So FAR:

quantized fields $\longrightarrow$ particles

non-linear terms $\longrightarrow$ interactions

non-derivative ϕ^2 term $\longrightarrow$ mass

Can we describe all particles with quantum fields?

Higher dimensions $\rightarrow$ similar

$$\phi(x) = \sum \phi_{n,m,l} \sin\left(\frac{n\pi x}{L}\right) \sin\left(\frac{m\pi x}{L}\right) \sin\left(\frac{l\pi x}{L}\right)$$

particles created by $a^\dagger_{\vec{n}}$ with energy

$$E^2 = m^2c^2 + \hbar^2c^2\left(\frac{\pi}{L}\right)^2(n^2+m^2+l^2)$$

Out of the box $\rightarrow$ just take $L \to \infty$

$\lambda^{-1} = \frac{n}{2L}$ becomes continuous

other particle types? Fermions?

What kinds of particles are there?
What kinds of fields can we have?
What is the physics of a general field theory?
What questions should we ask?

Today: Actions, symmetries, + conserved quantities in classical field theory

Action formulation of mechanics: Classical variables $q_\alpha(t)$ evolve in such a way to minimize an action
(extremize)

$q_\alpha(t)$ from t_i to t_f $\xrightarrow{\text{ACTION}}$ Number

possible time evolutions of classical variables

Usually: $$S[q_\alpha(t)] = \int_{t_i}^{t_f} dt \; L(q_\alpha(t), \dot{q}_\alpha(t))$$

e.g: $$S[x(t)] = \int_{t_i}^{t_f} dt \left\{ \tfrac{1}{2} m \dot{x}^2(t) - \tfrac{1}{2} V(x(t)) \right\} \quad \Leftrightarrow \; m\ddot{x} = -V'(x)$$
extemized

* functional extremized if 1st order variation is zero *

e.g. $f(x)$: $f(x_0 + \delta x) = f(x_0) + \tfrac{1}{2} f''(x_0)\, \delta x^2 + \ldots$

linear term in δx vanishes for extremum.

$$S[x_0(t) + \delta x(t)] = \int_{t_i}^{t_f} dt \left\{ \tfrac{1}{2} m (\dot{x}_0 + \delta\dot{x})^2 - V(x_0 + \delta x) \right\}$$

assume $\delta x(t_i) = \delta x(t_f) = 0$

$$= S[x_0(t)] + \underbrace{\int_{t_i}^{t_f} dt \left\{ m \dot{x}_0 \, \delta\dot{x}_0 - V'(x_0)\, \delta x \right\}}_{\delta S} + \ldots$$

Isolate δx via integration by parts: $$\delta S = \int_{t_i}^{t_f} dt \left\{ -m\ddot{x}_0 - V'(x_0) \right\} \delta x(t)$$

δS vanishes for all possible $\delta x(t)$ if and only if

$$-m\ddot{x}_0 - V'(x_0) = 0$$

$$\underset{ma}{m\ddot{x}_0} = \underset{F}{-V'(x_0)}$$

Exercise: E.O.M. for

$$S = \int dt \int_0^L dx \left\{ \frac{1}{2}\left(\frac{\partial \phi}{\partial t}\right)^2 - \frac{1}{2}\beta\left(\frac{\partial^2 \phi}{\partial x^2}\right)^2 \right\}$$

What good is an action:
- simple & elegant
- e.o.m.
- QM
- conserved quanties from symmetries

Symmetries: transformations $\phi_i \longrightarrow \tilde{\phi}_i$

that leave the action unchanged.

$$S[\tilde{\phi}_i(t)] = S[\phi_i(t)]$$

{so ϕ satisfies EOM $\Rightarrow$ $\tilde{\phi}$ satisfies EOM}

maps between physically equivalent (but distinct) configurations

e.g.

$$S = \int dt \int_0^L dx \left\{ \frac{1}{2}\rho\left(\frac{\partial \phi_y}{\partial t}\right)^2 + \frac{1}{2}\rho\left(\frac{\partial \phi_z}{\partial t}\right)^2 - \frac{1}{2}\tau\left(\frac{\partial \phi_y}{\partial x}\right)^2 - \frac{1}{2}\tau\left(\frac{\partial \phi_z}{\partial x}\right)^2 \right\}$$

time transl., rotations

EXERCISE: What are some symmetries?