LAST TIME:

transition amplitudes $\qquad H = H_0 + H_I$

$$T = \langle 0| a_{q_1} \cdots a_{q_m} \underbrace{e^{+iH_0 t} e^{-iH(t-t_0)} e^{-iH_0 t_0}}_{U(t,t_0)} a^\dagger_{p_1} \cdots a^\dagger_{p_n} |0\rangle$$

H_I with $\phi(x) \to \phi(x,t)$

$$U(t,t_0) = e^{-iH_I[t]dt} e^{-iH_I[t-dt]dt} \cdots e^{-iH_I[t_0]dt}$$

$$\equiv T\left\{ e^{-i\int_{t_0}^{t} H_I(t)dt} \right\}$$

time-ordered exponential

$$= 1 - i\int_{t_0}^{t} H_I(t)\,dt - \int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2 \, H_I(t_1) H_I(t_2) + \ldots$$

$$\shortparallel$$

$$-\frac{1}{2}\int_{t_0}^{t} dt_1 \int_{t_0}^{t} dt_2 \; T\{H_I(t_1) H_I(t_2)\}$$

time-ordered: ops at earlier times to the right.

Compute T using PERTURBATION THEORY (for small H_I).

Order n:

$$\langle 0| a \cdots a \, \frac{(-i)^n}{n!} \int dt_1 \cdots \int dt_n \, T\{H_I(t_1) \cdots H_I(t_n)\} \, a^\dagger \cdots a^\dagger |0\rangle$$

~~[illegible]~~ product of $\phi(x,t)$, $\pi(x,t)$

linear comb. of cr. + anih ops.

Need to compute

$$\langle 0 | A_1 \cdots A_n | 0 \rangle \qquad A_i = A_i^+ + A_i^-$$

linear comb. of cr. ops

linear comb. of annih. ops.

(assume bosons for now)

$$\langle 0 | A_1 \cdots A_n | 0 \rangle = \langle 0 | A_1^- A_2 \cdots A_n | 0 \rangle$$

$$= \langle 0 | [A_1^-, A_2] A_3 \cdots A_n | 0 \rangle \rightarrow \langle 0 | A_1 A_2 | 0 \rangle \langle 0 | A_3 \cdots A_n | 0 \rangle$$

$$+ \langle 0 | A_2 [A_1^-, A_3] \cdots A_n | 0 \rangle$$

$$\cdots + \langle 0 | A_2 A_3 \cdots [A_1^-, A_n] | 0 \rangle$$

$$[A_1^-, A_k] = [A_1^-, A_k^+] = \text{some number}$$

$$= \langle 0 | A_1 A_k | 0 \rangle$$

repeat until all As paired up

Q: $\langle 0 | A_1 \cdots A_6 | 0 \rangle = \langle 0 | [A_1^-, A_2] A_3 \cdots A_6 | 0 \rangle$
$+ \cdots$

$= \langle 0 | A_1 A_2 | 0 \rangle \langle 0 | A_3 \cdots A_6 | 0 \rangle$ → what is this term in terms of $\langle 0 | A_i A_j | 0 \rangle$?
$+ \cdots$

A: $\langle 0 | A_3 A_4 A_5 A_6 | 0 \rangle = \langle 0 | [A_3^-, A_4] A_5 A_6 | 0 \rangle$
$+ \langle 0 | A_4 [A_3^-, A_5] A_6 | 0 \rangle$
$+ \langle 0 | A_4 A_5 [A_3^-, A_6] | 0 \rangle$

$= \langle 0 | A_3 A_4 | 0 \rangle \langle 0 | A_5 A_6 | 0 \rangle + \langle 0 | A_4 A_6 | 0 \rangle \langle 0 | A_3 A_5 | 0 \rangle$
$+ \langle 0 | A_3 A_6 | 0 \rangle \langle 0 | A_4 A_5 | 0 \rangle$

WICK'S THEOREM: General correlation fns of fields in state $|0\rangle$ reduce to sum of products of 2pt fns.

