes open, move the eyepiece in and out until the crosswires are sharply in focus through one eye, while the other views the cardboard. Rotate the crosswires to form an X, rather than a +. If your partner's eyes are different than yours you will need to readjust the eyepiece

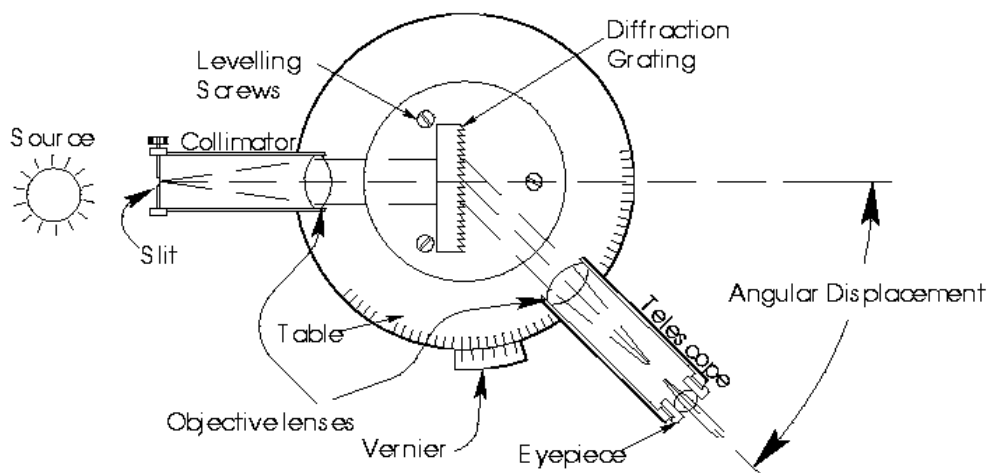


Figure 3: The Diffraction Grating Spectrometer

as you alternate who takes data.

2.4 Adjustment of the Telescope Objective

WARNING: The Diffraction gratings are made of soft material and are easily scratched. Handle the grating by the surrounding glass mount **only**.

Carefully remove the grating from its holder. Look at some distant object - **as far away** as possible (across the room or out a window) - and adjust the objective lens until the image is focused. Replace the glass mount, with the grating side towards the telescope.

2.5 Setting the Collimator Lens to Give Parallel Light from the Slit Source

Open the slit fairly wide, and illuminate it with the hydrogen lamp. Adjust the collimator lens so the slit is in sharp focus - **DO NOT TOUCH** the telescope setting! Moving your eye left and right, while watching to see if the crosswires move relative to the slit, check for parallax (motion for the image of the slit compared to the crosshairs). If you see any movement, repeat steps

2 to 4. Alternatively, you may adjust the telescope lens and the collimator lens iteratively to remove the parallax. That is move the telescope lens a **small amount** and adjust the Collimator to restore focus. Is the parallax better or worse? If it is better keep going. If it is worse go the other way.

Now note which side of the slit is **fixed** and which side moves as you adjust the slit width. You should make your angular measurements to this fixed side. Why? If your central slit image is not vertical, or if, at any angle diffracted images are not centered in the eyepiece, you will need to adjust the spectrometer table position. The table is adjusted using the three leveling screws shown in figure 4.

First, try to get the glass plate of your diffraction grating vertical. Then centre diffracted images on both sides of zero order. Adjusting one side will affect the centering of the other side. Thus, you may need to go back and forth between sides several times to converge to an appropriate table level. Finally, place the black cloth over the instrument, and reduce the slit width

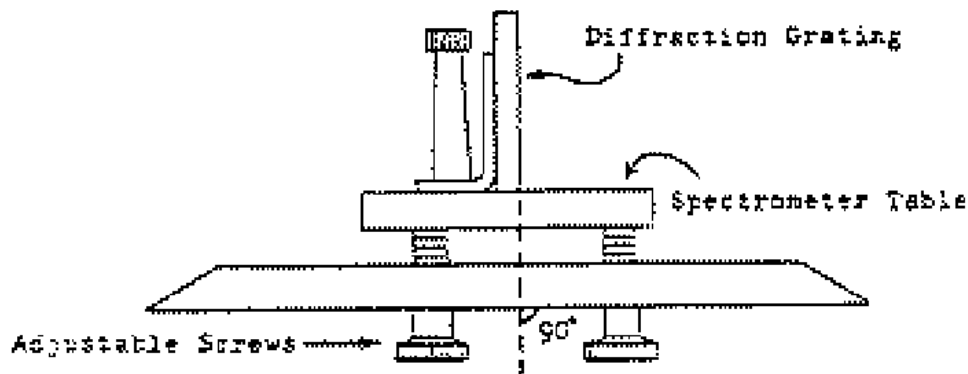


Figure 4: Correct Grating Alignment

so that the image is just bright enough for you to see the crosswires distinctly.

