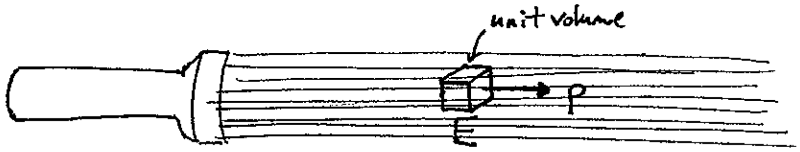
Name:

Physics 200 Tutorial 7:

Light, Energy, and Momentum II

In this tutorial, we will consider a few problems where it will be important to take into account the energy and momentum carried by light. As always, we can assume that the total energy and momentum are conserved.

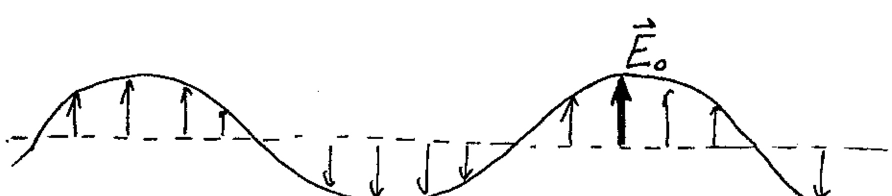


At any location where there is light or some other electromagnetic radiation, there will be some energy density E, and some momentum density **p**, which measure the energy and momentum per unit volume at that point. For light moving in a single direction, these are related by

$$E = |\mathbf{p}|c$$

This relation also applies to the total energy and momentum of some amount of light (e.g. a whole light pulse) travelling in one direction. The energy density and momentum density in a light wave are each proportional to the squared amplitude of the electric field:

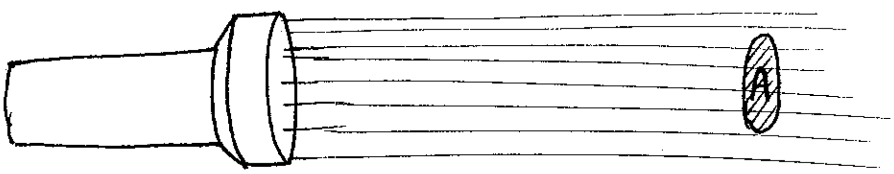
$$E \propto |\vec{E}_0|^2$$



Question 1

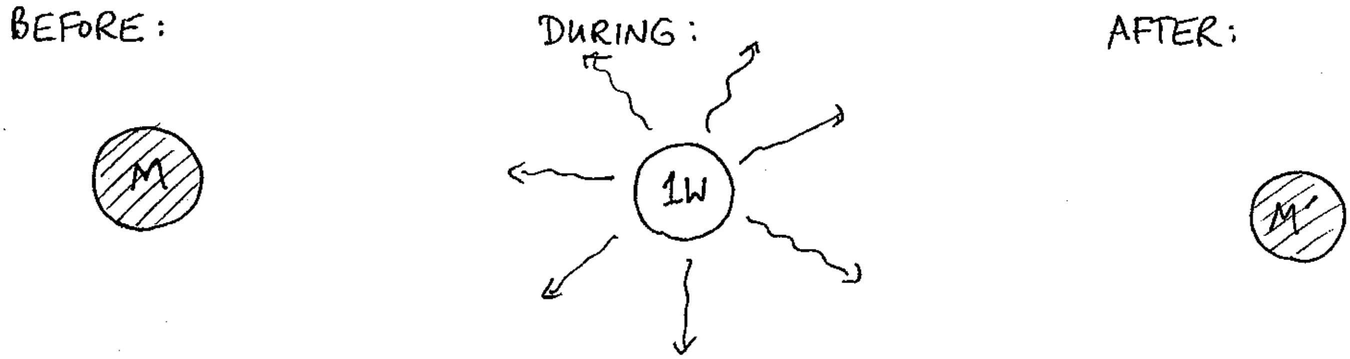
When talking about light, we often refer to the INTENSITY of the light rather than its energy or momentum. Intensity is defined to be the amount of energy that passes per unit time per unit area. So if we have an object with some cross-sectional area A in a light beam with intensity I, then the amount of energy that will hit the object in a time t is

$$E = I A t$$



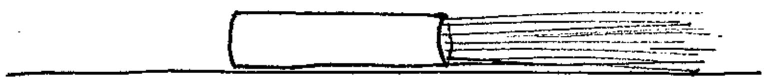
a) Determine the intensity of a uniform light beam with energy density e.

