A ROAD MAP OF PHYSICS 526

MOTIVATION: special relativity + QM → should be able to create particles from kinetic energy → what quantum systems have states w. different numbers of particles??

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quantum field theories! $\phi(x) \longrightarrow \phi(p)$ (change vars); $\phi(p)$ like harmonic oscillator with $\omega_p = \sqrt{p^2 + m^2}$

→ what kind of field theories are there? → summarize physics using an ACTION: choose some number of fields ϕ_α, functional $S(\phi_\alpha)$ determines what f.t. we're talking about.

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But how should we choose S? → - want locally conserved energy, momentum - want physics same in any reference frame → choose S to be LOCAL: $\int d^4x \, \mathcal{L}(\phi_\alpha(x))$ and invariant under translations, time translations & Lorentz transforms.

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BUT how do the fields transform under these symmetries that we want to impose? → generally $\tilde{\phi}_\alpha(\Lambda x) = M_{\alpha\beta}(\Lambda)\phi_\beta(x)$ but there are different possibilities for M (how the field components get mixed up). Need $M(\Lambda_1)M(\Lambda_2) = M(\Lambda_1\Lambda_2)$ → How do we find the possible M's?

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Ask a mathematician OR reduce the problem to one from undergrad quantum mechanics. ANSWER: can have SCALAR, SPINOR, VECTOR, etc... fields → okay, great. now how do I write invariant actions with these fields → bottom line: just contract up indices! lots of possible actions

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wasn't this supposed to be a physics course? what is the physics of these various field theories? → for theories with the simplest quadratic actions, each type of field gives us a different type of particle (spin 0, spin 1/2, spin 1, etc...). We see this by writing the field & conserved quantities in terms of a,s & a†s. → complete basis of energy eigenstates for these theories: $a^\dagger_{p_1 r_1} a^\dagger_{p_2 r_2} \cdots a^\dagger_{p_N r_N} |0\rangle$ (p: momentum, r: spin state)

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these theories are really boring. how do I study fun things like scattering & particle production? → need to add interactions (non-quadratic terms) to have particles interact w. each other. → okay, I've added interaction terms. now what do I do?

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calculate transition amplitude $\langle 0| a \ldots a \; \mathcal{U}(t_f, t_i) \; a^\dagger \ldots a^\dagger |0\rangle$ (final state; time evolution operator; initial state). Save work by using Wick's theorem or diagrams. Work order by order in $\mathcal{H}_I$.

→ How do these relate to things we can actually measure? → we can derive nice formulas for decay rates $d\Gamma$ and cross sections $d\sigma$ in terms of the transition amplitudes $\mathcal{M}_{fi}$ (which are most easily calculated using Feynmann rules) → Do these actually match with experimental results? → yes. really.