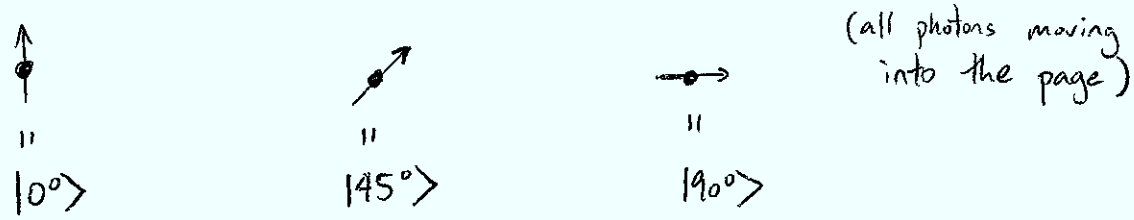
Name:

Physics 200 Tutorial 9:

Photons and Polarizers

This tutorial explores the photon picture of polarizer experiments. This is perhaps the simplest example of a system that displays all the essential features of quantum mechanics.

Let's first recall the mathematical model of photon polarizations that we developed in class. Classically, polarization is the direction that the electric field vector oscillates in. It is always perpendicular to the direction of the wave, so we can represent it by a unit vector which is perpendicular to the direction of motion. So to every photon, we can associate a unit vector that tells us what state the photon is in. We will use the notation $|\theta\rangle$ to describe the unit vector which points in a direction at an angle θ to some arbitrarily chosen axis, as shown in the picture below.

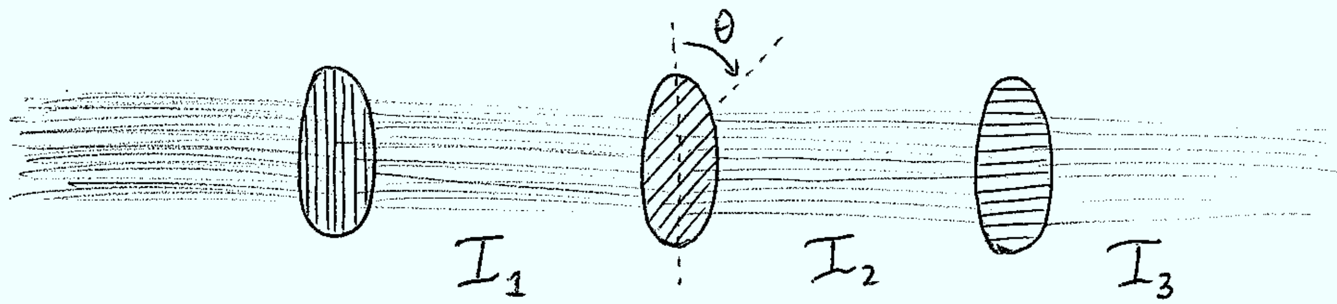


Now, suppose there is a photon that encounters a polarizer oriented in some direction. As we discussed in class, the only way to explain the classical reduction in intensity with the photon picture is to say that each photon has some probability of passing through the polarizer. The best we can do in this quantum mechanical system is to make a prediction for the probability of a given photon passing through. To get the right answer for the probability, we can do the following:

- Write the unit vector representing the initial state as a superposition of unit vectors parallel and perpendicular to the polarizer.
- The squared coefficient of the vector parallel to the polarizer gives the probability that the photon will pass through (and then have its polarization aligned with the polarizer)
- The squared coefficient of the vector perpendicular to the polarizer gives the probability that the photon will be absorbed.

