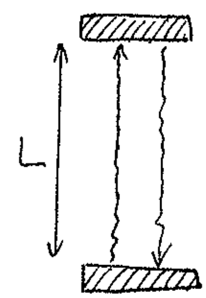
Name:

Physics 200 Tutorial 2

In this tutorial, you will explore some consequences of the principle of relativity and the postulate that the speed of light is the same in all frame of reference. You will see that observers in relative motion will not generally agree on distances or time intervals, but you will also derive precise formulae showing how the distance and time measurements for the two observers are related. These will reveal the famous LENGTH CONTRACTION and TIME DILATION effects of special relativity.

Question 1:

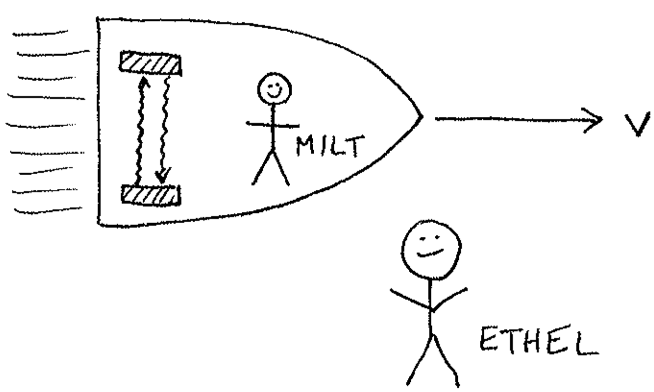
Consider a simple apparatus with two parallel mirrors separated by a distance L. and suppose there is a pulse of light that reflects back and forth between the mirrors.



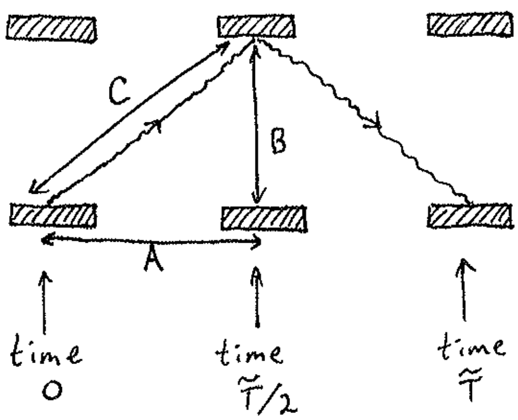
a) In the frame of the mirrors, how long does it take for the light to go from the bottom mirror to the top mirror and back?

ANSWER: T =

b) Now consider placing this apparatus in a rocket moving at speed v in a direction perpendicular to the line between the mirrors.



Milt rides in the rocket with the mirrors, so he observes the light to go up and down in time T. We would now like to determine the amount of time $\tilde{T}$ that Ethel measures for the light to go up and down.



Pictured at the left are the mirrors at three different times as observed by Ethel, and the path of the light pulse in Ethel's frame. In terms of $\tilde{T}$, v, c, and L, determine the distances A,B, and C on the diagram.

ANSWER:	A = B = C =

find C without using your answers for A and B

c) Using Pythagorus' Theorem, determine $\tilde{T}$ in terms of the other quantities.

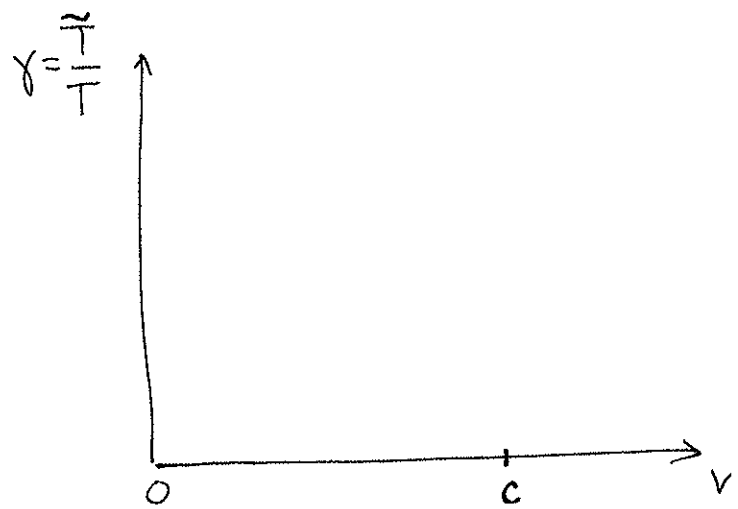
ANSWER:	$\tilde{T}$ =

d) Compare your results for $\tilde{T}$ and T. Do Milt and Ethel agree on the amount of time it takes for the light to go up and down? Express $\tilde{T}$/T in terms of v and c.

ANSWER:	$\tilde{T}$/T =

Congratulations! You have (hopefully) just discovered the formula describing TIME DILATION. The apparatus we have described can be thought of as a simple clock, since the pulse of light will hit the bottom mirror at regular intervals (the time T). What you have found is that Ethel observes the moving clock to run slow: you can check that the time $\tilde{T}$ she observes between the "ticks" is larger than T for any $v > 0$.

