Physics 400/506
Problem Set 3
Due Tuesday, February 7, 2006 at the start of class

1. A muon is captured in orbit around an α particle in a state with orbital angular momentum $l = 2$. If the total angular momentum quantum number j for the system is $5/2$, and the z component of the total angular momentum is $+\frac{1}{2}\hbar$, what is the probability of finding the muon with $m_s = +\frac{1}{2}$?

2. (This question will test your angular momentum skills and give you practice finding information in the Particle Data Booklet, or in the online version.) A $K_3^*(1780)$ particle decays into a K^0 and a $\rho^0(770)$ particle. What are the allowed values of orbital angular momentum between the two final state particles?

3. What are the total possible isospins for the following reactions: (a) $K^+ + p \rightarrow \Sigma^+ + \pi^+$; (b) $K^- + p \rightarrow \Sigma^+ + \pi^-$. Find the ratio of the two cross sections, assuming that either one or the other isospin channel dominates.

4. Explain why a $\rho^0(770)$ particle decays strongly into two pions but not three pions. What is the ratio of the decay rate into $\pi^0\pi^0$ to the decay rate into $\pi^+\pi^-$?

5. A Ξ^0 can decay by $\Xi^0 \rightarrow \Sigma^+ e^- \bar{\nu}_e$. Initially the Ξ^0 is at rest with its spin pointed in the $+z$ direction. The decay products are produced with no orbital angular momentum. The $\bar{\nu}_e$ is produced going in the $-z$ direction, and the electron's spin is measured to have $m_s = +\frac{1}{2}$. What is m_s for the Σ^+?

6. *Graduate students only:* The T2K experiment will study neutrino oscillations with a terrestrial beam. A particle accelerator in Tokai, Japan will produce a collimated beam of ν_μ's that will be aimed at the Super-Kamiokande detector 295 km away. Super-Kamiokande will measure the flavour content of the oscillated beam (i.e. what fractions of the beam are ν_μ, ν_e, ν_τ). We denote the oscillation probability by $P(\nu_a \rightarrow \nu_b)$: the probability that a neutrino of type a turns into a neutrino of type b at some later time.

A Show that conservation of CPT requires that $P(\nu_a \rightarrow \nu_a) = P(\bar{\nu}_a \rightarrow \bar{\nu}_a)$.

B Show that if neutrinos respect CP symmetry, then $P(\nu_\mu \rightarrow \nu_e) = P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$.

C We can define the CP asymmetry for any neutrino oscillation by:

$$A_{CP}(\nu_a \rightarrow \nu_b) = \frac{P(\nu_a \rightarrow \nu_b) - P(\bar{\nu}_a \rightarrow \bar{\nu}_b)}{P(\nu_a \rightarrow \nu_b) + P(\bar{\nu}_a \rightarrow \bar{\nu}_b)}$$

If muon neutrinos (antineutrinos) do not oscillate into electron neutrinos (antineutrinos), then prove that $A_{CP}(\nu_\mu \rightarrow \nu_\tau) = 0$. (Assume that CPT is conserved, and that there are only three neutrino flavours.)

7. *Graduate students only:* Griffiths 4.19