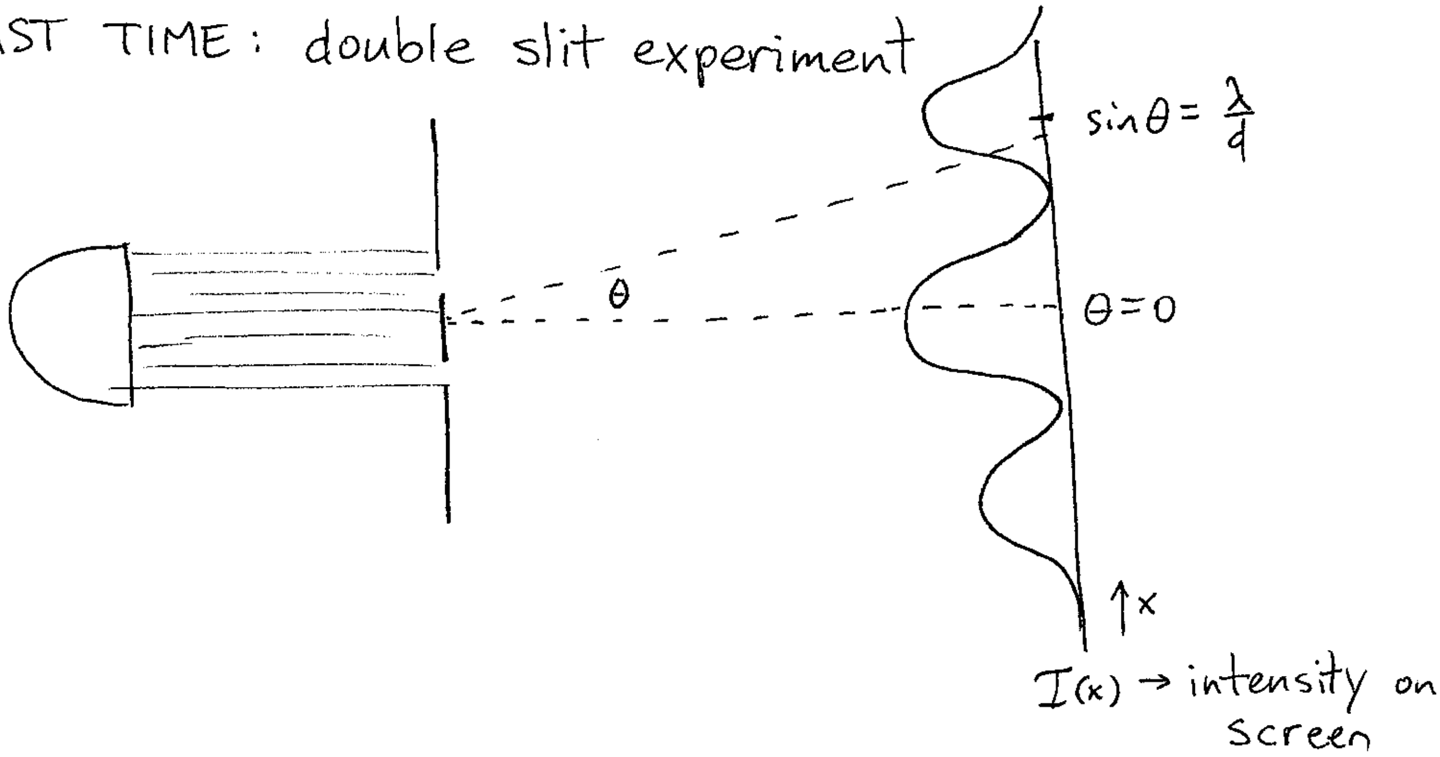
LAST TIME: double slit experiment



single photon picture:

→ each photon hits screen at specific spot with probability density $P(x) \propto I(x)$

2 slits essential → each photon spread out during experiment (sees both slits)

BUT: has specific location when we measure it.

Explanation: QUANTUM SUPERPOSITION

state of photon in flight = superposition of states w. definite location. ("position eigenstates")

when photon hits screen (measurement) → - becomes one of the eigenstates
- randomly "chooses" a position w. prob. given by squared coeffs in superposition.

Surprise: same results with electrons!

Get diffraction pattern corresponding to

$$\lambda = \frac{h}{|\vec{p}|}$$ DE BROGLIE WAVELENGTH

e.g. 100 eV $\quad \lambda \approx 10\,\text{nm}$

$\rightarrow$ quantum description of light & matter essentially the same.

Mathematical description of quantum superpositions

recall: all polarization states superpositions of 2 eigenstates

$$|\theta\rangle = a_{0^\circ}|0^\circ\rangle + a_{90^\circ}|90^\circ\rangle$$

choice of basis states

$$|a_{0^\circ}|^2 + |a_{90^\circ}|^2 = 1$$

position: one eigenstate (basis vector) for every point in space $\quad |\vec{x}\rangle = |(x,y,z)\rangle$

more general: $\quad a_{\vec{x}_1}|\vec{x}_1\rangle + a_{\vec{x}_2}|\vec{x}_2\rangle + \dots$

most general state: $$\int d^3\vec{x} \;\; \psi(\vec{x}) \;\; |\vec{x}\rangle$$

integral = "sum" over all possible positions

coefficient in superposition.

If we measure position:

$|\psi(x)|^2$ gives probability density for finding electron at $\vec{x}$

e.g.

prob. that we will find electron in volume V $= \int_V d^3x \, |\psi(\vec{x})|^2$ (just like finding mass from mass density)

Net probability must be 1 $\therefore$ require $\int_{\text{all space}} d^3x \, |\psi(x)|^2 = 1$

$\psi(\vec{x})$ = WAVEFUNCTION