Problem Set 0

Problem 1

a) For a system of two harmonic oscillators with frequencies ω and 2ω, what are the three lowest energy eigenvalues for the system and what are the degeneracies for each of these?
b) Write a general formula for the allowed energy eigenvalues.

Problem 2

Using only your knowledge of how creation and annihilation operators act on states of the harmonic oscillator (see harmonic oscillator notes), calculate the expectation value of the x^2 operator in the second excited state $|2\rangle$ of a harmonic oscillator system with mass m and frequency ω. *Hint: can you express the x^2 operator completely in terms of a and $a^\dagger$?*

Problem 3

If we start in a state $|0\rangle$ for the 1D harmonic oscillator and add to the Hamiltonian a perturbation $V(x,t) = \lambda x^4 f(t)$ starting at $t = 0$, how do we calculate the probability that the system will be in the state $|N\rangle$ after time T?

Problem 4

For a two dimensional harmonic oscillator, we can label states by

$$|n_x, n_y\rangle = \frac{1}{\sqrt{n_x! n_y!}} (a_x^\dagger)^{n_x} (a_y^\dagger)^{n_y} |0\rangle$$

What is $\langle 0,1|xy|1,0\rangle$?