b) How is the intensity related to the TOTAL POWER of the light source (i.e. the energy of light per unit time leaving the source) for the source pictured above?



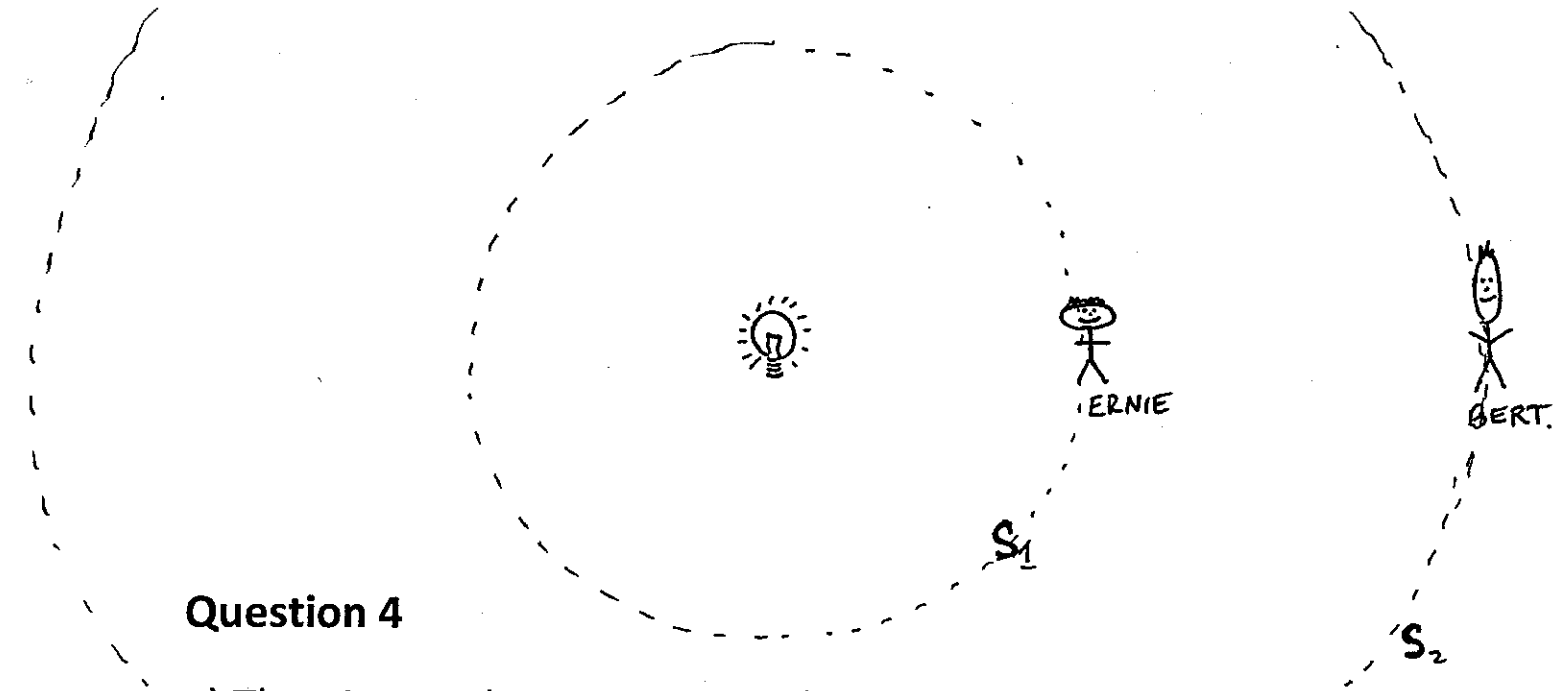
Question 2

An object with initial mass M = 1g has lights all over its surface and a battery inside it to power the lights. If the lights are turned on for one hour and the total power of the lights is 1W, how much less is the mass of the object after the hour? Assume that all the power goes into producing light.



