Physics 400/506
Problem Set 8
Due Thursday April 6, 2006 by the end of class

1. The Ω^- baryon has only weak decays. However, if the Ω^- were modestly heavier, it could have one strong decay mode, and if it were just a tiny bit heavier still, it could have another. What are these two decay modes? What minimum mass would an Ω^- have to have in order for these modes to be possible? Show explicitly that these two decay modes satisfy all of the conservation laws satisfied in strong force decays.

2. In HW 7 you were asked to calculate the branching ratios of the decays of a W boson. Let's do a similar thing for the decay of a Z^0. The vertex factor for Z^0's coupling to a fermion-antifermion pair is proportional to $c_V^f - c_A^f \gamma^5$.

A We can rewrite this expression as

$$g_L(1-\gamma^5) + g_R(1+\gamma^5)$$

Calculate g_L and g_R in terms of c_V^f and c_A^f.

Just as a vertex factor of the form $(1-\gamma^5)$ couples only to left-handed states, a vertex factor of the form $(1+\gamma^5)$ couples only to right-handed states. So in part A, we've decomposed the coupling into a left-handed part and a right-handed part. The matrix element squared ($|M^2|$) for coupling to a right-handed final state is therefore proportional to g_R^2, while for the left-handed state, it's proportional to g_L^2.

B By summing over the two possible final chiralities (choices of handedness), use the above decomposition show that the total decay rate of the Z^0 to a fermion-antifermion is proportional to $(c_V^f)^2 + (c_A^f)^2$.

C Write down all of the different fermion-antifermion states that a Z^0 can decay into.

D Look at Table 10.1 on page 324 of Griffiths. Before going any further, correct the typo in the second line of the table by turning $\sin\theta_w$ into $\sin^2\theta_w$. Now, using the values in this table and substituting $\sin^2\theta_w = 0.23$, calculate the relative decay rate for each fermion-antifermion pair you listed in part C. Don't forget to count each $q\bar{q}$ pair three times, for the three different colours.

E Finally, use the relative decay rates from part D to calculate the relative branching ratio for each decay mode. Compare your results to what's in the Particle Data Booklet. How well do they compare?

More HW on the back of this page!

3. A neutrino beam with an energy of 3 GeV is created, and sent through a distance L. At what distance L would the oscillation probability reach its first maximum if the oscillation involves the second and third neutrino mass eigenstates? State clearly what value of Δm^2 you used in the calculation—consult the Particle Data Booklet to get the range of plausible values.

4. (Graduate students only): The Proca equation for a *massive* photon is $(\partial^\nu \partial_\nu + m^2) A^\mu = (4\pi) J^\mu$, where $A^\mu = (A^0, \vec{A})$ is the 4-potential and $J^\mu = (\rho, \vec{J})$ is the 4-current. Suppose that the 4-current consists of a single electron at rest at the origin, with $\rho = \delta(\vec{r})$ and $\vec{J} = 0$. Show that, in units with $\hbar = c = 1$, the time-independent solution to this equation is:

$$A^0 = \frac{C}{r} e^{-mr}$$

where C is a normalization constant you don't need to worry about.

Suppose that $m = 10^{-19}$ eV. Insert appropriate factors of $\hbar$ and c into the exponential, then calculate the distance at which A^0 differs from a Coulomb potential by a factor of $1/e$. (Hint: insert $\hbar$ and c factors somehow in the exponent to convert m into units of 1/distance.)