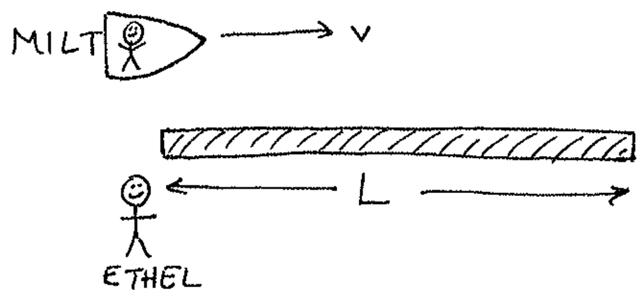
e) The ratio $\tilde{T}/T$ is often called γ. Plot γ as a function of v for v between 0 and c.



f) What happens to γ for $v \to 0$ and $v \to c$?

Question 2:

In this question, we would like to consider an object which has some length L as measured in its own frame (this is called the PROPER LENGTH) and determine how long it would be measured to be by an observer moving relative to it (in the direction along its length).



Consider the setup as shown above and suppose Milt and Ethel both set their clocks to zero when Milt's rocket passes the left side of the object. Milt can determine the length of the object in his frame by multiplying his speed relative to the object by the amount of time it takes him to go from one end to the other

(i.e. distance $L' = v \times T$ = speed × time)

a) What is the time $\tilde{T}$ on Ethel's clock when Milt's rocket reaches the end of the object?

ANSWER: $\tilde{T}$ =

b) What is the time T on Milt's clock when he reaches the end of the object? (Hint: use your result from Question 1d)

ANSWER: T =

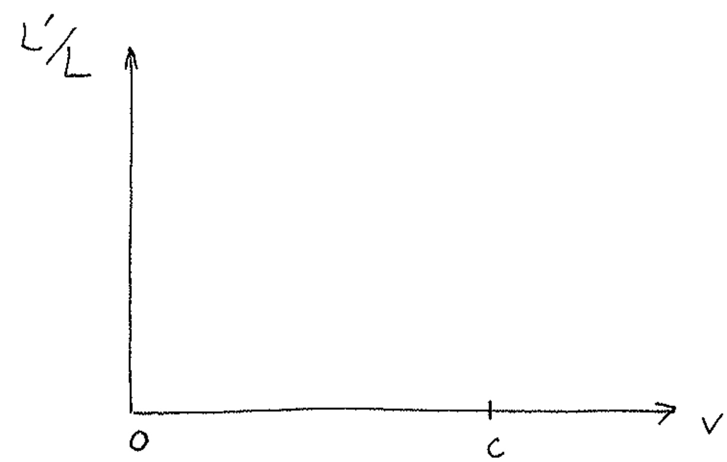
c) What is the length L′ that Milt determines the object to have?

ANSWER: L′ =

d) Compare the distance L (the proper length of the object) to L′, the length that an observer in relative motion measures for the object. Express L′/L in terms of v and c:

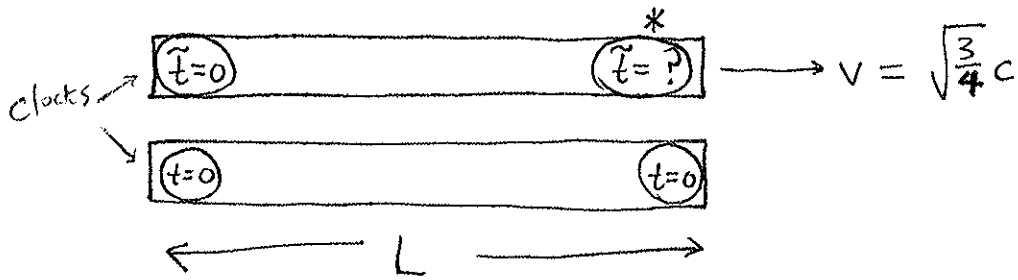
ANSWER: L′/L =

e) Plot this ratio for v between 0 and c. How does it behave at the two limits?



The effect you have just discovered is LENGTH CONTRACTION. An object in motion relative to you will appear (i.e. be measured) to be shorter in its direction of motion than its actual length.

Question 3 (Challenge Question):



The picture above shows a moving object and a stationary object which are observed to have the same length in some frame of reference (the frame in which the picture is drawn).

a) What is the proper length (i.e. the actual length) of the moving object?

b) Draw a picture of the two rulers as see from the frame of the upper ruler, when the left hand sides of the two rulers line up. Indicate the velocity of the two rulers in this frame.

c) What do observers in the frame of the lower object observe for the time on the clock indicated by * above? Assume that the clocks on each object are synchronized with each other in the frame of the object.

(Simpler question: is the time greater than, equal to, or less than zero?)

(Hint: An equivalent question is: what does the clock on the right side of the upper ruler read when the right hand sides of the two rulers line up)

(Hint 2: Look at your picture in part b) and then read the previous hint)