The Vernier Scale pointer is at 182.68 on the main scale.

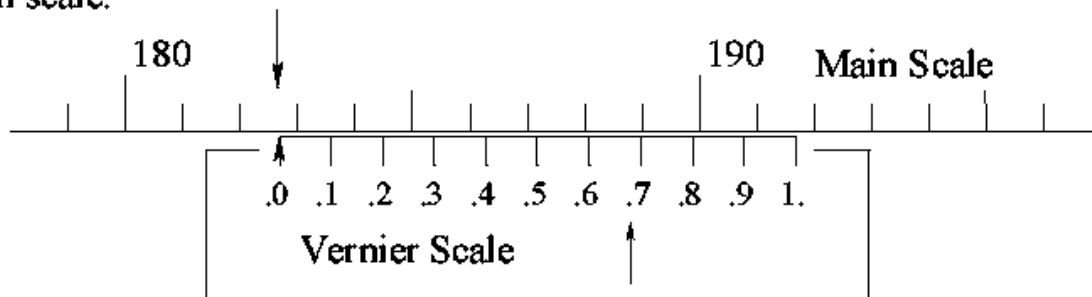


Figure 5: The Vernier Scale Arrangement

2.6 Reading The Spectrometer Vernier

In Figure 5, you can see that vernier scale divisions are spaced slightly closer together than main scale division. In fact, the length of 10 vernier scale divisions is one main scale division shorter than the length of ten main scale divisions.

Rotate the spectrometer telescope, noting that as the pointer moves through one main scale division, each vernier scale mark lines up with a mark on the main scale in succession. As a result, you can read the scale to one tenth of a main scale division. To demonstrate this to yourself:

- Find the main scale mark indicated by the zero or pointer.
- Get the precise subdivision of the interval by finding the vernier scaling that coincides with a main scale mark.

What is the vernier of Figure 5 reading? ANS = 182.68 units. (182.7 is an acceptable reading.) Play with the vernier until you can read it consistently. Notice that there is a micrometer which can rotate the telescope by a very small and easily measured amount. You may or may not wish to use this.

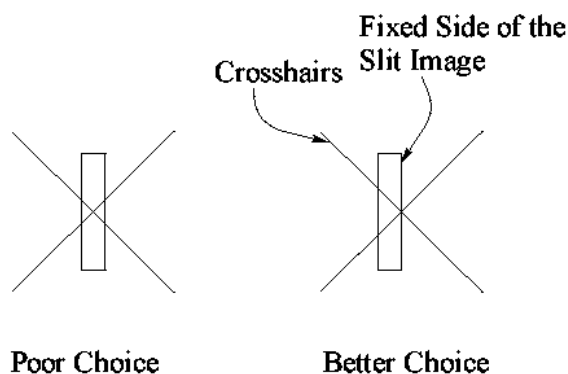


Figure 7: Measuring to the Fixed Side of the Slit

2.7 Checking Alignment and Grating Spacing

Assume that the apparatus is aligned properly. Check the alignment as described below. If you feel adjustment is required call your lab instructor.

Set up the lamp close to the collimator slit. Make sure your diffraction grating is aligned at right angles to the collimator; this will ensure that the angle of incidence is close to zero. You can check the alignment by verifying that the spectral lines appear symmetrically on either side of the zeroth order. Measure the angle between zeroth order and 1st order for the red line and confirm that the angle is the same on both the left and the right. Adjust the grating table if necessary. *Remember to measure first and second order angles to the fixed side of the slit as above.*

2.8 Determination of the Hydrogen Spectrum

In the visible regions there are three strong lines - red, light blue and violet - along with one weaker violet line. Sketch the spectrum, putting color and intensity (scale of 1 to 5) against the lines. (Note: you might see some other lines that are due to impurities; if such lines dominate the spectrum you observe, ask your TA for advice.)

For the first order, measure each line by determining the total angle between the left and right side and then divide by 2. The advantage of this is that you reduce the relative uncertainty by a factor of 2. Calculate the wavelengths. Repeat the measurements for second order and, if possible, third order as well. Estimate the uncertainties as you proceed - for some of the fainter lines the uncertainty might be large, but for the brightest lines they might only be determined by the precision of the vernier scale. **Justify the uncertainty that you assign to a measurement.**

2.8.1 Testing of Bohr's Theory for Hydrogen

Refer to the hydrogen energy level diagram (Fig. 4). The Balmer series actually contains an infinite number of spectral lines, six of which are shown in the diagram, and four of which have the colors described above. You will only see the brightest four lines. Calculate the theoretical wavelength of each of the first six transitions by applying Rydberg's expression. Compare these theoretical values to your experimentally determined wavelengths. Do they agree within your estimated errors? Is Bohr's Theory consistent with your data?

Use your data to produce a linearized plot and fit it to a straight line. What is your best estimate for the value of R_H based on the results of this fit?