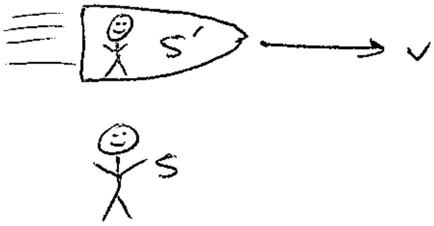
LAST TIME:

Lorentz Transformation:



★ Let (x, y, z, t) : coordinates of SINGLE EVENT as measured in frame S.

Coordinates of same event as measured in frame S′ moving at velocity v in $\hat{x}$ direction relative to S:

$$\boxed{\begin{aligned} t' &= \gamma\left(t - \frac{v}{c^2}x\right) \\ x' &= \gamma(x - vt) \\ y' &= y \\ z' &= z \end{aligned}}$$

assumes: observers agree on origin $(x,y,z,t) = (0,0,0,0) \updownarrow (x',y',z',t') = (0,0,0,0)$

+ $\hat{x}$ direction

$\xrightarrow{\frac{v}{c} \ll 1}$

$$t' \approx t$$
$$x' \approx x - vt$$
$$y' = y$$
$$z' = z$$

ordinary transforms. from 1st lecture

Inverse transform:

$$t = \gamma\left(t' + \frac{v}{c^2}x'\right)$$
$$x = \gamma(x' + vt')$$
$$y = y'$$
$$z = z'$$

★ get by solving for (t, x, y, z)

OR

notice that frame S moves at velocity $-v$ relative to frame S′

example:

What velocity does Pip observe for the small ship?

Assume: both observers agree small ship is at position 0 at time 0. $(x=0, t=0) \Longleftrightarrow (x'=0, t'=0)$

Time T in Jim's frame: small ship at $u \cdot T$
$x = uT \quad t = T$

Pip's frame:

$$x' = \gamma(x - vt) \qquad t' = \gamma\left(t - \frac{v}{c^2}x\right)$$
$$= \gamma(uT - vT) \qquad = \gamma\left(T - \frac{vuT}{c^2}\right)$$

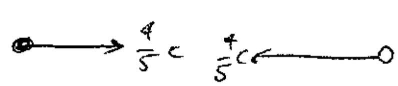
velocity in Pip's frame: $\dfrac{\Delta x'}{\Delta t'} = \dfrac{u - v}{1 - \frac{uv}{c^2}}$

VELOCITY TRANSFORM:

$$u'_x = \frac{u_x - v}{1 - \frac{u_x v}{c^2}} \qquad u'_y \quad \text{(EXERCISE)}$$

observed velocity always less than c

e.g.



frame of left ball