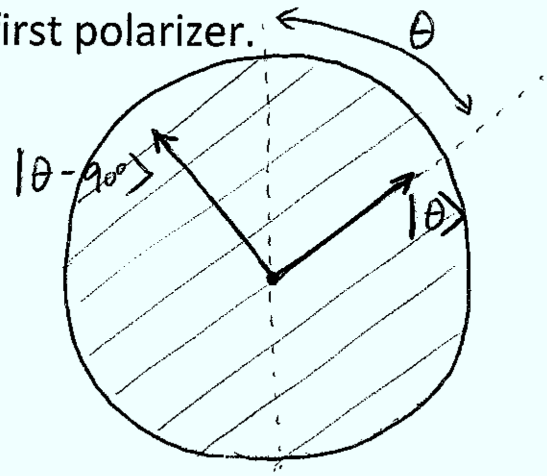
Question 1

One of the most remarkable experiments to do with polarizers is to start with two polarizers at 90° relative to each other, so that no light passes through, and then insert a third polarizer between them. If the third polarizer is not lined up with the first or second, some light will be observed to pass through all three polarizers (this experiment is set up at the front of the room for you to see). In this question, we'll try to predict how much light will make it through using our model for calculating transmission probabilities. The basic setup is shown below:



Our goal will be to predict the intensity I_3 of light passing through the final polarizer in terms of the intensity I_0 of the original light. Let's start by considering the photons that have made it though the first polarizer.

a) The diagram below shows the second polarizer, and the unit vectors $|\theta\rangle$ and $|\theta - 90°\rangle$ representing photons that will definitely pass through and definitely not pass through. On this diagram, draw the unit vector representing a photon that has come from the first polarizer.



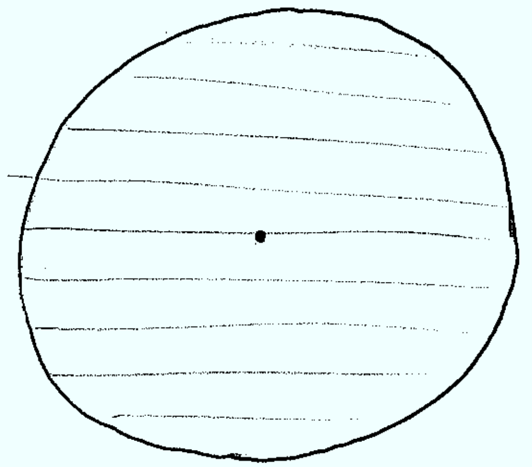
b) Let's call the polarization direction of this photon 0°. In order to calculate the probability that the photon will pass through, we want to write the unit vector for our photon as a superposition of the unit vectors $|\theta\rangle$ and $|\theta - 90°\rangle$ (the eigenstates for this polarizer):

$$|0°\rangle \quad = \quad a\,|\theta\rangle + b\,|\theta - 90°\rangle$$

What are a and b?

c) What is the probability that this photon will pass through the second polarizer?

d) The diagram below shows the third polarizer. On this diagram, draw the unit vectors representing the eigenstates (i.e. the photons that will definitely pass through and definitely not pass through. Also draw the unit vector for a photon that has made it through the second polarizer.

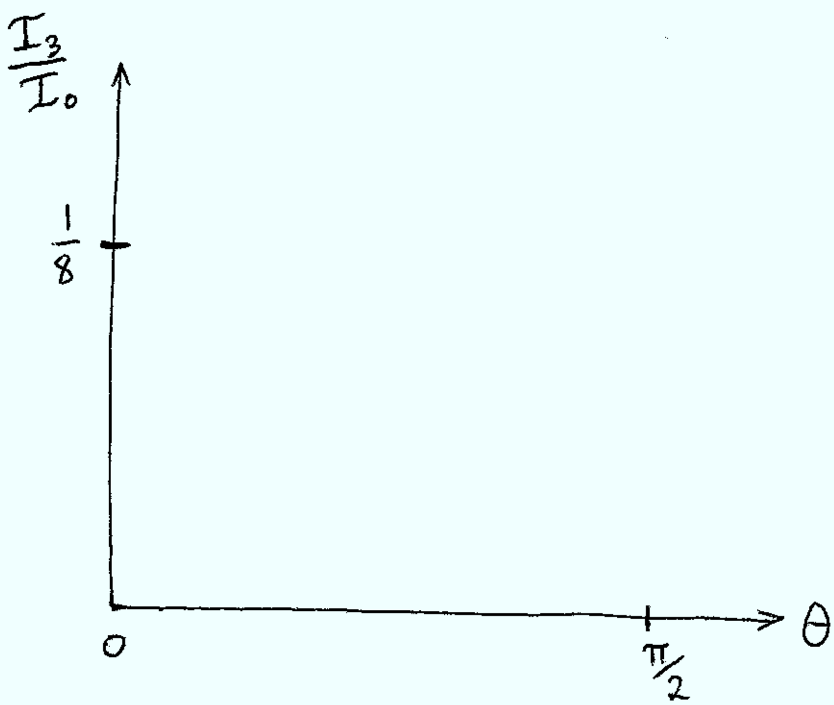


e) Using the same steps as before, calculate the probability that such a photon will pass through the third polarizer.

