Physics 400/506
Problem Set 6
Due Thursday, March 16, 2006 by the end of class

1. This problem asks you to do an "order of magnitude" calculation of proton decay. You don't need to do detailed calculations, but should be able to estimate things using the techniques of Chapter 6.

A Draw the Feynman diagram for the weak decay $\pi^+ \to \mu^+\nu_\mu$ (See Section 2.4 if you need help—you *did* read Chapter 2, didn't you?).

B In a Grand Unified Theory, there exists a supermassive gauge boson called X. X is hypothesized to have a charge of $-\frac{4}{3}$, and can decay as $X \to de^-$ or as $X \to \bar{u}\bar{u}$. By drawing the appropriate Feynman diagram, show that the existence of the X particle permits the proton decay mode $p \to e^+\pi^0$.

C The lifetime of the π^+ is 2×10^{-8} seconds. Assume that the X particle has a mass of 10^{14} GeV, and that its coupling constants to quarks and leptons are of the same magnitude as the couplings of the weak force carriers to quarks and leptons. Do an order of magnitude calculation of the livetime of a proton by scaling the pion's lifetime appropriately using the amplitudes of the matrix elements for the two decays. (Hint: consider the form of the propagator for a massive force carrier, as in Chapter 6. Ignore phase space factors and the spins of the particles, and differences in the internal structure of a pion and a proton.)

2. Griffiths: 7.24

3. Why can't an η particle decay electromagnetically into an e^+e^- final state through a virtual photon?

4. A neutral meson with $J^{PC} = 1^{--}$ can decay electromagnetically by $q\bar{q} \to \gamma \to e^+e^-$. Here are the flavour wavefunctions for four such mesons:

$$\begin{array}{ll} \rho & (u\bar{u} - d\bar{d})/\sqrt{2} \\ \omega & (u\bar{u} + d\bar{d})/\sqrt{2} \\ \phi & s\bar{s} \\ \psi & c\bar{c} \end{array}$$

Neglecting a possible dependence on the mass of the mesons (such a dependence should be negligible, since all are much heavier than an electron), show that the decay widths into e^+e^- of the four mesons are in the ratio

$$\rho : \omega : \phi : \psi = 9 : 1 : 2 : 8$$

(Hint: remember that the coupling between a photon and a quark is proportional to the quark's charge.)

(Graduate students: see back of page for more HW).

5. (Graduate students only) Start with Equations 7.36 of Griffiths, which is a form of the Dirac equation written in terms of u_A and u_B. Show that in the nonrelativistic limit the Dirac equation for an electron (charge $-e$) in an EM field $A^\mu = (A^0, \vec{A})$ reduces to the Schrödinger-Pauli equation:

$$\left(\frac{1}{2m}(\vec{P} + e\vec{A})^2 + \frac{e}{2m}\vec{\sigma} \cdot \vec{B} - eA^0 \right) \psi_A = E_{NR}\psi_A$$

where the magnetic field $\vec{B} = \nabla \times \vec{A}$ and $E_{NR} = E - m$. Assume $|eA^0| \ll m$, and set $c = 1$.

Hint: Make the substitution $P^\mu \to p^\mu + eA^\mu$ into Equations 7.36 written in terms of $\psi_{A,B}$, and use

$$\vec{P} \times \vec{A} + \vec{A} \times \vec{P} = +i\nabla \times \vec{A}$$

where $\vec{P} = -i\nabla$.

Finally, whether or not you succeed in deriving the Schrördinger-Pauli equation above, note that the magnetic moment of the electron is:

$$\vec{\mu} = -\frac{e}{2m}\vec{\sigma} = -g\frac{e}{2m}\vec{S}$$

where $\vec{S}$ is the angular momentum operator (with $\hbar = c = 1$) and $g = 2$. The prediction that $g = 2$ is one of the crowning triumphs of the Dirac equation, since classical theory got this wrong.