Physics 400/506
Problem Set 7
Due Thursday, March 23, 2005 by the end of class
Don't forget that your paper is due on March 28!

1. Draw the lowest-order (tree-level) Feynman diagrams for the following decays:

A $\tau^- \to K^- \nu_\tau$

B $\Lambda^0 \to n\gamma$

C $\Omega^- \to \Lambda^0 K^-$

D $\pi^+ \to e^+ \nu_e \pi^0$

2. Calculate the ratio of decay rates for $\tau^- \to \pi^- \nu_\tau$ versus $\tau^- \to K^- \nu_\tau$:

$$\frac{\Gamma(\tau^- \to \pi^- \nu_\tau)}{\Gamma(\tau^- \to K^- \nu_\tau)}$$

To work this out, draw the Feynman diagrams for both, assign the proper vertex factors for all vertices, figure out what's different between the two diagrams, then calculate the ratio of the amplitudes, M_1/M_2, between the two decays. You will also need to use Fermi's Golden Rule for decays from Chapter 6 to figure out any difference in the phase space factors for the two decays. Don't worry about details such as differences in the internal structure of the mesons. You shouldn't need to do any matrix manipulations either—just some algebra. Compare your answer to the experimental value.

(More homework on the back of this page!)

3. Pretend for a second that weak interactions with a W boson did *not* mix different quark generations. That is, assume that a W couples a u quark to a d quark, or a c to an s, or a b quark to a t, but, for example, never couples s to u. (In other words, ignore the fact that in real life the W can couple between generations by a small amount.) Assume that the W's couplings to charged leptons and neutrinos are unchanged.

A Look up the mass of a W, and the masses of all the quarks and leptons, using the Particle Data Booklet. Write down all the possible decay products of a W^+ into elementary particles —that is, all the combinations of leptons and/or quarks that the W^+ can decay into. (Hint: there are five such combos.)

Quarks come in three colours. Therefore, if a W decays into a quark and a (possibly different) antiquark, it can do so in three ways. It can make an $r\bar{r}$ state, a $g\bar{g}$ state, or a $b\bar{b}$ state.

B The coupling constant of the W to fermions is the same for all of the final states in part A. Remember that the decay rate is proportional to the square of the coupling constant (because the rate goes as the matrix element squared). What are the branching ratios for a W to decay into each of the five combinations in part A? (Hint: remember that each quark final state can come in three choices of colour, so must get counted three times. Note as well that since the W is so heavy, the kinematic factors are approximately equal for all allowed decay modes, and can be neglected–i.e. all the final state particles can be regarded as effectively massless, since they'll have momenta much larger than their masses.)

C Look up the branching ratios of the W in the Particle Data Booklet. How do your answers from part B compare to those in the particle data booklet for the lepton decay modes, and for the total branching ratio into quarks?

4. Adapt Equation (10.39) of Griffiths to estimate the partial decay width for the decay $\tau^- \to \nu_\tau \mu^- \bar{\nu}_\mu$. Look up the observed branching ratio for this decay, then use that along with the partial decay width to estimate the mean lifetime of a τ. Compare to the observed value.

5. Graduate students: no extra problems this week