Problem Set 3

Problem 1

a) A field theory is defined by an action

$$S = \int dt \int_{-\infty}^{\infty} dx \{ \frac{1}{2}\dot{\phi}^2(1+\lambda\phi^2) + \alpha\phi^2(\phi')^2 \}$$

Derive the classical equations of motion for the theory.

b) Derive expressions for the energy density, energy current, momentum density and momentum current for this theory.

Problem 2

Consider the field theory defined by an action

$$S = \int dt \int dx \left\{ \frac{1}{2}\dot{\phi}^2 - \frac{1}{2}(\phi')^2 \right\}$$

(we are using units where space and time have the same dimensions). This theory has a scaling symmetry of the form

$$\tilde{\phi} = \Lambda^n \phi(x/\Lambda, t/\Lambda)$$

for some integer n.

a) Determine n, and write the infinitesimal form of this symmetry.

b) Determine the density and current for the conserved quantity associated with this symmetry

Problem 3

The field theory with action

$$S = \int dt \int_0^L dx \left\{ \frac{1}{2}\rho \left(\frac{\partial \phi_y}{\partial t} \right)^2 + \frac{1}{2}\rho \left(\frac{\partial \phi_z}{\partial t} \right)^2 - \frac{1}{2}\tau \left(\frac{\partial \phi_y}{\partial x} \right)^2 - \frac{1}{2}\tau \left(\frac{\partial \phi_z}{\partial x} \right)^2 \right\}$$

has a symmetry under rotations of the vector (ϕ_x, ϕ_y).

a) Write the expression for the conserved charge (i.e. the charge density integrated over space) associated with the symmetry in terms of the modes ϕ_n^y and ϕ_n^z

b) Write the quantum version of this charge (i.e. the corresponding operator) in terms of the creation and annihilation operators associated with ϕ_n^y and ϕ_n^z.

c) For which choices of α and β is the single particle state $\alpha a_{1,y}^\dagger|0\rangle + \beta a_{1,z}^\dagger|0\rangle$ an eigenstate of the charge operator and what is the corresponding eigenvalue in each case?