LAST TIME: wavefunctions for free electrons evolve via

$$i\hbar \frac{d\psi}{dt} = -\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2}$$

$\hbar = \frac{h}{2\pi}$

"h-bar"

- states $\leftrightarrow \psi(x,t)$
- superposition principle
- solutions are superpositions of $e^{\frac{i}{\hbar}(px - \frac{p^2}{2m}t)}$

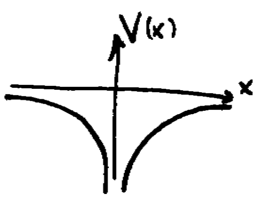
$\lambda = \frac{h}{p}$ $\qquad hf = \frac{p^2}{2m}$ energy

CLICKER

TODAY: what if we have forces or potentials?

e.g. proton at $x=0$

$$V(x) = -\frac{ke}{|x|}$$



→ need to incorporate V into Schrödinger equation.

CLICKER

simple: region of constant potential V

$$\text{energy} = \frac{p^2}{2m} + V$$

expect: frequency $\times h = \frac{p^2}{2m} + V$

$$e^{\frac{i2\pi}{h}(px - (\frac{p^2}{2m} + V)t)}$$

$$\boxed{i\hbar \frac{d\psi}{dt} = -\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V\psi}$$

right equation even when V depends on x!

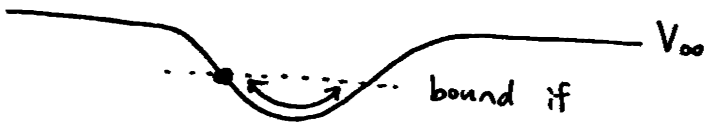
general 1D Schrödinger equation

3D: replace $\frac{d^2\psi}{dx^2} \rightarrow \frac{d^2\psi}{dx^2} + \frac{d^2\psi}{dy^2} + \frac{d^2\psi}{dz^2}$

1D simulation of "atom": electron gets trapped in region with $V < 0$ BOUND STATE

classically: bound state if $E_{tot} < V(x\to\infty)$

e.g. ball in valley



bound if $E_{kin} + V < V_\infty$

quantum: general states don't have definite energy

BUT: can be written as superposition of ENERGY EIGENSTATES

recall: energy ~ frequency × h

states w. definite energy

look like: $\psi(x,t) = \psi_E(x)\, e^{-\frac{iE}{\hbar}t}$

Special property: $|\psi(x,t)|^2$ indep of time

"STATIONARY STATES"

S.E. gives this time dependence only for special initial wavefunctions $\psi_E(x)$

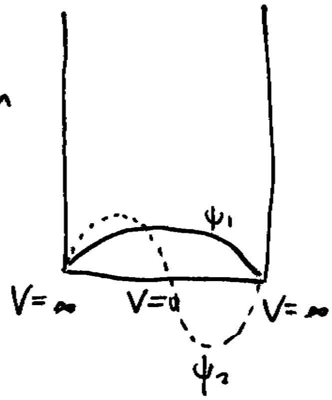
Need $$-\frac{\hbar^2}{2m}\frac{d^2\psi_E}{dx^2} + V\cdot\psi_E = E\,\psi_E$$

TIME INDEP. SCHRÖDINGER EQUATION.

QUANTUM BOUND STATES → solutions of this with $E < V_\infty$
normalizible wavefns with electron trapped in region.

** THESE EXIST ONLY FOR CERTAIN SPECIFIC ENERGIES **

e.g.
particle in "a box"



$$E_n = \frac{h^2 n^2}{8mL^2}$$