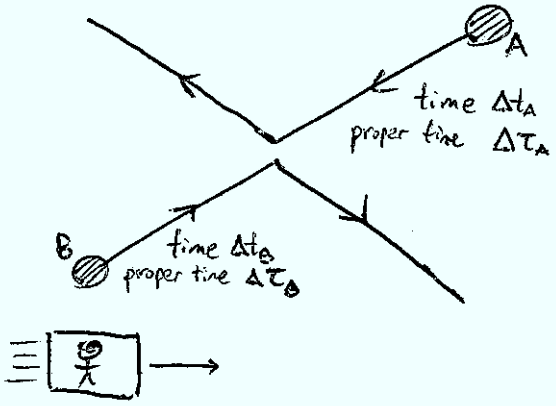
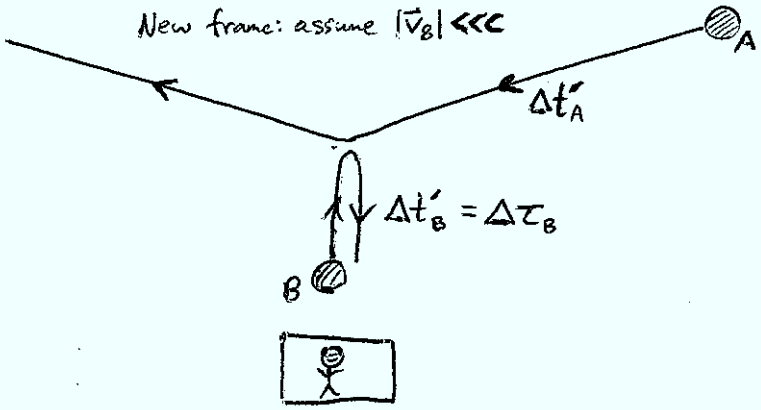
# LAST TIME: momentum in relativity




y velocities, momenta reversed after collision

$$p_y^{TOT} \rightarrow -p_y^{TOT}$$

conserved only if $p_y^{TOT} = p_y^A + p_y^B = 0$

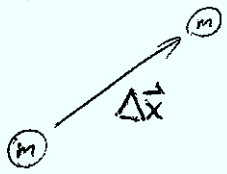
$$\therefore \quad p_y^A = -p_y^B = -m\frac{\Delta y_B}{\Delta t'_B} = -m\frac{\Delta y_B}{\Delta \tau_B} = m\frac{\Delta y_A}{\Delta \tau_A}$$

small $|\vec{v}_B|$ $\therefore$ use regular formula

equal since small $|\vec{v}_B|$

equal since symmetrical setup.

Final result:




momentum defined to be:

$$\vec{p} = m\frac{\Delta \vec{x}}{\Delta \tau}$$

$\Delta \vec{x}$ ← change in position

$\Delta \tau$ ← change in proper time

BUT $\Delta \tau = \Delta t\sqrt{1 - \frac{\vec{u}^2}{c^2}} = \frac{\Delta t}{\gamma(\vec{u})}$ ← full velocity of object.

$$\therefore \quad \vec{p} = m\frac{\Delta \vec{x}}{\Delta t}\gamma$$

$$\Rightarrow \boxed{\vec{p} = \gamma_{\vec{u}}\, m\, \vec{u}} \qquad \text{RELATIVISTIC MOMENTUM}$$

Properties:

- $\vec{p} \to m\vec{u}$ for $|\vec{u}| \ll c$
- $|\vec{p}| \to \infty$ for $|\vec{u}| \to c$
- sum of $\vec{p}$ for all object same before and after collision (CONSERVATION OF MOMENTUM)

Conservation of momentum holds in any frame, though $\vec{p}$ is different in different frames.

**Aside: for any system where the laws of physics are the same at every location, there is a conserved momentum**

SYMMETRIES $\leftrightarrow$ CONSERVATION LAWS

(Proven by Emmy Noether)