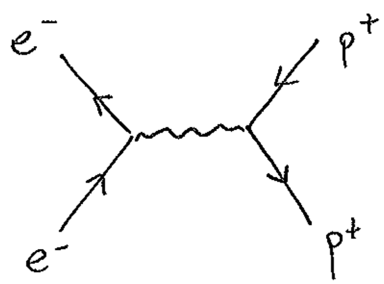
Other processes very similar.

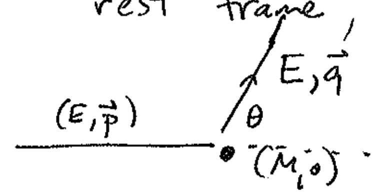
e.g. $e^- p^+ \to e^- p^+$ RUTHERFORD $\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2} \frac{e^4 m^2}{p^4 \sin^4\left(\frac{\theta}{2}\right)}$

treat proton as point particle if $\lambda_{electron} \gg$ size of proton

$E \ll 1\,\text{GeV}$

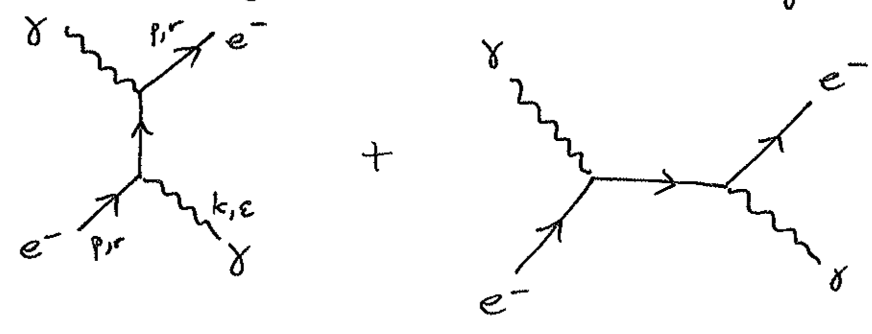


best to work in proton rest frame, approximate $M_p \to \infty$



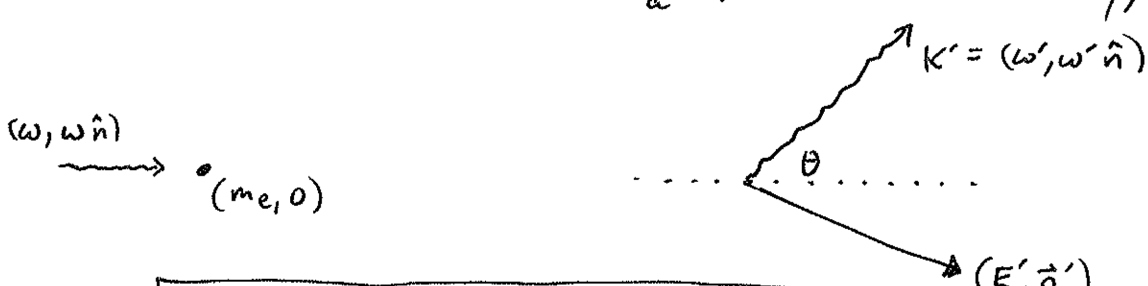
$$\boxed{\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2} \frac{e^4}{p^4 \sin^4\left(\frac{\theta}{2}\right)} \left(m^2 + p^2 \cos^2\left(\frac{\theta}{2}\right)\right)} \quad \text{MOTT CROSS SECTION}$$

External photons: eg. COMPTON SCATTERING $e^-\gamma \to e^-\gamma$



want to sum/average $|M|^2$ over electron spin & photon polarization

use $\sum_a \varepsilon_\mu^a(k)\, \varepsilon_\nu^{*a}(k) = -\eta_{\mu\nu}$



$\omega' = \frac{\omega}{1 + \frac{\omega}{m}(1-\cos\theta)}$

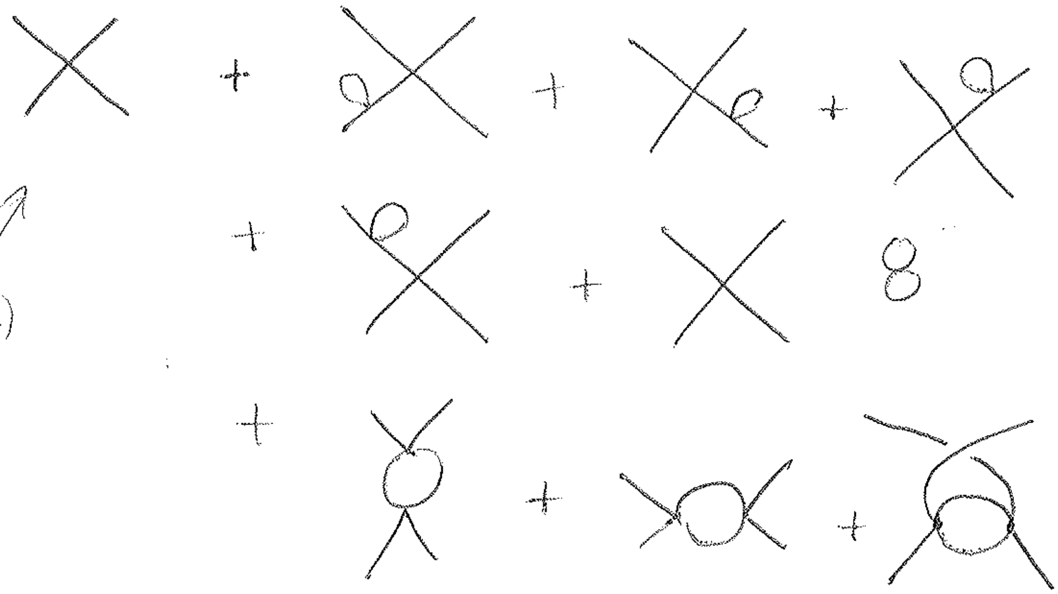
$$\boxed{\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{2m_e^2}\left(\frac{\omega'}{\omega}\right)^2 \left(\frac{\omega'}{\omega} + \frac{\omega}{\omega'} - \sin^2\theta\right)}$$

