Expectation Values and Uncertainty

In statistical experiments where the results are governed by probabilities, it is often interesting to ask what the average value will be if the measurement is repeated a very large number of times. In quantum mechanics, this is known as the EXPECTATION VALUE and is simply the average of all the possible outcomes weighted by the probability of each outcome. In cases where the possible outcomes are described by some continuous variable x (like positions in space) we denote the expectation value by $\langle x \rangle$

$$\langle x \rangle = \int dx x P(x) \ .$$

More generally, the expectation value of some function of x will be

$$\langle f(x) \rangle = \int dx f(x) P(x) \ .$$

The expectation value tells us only one bit of information about the full probability distribution, basically giving the point for which we're equally likely to find the value to the left or to the right. This gives us no measure of how spread out the probability distribution is. A simple measure of this spreading is known as the standard deviation, which tells us on average how far a given measurement will be from the average. More precisely, we define

$$\Delta x = \langle (x - \langle x \rangle)^2 \rangle^{\frac{1}{2}} \ .$$

It is this quantity that is used as the precise definition of UNCERTAINTY in quantum mechanics, and this is what comes in to the precise version of the Heisenberg Uncertainty Principle

$$\Delta x \Delta p \geq \frac{h}{4\pi} \ .$$

It is an interesting exercise to calculate Δx and Δp for the example in question 3 of problem set 8 to check how close we come to the uncertainty bound.