Question 3

A powerful laser weighing 1kg sits on a frictionless table. If the laser has a power of 1 kW = 10^3 W and we turn it on at time t=0, at what time will the speed of the laser be 1mm/s?



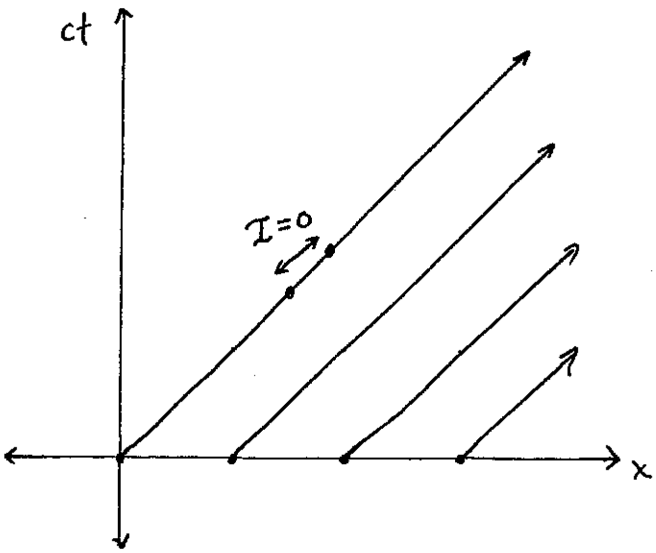
Question 4

a) The picture shows a source of light with a constant energy output that has been on a long time. What is the relation between the total energy passing though the spherical surface S_1 and the spherical surface S_2 with twice the radius?

b) Using your result from part a), how is the intensity of the light that Ernie measures related to the intensity of the light that Bert measures?

c) If Ernie measures the amplitude of the electric field from the light to be $|E_1|$, what does Bert measure the amplitude of the electric field to be?

Question 5: Black holes



The picture at the left shows a spacetime diagram in which a bunch of light pulses are emitted from various points along the x axis at t=0, all moving in the positive x direction. The path of the light pulses on the diagram can be characterized by saying that the invariant interval ***I*** between any two points on the path is zero:

