LAST TIME: Spinor fields

$$\psi(x,t) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \left(a^r_{\vec{p}}\, u_r(\vec{p})\, e^{-ip\cdot x} + b^{\dagger r}_{\vec{p}}\, v_r(\vec{p})\, e^{ip\cdot x} \right)$$

$$\langle 0 | \psi_\alpha(x) \psi_\beta(y) | 0 \rangle \quad aa \quad ab^\dagger \quad b^\dagger a \quad b^\dagger b^\dagger$$
$$= 0$$

$$\langle 0 | \psi_\alpha(x) \bar{\psi}_\beta(y) | 0 \rangle = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \int \frac{d^3q}{\sqrt{2E_q}}$$

$$u^r_\alpha(\vec{p})\, e^{-ip\cdot x}\, u^{\dagger \beta}_s(\vec{q})\, e^{-iq\cdot x}$$

$$\langle 0 | a^r_{\vec{p}}\, a^{\dagger s}_{\vec{q}} | 0 \rangle$$

$$= \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_p} e^{-ip\cdot(x-y)} \sum_r u^r_\alpha(\vec{p}) \bar{u}^r_\beta(\vec{p})$$

$$\parallel$$

$$(\gamma^\mu p_\mu + m\mathbb{1})_{\alpha\beta}$$

$$= \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_p} (\gamma^\mu p_\mu + m\mathbb{1})_{\alpha\beta}\, e^{-ip(x-y)}$$

$$= \left(i\gamma^\mu_{\alpha\beta} \partial_\mu + m\,\delta_{\alpha\beta} \right) \int \frac{d^3p}{(2\pi)^3} e^{-ip\cdot(x-y)} \frac{1}{2E_p}$$

$$= \left(i\gamma^\mu \partial_\mu + m \right)_{\alpha\beta} D(x-y)$$

$$\uparrow$$

scalar propagator

generally more interested in correlators of physical observables

e.g. $\langle J^0(x) J^0(y) \rangle \qquad J^\mu = \bar{\psi}\gamma^\mu\psi$

TODAY: The real world.

Q: Write down a field theory for nature. Treat nuclei as elementary, assume periodic table is

H	He.

charged particles interacting w. electromagnetic fields $\longrightarrow$ vector field w. $m=0$ $\quad \int d^4x \{ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \}$

electron $\rightsquigarrow$ charged, spin $\frac{1}{2}$, m_e $\quad i\bar{\psi}_e \gamma^\mu \partial_\mu \psi_e - m_e \bar{\psi}_e \psi_e$

proton $\rightsquigarrow$ charged, spin $\frac{1}{2}$, M_p $\quad i\bar{\psi}_p \gamma^\mu \partial_\mu \psi_p - M_p \bar{\psi}_p \psi_p$

alpha particle $\rightsquigarrow$ charged, spin 0, M_α $\quad \frac{1}{2}\partial_\mu \phi^* \partial^\mu \phi - \frac{1}{2} M_\alpha^2 \phi^* \phi$

interactions: $Q_1 \bar{\psi}_e \gamma^\mu \psi A_\mu \quad Q_2 \bar{\psi}_p \gamma^\mu \psi_p A_\mu \quad \phi^* \partial_\mu \phi A^\mu$

$\bar{\psi} \gamma^{\mu\nu} \psi F_{\mu\nu} \quad \phi^* \phi A_\mu A^\mu$ $\rightarrow$ need to understand more about $m=0$ vector fields

$\bar{\psi}\psi A_\mu A^\mu$?

A^μ $m \neq 0$ 3 states for each $\vec{p}$ (spin 1)

light (photons) only 2 indep states (polarizations)

ALSO $\langle 0 | A^\mu(x) A^\nu(y) | 0 \rangle = \left(-\eta^{\mu\nu} - \frac{1}{m^2} \partial_x^\mu \partial_x^\nu \right) D(x-y)$

singular as $m \to 0$

CRUCIAL POINT: ☆ MANY $A_\mu(x)$ GIVE RISE TO SAME $\vec{E}$ AND $\vec{B}$ ☆

For $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$ $\qquad F^{\mu\nu} = \begin{pmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & -B_z & B_y \\ -E_y & B_z & 0 & B_x \\ -E_z & B_y & -B_x & 0 \end{pmatrix}$

