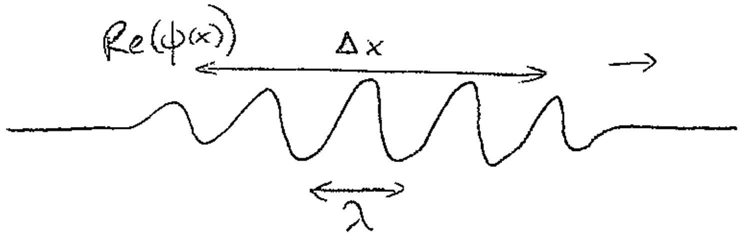
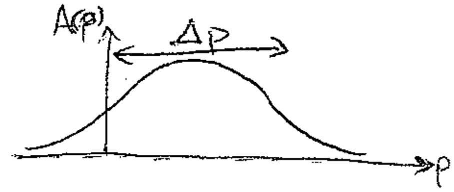
SO FAR: describe traveling electrons by wavepackets





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superposition of pure waves

$e^{i\frac{2\pi p}{h}x}$ → one of these for each momentum p

→ add with coeff A(p)

3D:

$\Delta x \, \Delta p_x \geq \frac{h}{4\pi}$ $\quad \Delta y \, \Delta p_y \geq \frac{h}{4\pi}$ $\quad \Delta z \, \Delta p_z \geq \frac{h}{4\pi}$

What about time dependence? given $\phi(x)$ at $t=0$ what is $\phi(x,t)$

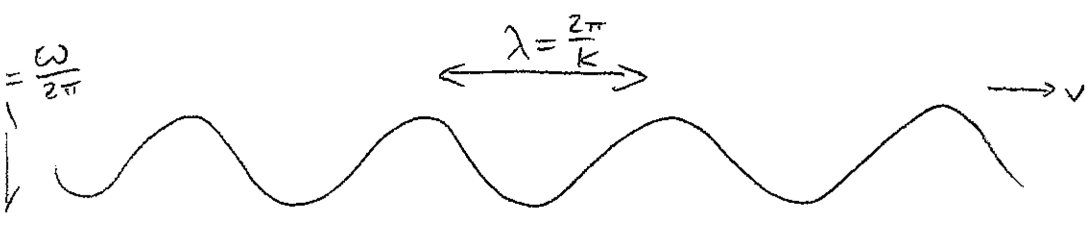
CLICKER:

- expect packet to travel at velocity

$$v = \frac{p}{m} = \frac{h}{\lambda m}$$

v depends on wavelength → typical of most waves (except light in vacuum)

Mathematically: time dependent pure wave: $e^{i(kx-\omega t)}$ $\quad k = \frac{2\pi}{\lambda}$ $\quad \omega = 2\pi f$



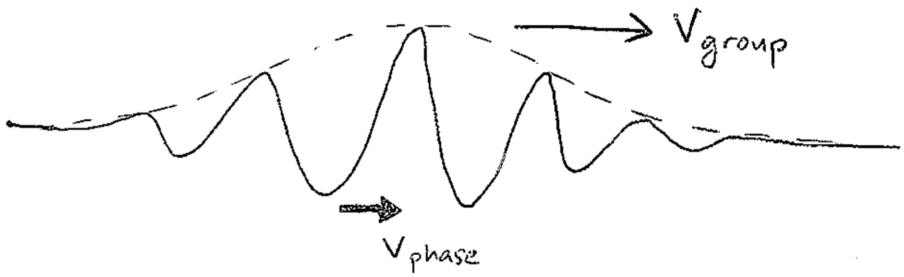
light:

$\omega = kc$

general waves: $\omega(k)$ not prop. to $k \Rightarrow v$ depends on λ

↑ DISPERSION RELATION

Add up time-dep pure waves with k near k_0



Get 2 different velocities

phase velocity = velocity of ripples $= \dfrac{\omega(k_0)}{k_0}$

group velocity = velocity of packet $= \left.\dfrac{d\omega}{dk}\right|_{k=k_0}$

We want: group velocity = electron velocity $= \dfrac{p}{m} = \dfrac{hk}{2\pi m}$

$\therefore$ Must have $\dfrac{d\omega}{dk} = \dfrac{hk}{2\pi m}$

True if: $\omega = \dfrac{hk^2}{4\pi m} = \dfrac{2\pi}{h} \cdot \dfrac{p^2}{2m}$

$\Rightarrow hf = \dfrac{p^2}{2m} = E$ (non-rel. energy)

Final result: wavepacket will have velocity $\frac{p}{m}$ if made of pure waves with frequency $= \dfrac{\text{energy}}{h}$

$$\psi(x,t) = e^{i\frac{2\pi}{h}\left(px - \frac{p^2}{2m}t\right)}$$

pure wave for elec with momentum p

→ Using superposition, can now determine time dependence for any wavefunction.