

So FAR:

quantized fields \rightarrow particles

non-linear terms \rightarrow interactions

non-derivative ϕ^2 term \rightarrow mass

Can we describe all particles with quantum fields?

Higher dimensions \rightarrow similar

$$\phi(x) = \sum \phi_{n,m,l} \sin\left(\frac{n\pi x}{L}\right) \sin\left(\frac{m\pi x}{L}\right) \sin\left(\frac{l\pi x}{L}\right)$$

particles created by a_n^\dagger with energy

$$E^2 = m^2 c^2 + \hbar^2 c^2 \left(\frac{\pi}{L}\right)^2 (n^2 + m^2 + l^2)$$

Out of the box \rightarrow just take $L \rightarrow \infty$

$\lambda^{-1} = \frac{\hbar}{2L}$ becomes continuous

other particle types? Fermions?

What kinds of particles are there?

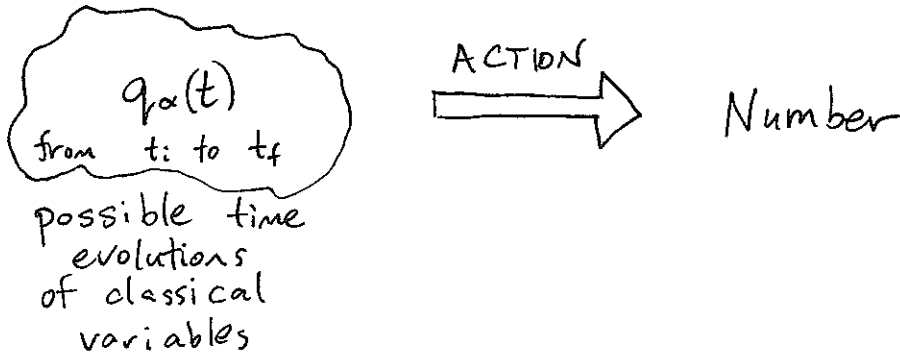
What kinds of fields can we have?

What is the physics of a general field theory?

What questions should we ask?

Today: Actions, symmetries, + conserved quantities in classical field theory

Action formulation of mechanics: Classical variables $q_\alpha(t)$ evolve in such a way to minimize an action (extremize)



Usually: $S[q_\alpha(t)] = \int_{t_i}^{t_f} dt L(q_\alpha(t), \dot{q}_\alpha(t))$

e.g.: $S[x(t)] = \int_{t_i}^{t_f} dt \left\{ \frac{1}{2} m \dot{x}^2(t) - \frac{1}{2} V(x(t)) \right\} \Leftrightarrow m\ddot{x} = -V'(x)$
extremized

* functional extremized if 1st order variation is zero *

e.g. $f(x): f(x_0 + \delta x) = f(x_0) + \frac{1}{2} f''(x_0) \delta x^2 + \dots$



↑ linear term in δx vanishes for extremum.

$S[x_0(t) + \delta x(t)] = \int_{t_i}^{t_f} dt \left\{ \frac{1}{2} m (\dot{x}_0 + \delta \dot{x})^2 - V(x_0 + \delta x) \right\}$
assume $\delta x(t_i) = \delta x(t_f) = 0$
 $= S[x_0(t)] + \underbrace{\int_{t_i}^{t_f} dt \left\{ m \dot{x}_0 \delta \dot{x}_0 - V'(x_0) \delta x \right\}}_{\delta S} + \dots$

Isolate δx via integration by parts: $\delta S = \int_{t_i}^{t_f} dt \left\{ -m \ddot{x}_0 - V'(x_0) \right\} \delta x(t)$

δS vanishes for all possible $\delta x(t)$ if and only if

$$-m\ddot{x}_0 - V'(x_0) = 0$$

$$m\ddot{x}_0 = -V'(x_0)$$

$$\begin{array}{ccc} \uparrow & & \uparrow \\ m a & & F \end{array}$$

Exercise: E.o.m. for

$$S = \int dt \int_0^L dx \left\{ \frac{1}{2} \left(\frac{\partial \phi}{\partial t} \right)^2 - \frac{1}{2} \beta \left(\frac{\partial^2 \phi}{\partial x^2} \right)^2 \right\}$$

What good is an action:

- simple & elegant
- e.o.m.
- QM
- conserved quantities from symmetries

Symmetries: transformations $\phi_i \rightarrow \tilde{\phi}_i$

that leave the action unchanged.

$$S[\tilde{\phi}_i(t)] = S[\phi_i(t)] \quad \left\{ \text{so } \phi \text{ satisfies EoM} \Rightarrow \tilde{\phi} \text{ satisfies EoM} \right\}$$

maps between physically equivalent (but distinct) configurations

e.g.

$$S = \int dt \int_0^L dx \left\{ \frac{1}{2} \rho \left(\frac{\partial \phi_y}{\partial t} \right)^2 + \frac{1}{2} \rho \left(\frac{\partial \phi_z}{\partial t} \right)^2 - \frac{1}{2} \tau \left(\frac{\partial \phi_y}{\partial x} \right)^2 - \frac{1}{2} \tau \left(\frac{\partial \phi_z}{\partial x} \right)^2 \right\}$$

time transl., rotations

EXERCISE: What are some symmetries?