$$\langle 0|A_1 A_2 A_3 A_4|0\rangle = \langle 0|\underbracket{A_1 A_2}\,\underbracket{A_3 A_4}|0\rangle$$

$$+ \langle 0|A_1 A_2 A_3 A_4|0\rangle$$

$$+ \langle 0|A_1 A_2 A_3 A_4|0\rangle$$

$$\underbracket{A_1 A_2} \sim \langle 0|A_1 A_2|0\rangle$$

Fermions: get $-$ signs when moving fields past one another

overall: $(-1)^N$ $\quad N = \#$ of crossings of lines connecting two fermions

Field theory: e.g. $H_I(t) = \int dt\, d^3x\, \phi(x,t)\,\phi(x,t)\,\phi(x,t)\,\phi(x,t)$

$$\langle 0| a \ldots a\; T\left\{ \int dt_1 H_I(t_1) \ldots \int dt_n H_I(t_n) \right\} a^\dagger \ldots a^\dagger |0\rangle$$

$$\langle 0| a \ldots a\; T\left\{ \int d^4x_1\, \phi^4(x) \ldots \int d^4x_2\, \phi^4(x_n) \right\} a^\dagger \ldots a^\dagger |0\rangle$$

Q:

$$\langle 0| a_{\vec p}\, \phi(x^\mu) |0\rangle = \frac{1}{\sqrt{2E_p}} e^{ip\cdot x}$$

$$\langle 0| \phi(x^\mu)\, a^\dagger_{\vec p} |0\rangle = \frac{1}{\sqrt{2E_p}} e^{-ip\cdot x}$$

$$\langle 0| T\{ \phi(x_1^\mu)\, \phi(x_2^\mu) \} |0\rangle$$

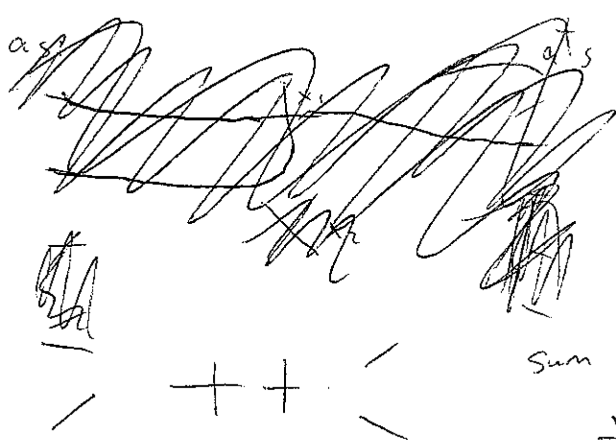
These are the only correlators we need to compute.

$$= \begin{cases} \langle 0| \phi(x_1)\phi(x_2)|0\rangle & x_1^0 > x_2^0 \\ \langle 0| \phi(x_2)\phi(x_1)|0\rangle & x_2^0 > x_1^0 \end{cases}$$

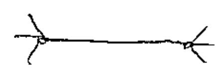
$$= \theta(x_1^0 - x_2^0)\, D(x_1 - x_2) + \theta(x_2^0 - x_1^0)\, D(x_2 - x_1)$$

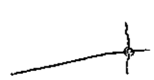
$$\equiv D_F(x-y) \qquad \text{FEYNMAN PROPAGATOR}$$

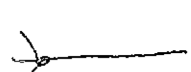
Feynman Diagrams:



sum over ways of joining up lines

 $\to D_F$

 $e^{ip\cdot x}$

 $e^{-ip\cdot x}$

HOMEWORK: relevant interaction terms for scalar, spinor, vector

$$\left.\begin{aligned}
&\phi^3 , \quad \phi^4 \\
&\phi \bar{\psi} \psi , \quad A_\mu \bar{\psi} \gamma^\mu \psi \\
&\partial_\mu \phi A^\mu \phi , \quad A^\mu \phi A_\mu \phi \\
&\partial_\mu A_\nu A^\mu A^\nu , \quad A_\mu A^\mu A_\nu A^\nu
\end{aligned}\right\}$$

all observed particle interactions in nature described by interactions of this type.

$\Rightarrow$ If true action has irrelevant terms, energy scale associated w. couplings must be much higher than accessible energies.