f) If the initial light has photons with a random assortment of polarizations, we can show that the average probability of passing though the first polarizer is 1/2 (can you prove this?). In parts c and e, you have calculated the probability for a photon that has passed through the first polarizer to pass through the second polarizer, and the probability for a photon that has passed through the second polarizer to pass through the third polarizer. Putting this all together, what is the probability that one of the original photons will pass through all three polarizers. (Hint: the probability is the same as the fraction of photons that will pass though.)

g) In terms of the initial intensity I_0, what is the intensity of I_3 of the light that comes out of the final polarizer (as a function of the angle θ? Sketch your result on the graph below:

(check your answer with a TA)



Quantum mechanics interpretation:

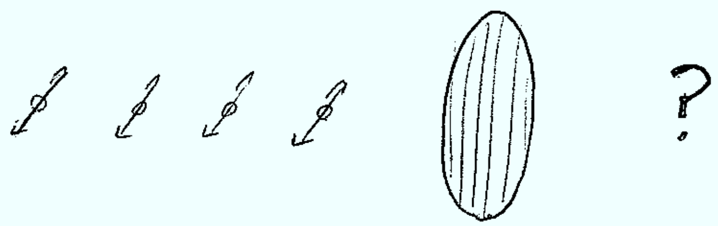
You may have realized that after drawing the pictures, writing the vector as a superposition of other vectors and calculating the square of the coefficient for the eigenstates that will pass through, the answer we get for the probability is always just $\cos^2(\phi)$, where ϕ is the angle between the photon polarization direction and the polarizer direction. We could have saved some time just by using this fact.

The reason that we took the long-winded approach is that this method (writing the original state as a superposition of eigenstates and calculating the probabilities based on the squared coefficients) is how we predict probabilities in all quantum mechanical experiments. Like the unit vectors in our example, quantum states for any system can always be added together to get new quantum states. We interpret the equation in part b as telling us that the photon with polarization 0° is a QUANTUM SUPERPOSITION of a photon with polarization θ and a photon with polarization (θ - 90°). This means that when we do the experiment, the photon will completely change into one of these two special photon states for which the outcome of the experiment is definite.

In the same way, we can have electron states that are quantum superpositions of states where the electron has a definite position. For example, if we can have an electron at position x_1 and an electron at position x_2, we can also have an electron in a state which is a superposition of these, such that a measurement of the position might yield x_1 or x_2 with some definite probability determined by the coefficients in the superposition.

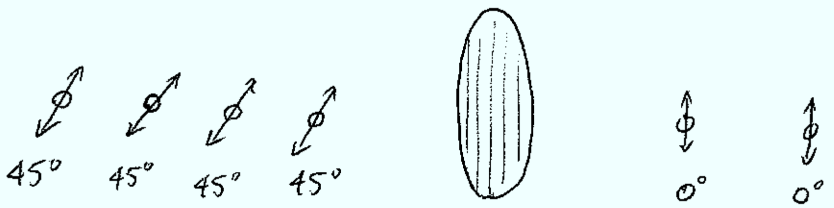
Question 2

Four photons with polarization state $|45°\rangle$ are sent towards a polarizer oriented at 0°. What are the possible outcomes of this experiment, and what is the probability for each of the outcomes?