HIGHER ORDERS:

All amplitudes receive perturbative corrections

e.g. ϕ^4 scattering

$i\mathcal{M} =$



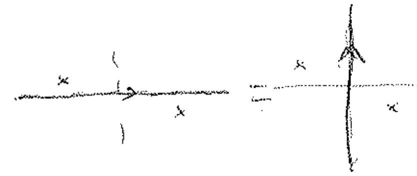
$\mathcal{O}(\lambda)$

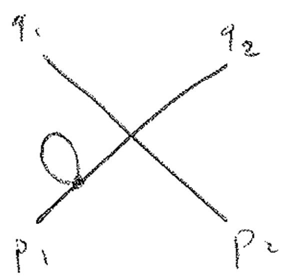
$\mathcal{O}(\lambda^2)$

1st correction to do: cross term $\mathcal{M}_\lambda^* \mathcal{M}_{\lambda^2} + \mathcal{M}_{\lambda^2}^* \mathcal{M}_\lambda$

Higher order terms involve integrals over momenta of internal lines.

These are often DIVERGENT





$$-\frac{\lambda^2}{2}\,\frac{1}{p_1^2-m^2+i\varepsilon}\int\frac{d^4k}{(2\pi)^4}\,\frac{1}{k^2-m^2+i\varepsilon}$$

$$\downarrow$$

$$\int\frac{d^4k_E}{(2\pi)^4}\,\frac{1}{k_E^2+m^2}$$

$$\sim\int_0^\infty dk\,\frac{k^3}{k^2+m^2}=\infty$$

Divergence due to large k part of integral

= UV DIVERGENCE

$\longrightarrow$ comes from non-physical assumption that fields can have arbitrarily short wavelength excitations

$\longrightarrow$ Need to impose CUTOFF

e.g. $$L = \int d^4x \left(\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - m^2 \phi^2 - \frac{\lambda}{4!}\phi^4 \right)$$

take $$\phi(x) = \int^{|k|<\Lambda} \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2E_k}} \left(a_k e^{-ik\cdot x} + a_k^\dagger e^{ik\cdot x} \right)$$

OR $$\int_0^\infty \frac{d^3k}{(2\pi)^3} e^{-k^2/\Lambda^2} \{ \dots \}$$

Answers to physical questions expressed in terms of m, λ, Λ (finite for finite Λ)

BUT: m, λ not physical mass & coupling

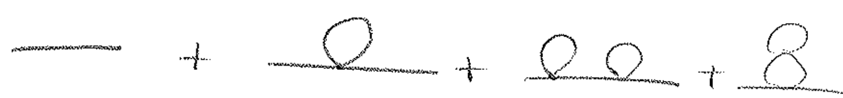
m_{phys} ~ determined by how fast correlation fns fall off

$\updownarrow$

location of pole in propagator

$\langle \phi(x) \phi(0) \rangle \sim e^{-m|x|}$ (large x)

$\langle 0|\phi(p)\phi(-p)|0\rangle$



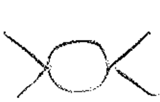
+ ...

$\frac{1}{p^2 - m^2} + \dots$

find p^2 where full expression has pole

$m_{phys} = f(m, \lambda, \Lambda)$

λ_{phys}: can define as $\vec{p} \to 0$ scattering amplitude

$$iM = $$  $$+$$  $$+ \ldots$$

$$= -i\lambda + \ldots$$

$$\equiv -i\lambda_{phys}$$

$$\lambda_{phys} = f_2(m, \lambda, \Lambda)$$

m_{phys}, λ_{phys} "RENORMALIZED" parameters.

~~CENTRAL RESULT IN QFT:~~

~~For any theory S_Λ, physical quantities can be expressed~~

Physical quantities expressed in terms of renormalized parameters have finite $\Lambda \to \infty$ limits indep. of how cutoff was imposed.

(for theories where all interactions get weaker/stay same at high energies = RENORMALIZIBLE field theori

generally: ~~[illegible] S_Λ~~

For ANY field theory w. cutoff Λ, can express low-energy ($E_i \ll \Lambda$) quantities as

$$F(\lambda_i, m_i; E_i) + \mathcal{O}\left(\frac{1}{\Lambda}\right)$$

finite # of parameters = # of interaction terms that don't get weaker at low energies.

Given S_Λ $\exists$ S_Λ^R with only RENORMALIZIBLE terms such that all Fs are the same as for S_Λ.