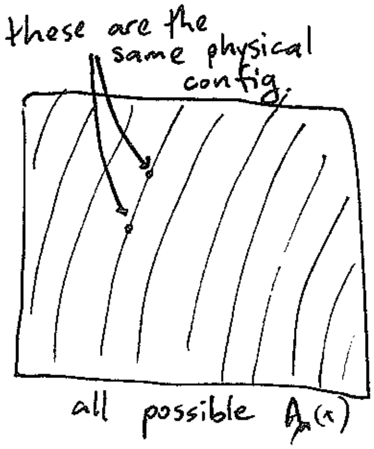
if $\tilde{A}_\mu = A_\mu - \partial_\mu \lambda \qquad \tilde{F}_{\mu\nu} = F_{\mu\nu}$

$\therefore\ S = \int d^4x \left\{ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right\}$ invariant under $A_\mu \to A_\mu - \partial_\mu \lambda$ for any $\lambda(x)$!

This is a GAUGE SYMMETRY or LOCAL SYMMETRY
→ looks like ∞ number of symmetries.

really: NOT A SYMMETRY.

A_μ and $A_\mu - \partial_\mu \lambda$ represent PHYSICALLY IDENTICAL configurations.



How do we quantize?

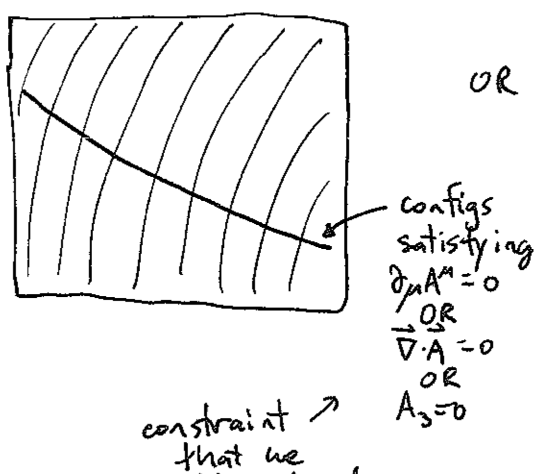
① Take $m \to 0$ limit of massive theory. Compute only observables invariant under $A_\mu \to A_\mu - \partial_\mu \lambda$

OR

② Choose a "gauge" i.e. impose additional restriction that picks out a unique A_μ for each config.

OR

③ Fancy methods Weinberg Ch 15
Tong Ch 6.2.2



→ removes component of A → 2 particles for each momentum.

INTERACTIONS: Full action must be invariant under gauge transforms.

~ should reproduce Maxwell's equs.

$$S = \int d^4x \left\{ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + J^\mu A_\mu \right\} \quad \text{gives} \quad \partial_\mu F^{\mu\nu} = J^\nu$$

Maxwells eqns $\quad J^0$ charge density

$\vec{J}$ current.

Dirac field

$$J^\mu = \bar{\Psi} \gamma^\mu \psi$$

TRY: $$S = \int d^4x \left\{ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i \bar{\Psi} \gamma^\mu \partial_\mu \psi - m \bar{\Psi} \psi + q A_\mu \bar{\Psi} \gamma^\mu \psi \right\}$$

Gauge symmetry $\quad \tilde{A}_\mu = A_\mu - \partial_\mu \lambda(x)$

$\tilde{\psi} = ?$

can we pick this so action is invariant.

YES: $\tilde{\psi} = e^{-iq\lambda(x)} \psi$

CHECK: $\quad \partial_\mu \psi - iqA_\mu \psi$ is invariant $\rightarrow i \bar{\Psi} \gamma^\mu \partial_\mu \psi$

scalar $\quad \partial_\mu \phi - iqA_\mu \phi$

$\equiv D_\mu \phi$ invariant. $\longrightarrow$ $D_\mu \phi D^\mu \phi$ invariant.

$$\int d^4x \left\{ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i \bar{\Psi} \gamma^\mu D_\mu \psi - m \bar{\Psi} \psi \right\}$$

QUANTUM ELECTRODYNAMICS ACTION

more generally: one ψ for each charged particle type (spin $\frac{1}{2}$)

for spin 0 charge particles $-\frac{1}{2} D_\mu \phi^* D^\mu \phi - \frac{1}{2} m^2 \phi^* \phi$