Problem 1

In Monday's class, we applied quantum mechanics to a simple field theory with one field $\phi(x,t)$ obeying boundary conditions $\phi(x=0,t) = \phi(x=L,t) = 0$. We found that the quantum energy spectrum for this system is the same as for a system of massless particles (photons) confined to the region $0 \le x \le L$. For this problem, we want to show that a simple change in the wave equation allows us to describe particles with mass. Suppose the field theory energy has a term proportional to ϕ^2 without any derivatives,

$$E = \int_0^L dx \{ \frac{1}{2}\rho\dot{\phi}^2 + \frac{1}{2}\tau(\phi')^2 + \frac{1}{2}\mu\phi^2 \} \ .$$

In the string picture, this means it costs energy to displace the string at all, not just to stretch it. For this system, the equations of motion are[1]

$$\rho\ddot{\phi} = \tau\phi'' - \mu\phi \ .$$

- **For this system, write a general formula for the allowed quantum energies relative to the ground state energy.** (Hint: look at the worksheet from Friday's class!)
- **What are then allowed energies for a system whose states include arbitrary numbers of particles of mass M confined in the region $0 \le x \le L$?** (Hint: compared to page 1 of Friday's worksheet, the allowed wavelengths are the same, but the relation between energy and wavelength will be different.
- **If we take $\sqrt{\tau/\rho} = c$, what should we take μ to be if we want the field theory to have exactly the same energies as the system of particles?**

[1] We'll soon see in class how the equations of motion and the energy can be derived from the action for the system, and therefore cannot be specified independently.