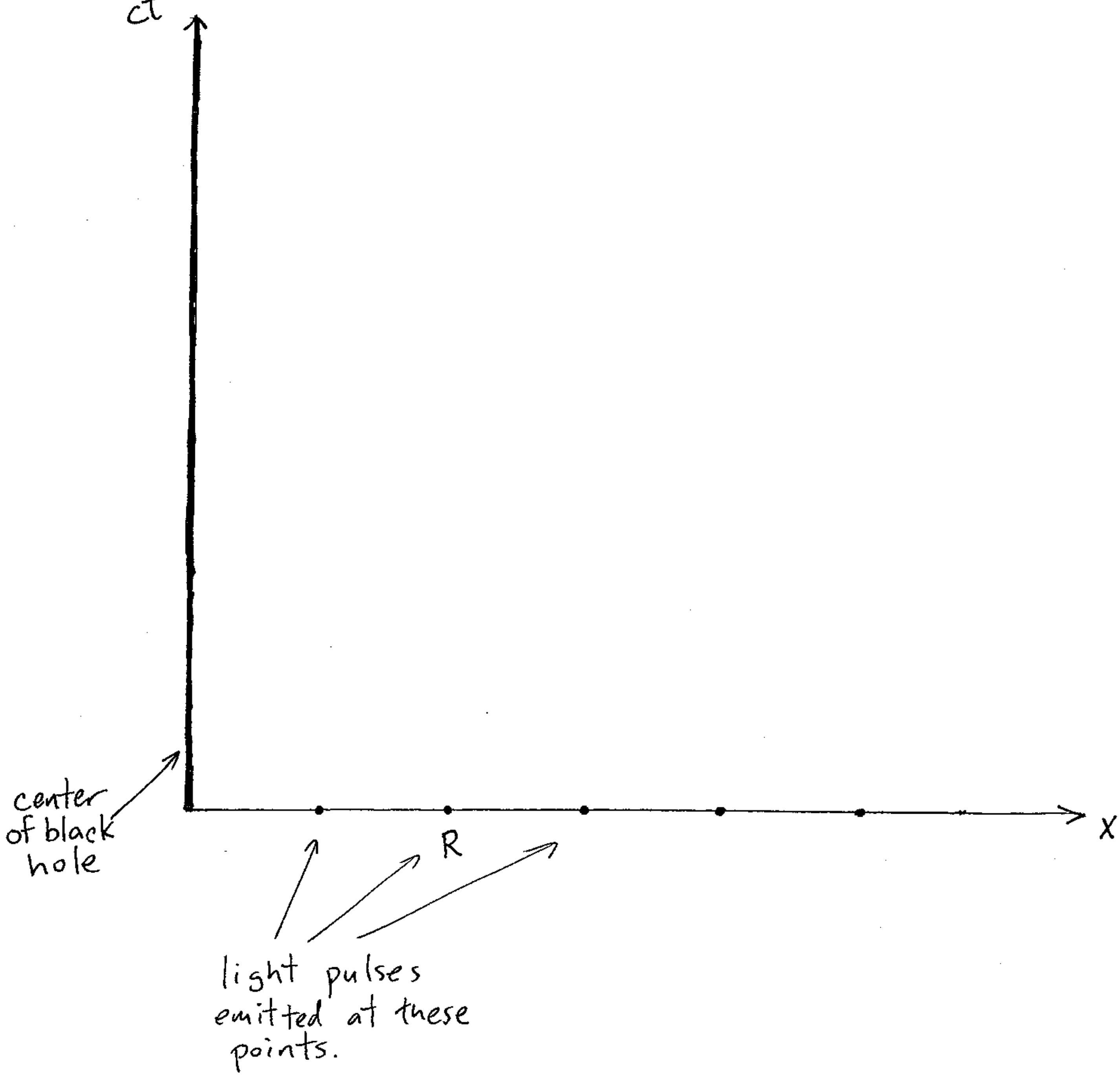
$$\boldsymbol{I} = (dx)^2 - c^2(dt)^2 = 0$$

In general relativity, the basic idea is that mass and energy warp spacetime. In practice, what this means is that the invariant interval is modified (e.g. a coordinate distance Δx might correspond to a larger proper length in one region of space than another due to the bending). Einstein's theory predicts that a sufficiently large (and dense) mass will warp spacetime in such a way that there is a region where not even light can escape. Such a region is known as a BLACK HOLE. In this problem, we'll see how the diagram above would be modified if there were a black hole sitting at x=0. In this case, the invariant interval would be changed due to the warping of spacetime to

$$\boldsymbol{I} = (1\text{-}R/x)^{-1}(dx)^2 - (1\text{-}R/x)\, c^2\, (dt)^2$$

Here, we will just consider light moving along the x axis at y=z=0, so we will just ignore the y and z directions. In the presence of the black hole, light pulses still travel along paths for which $\boldsymbol{I} = 0$ for nearby points. Using this information, draw the trajectories of the light rays that are emitted from the various points on the spacetime diagram on the next page.

Hint: for a light pulse at a position x, what will the slope of the trajectory on the spacetime diagram be if $\boldsymbol{I} = 0$? It may be helpful to draw a graph below of this slope vs the position x.


ct
center of black hole
R
x
light pulses emitted at these points.