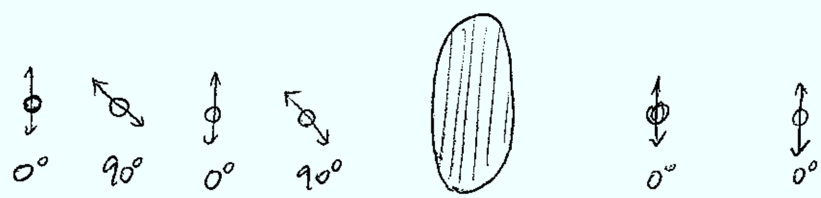


Question 3

In class, we discussed the photon interpretation of the observation that light with a polarization of 45° relative to a polarizer direction passes through with intensity reduced by half. We said that the only explanation was to say that each photon passes through with 50% probability, and that the ones passing through change their state to be polarized in the direction of the polarizer.

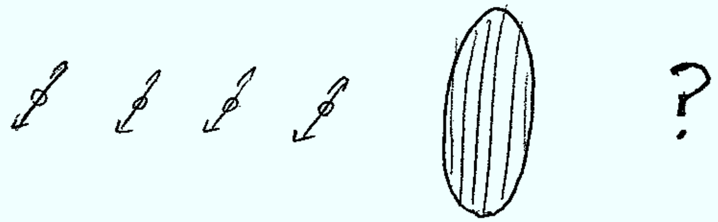


What's wrong with the following alternative model, that 45° polarized light is actually made up of a mixture of 0° and 90° polarized photons, in equal proportions, and the observed reduction in intensity is just due to the fact that only the 0° photons go through?



Question 2

Four photons with polarization state $|45°\rangle$ are sent towards a polarizer oriented at 0°. What are the possible outcomes of this experiment, and what is the probability for each of the outcomes?



None go through: Prob $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \boxed{\frac{1}{16}}$

1 goes through: NNNY, NNYN, NYNN, YNNN each w. prob $\frac{1}{16} \Rightarrow \boxed{P_1 = \frac{1}{4}}$

2 go through: NNYY, NYNY, NYYN, YNNY, YNYN, YYNN each w. prob $\frac{1}{16} \Rightarrow \boxed{P_2 = \frac{3}{8}}$

3 go through: YYYN, YYNY, YNYY, NYYY : each with prob $\frac{1}{16} \Rightarrow \boxed{P_3 = \frac{1}{4}}$

4 go through: Prob $\boxed{\frac{1}{16}}$

Question 4

An experiment is set up to measure the z-component of the angular momentum for a new kind of atomic nucleus. It is found that the result is always 1, 0, or -1 (in some units). According to our general rules for how quantum mechanics systems work, there should be special states of this nucleus, which we call $|1\rangle$, $|0\rangle$, and $|-1\rangle$, for which the angular momentum is predetermined (e.g. the state $|1\rangle$ has the definite value 1 for it's angular momentum even before we measure it). A more general state of this nucleus can be written as a superposition:

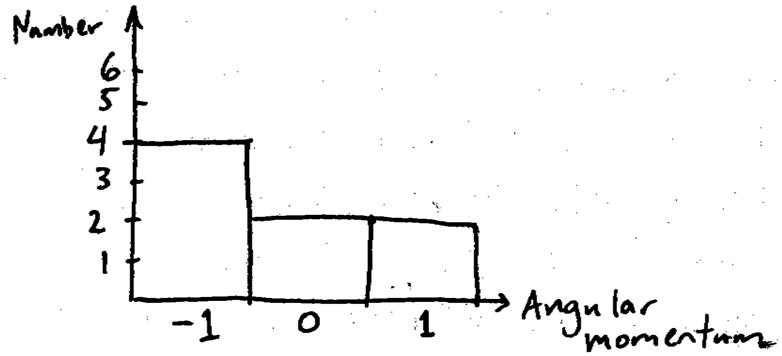
$a|1\rangle+b|0\rangle+c|-1\rangle$

a) For this state, if me measure the angular momentum, what results might we find, and what is the probability of each result, according to our general rules?

b) What conditions must the numbers a, b, and c satisfy (i.e. can they be any numbers at all or are there restriction on the values that they can take)?

c)Write down a state for which the probability of measuring angular momentum of 1 is twice the probability of measuring angular momentum 0 which in turn is twice the probability of measuring angular momentum -1.

d) A collection of nuclei are set up in the same initial state. After measuring the spin of each nucleus, we find the following histogram of results:



What can we conclude about the original state?

e) What would we have to do to determine the initial state more precisely?