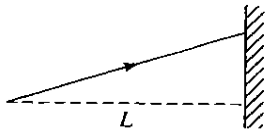
CHAPTER 2

PROBLEMS

1. 

Velocity along $L : C \pm v_{\|}$
Velocity $\perp L : v_{\perp}$

To complete trip

$$T = t_1 + t_2 = \frac{L}{c + v_{\|}} + \frac{L}{C - v_{\|}} = \frac{2L/C}{(1 - v_{\|}^2/C^2)}$$

If $\theta = 0 (v_{\perp} = 0) v = v_{\|}$

$$T_L = \frac{2L/C}{(i - v^2/C^2)} \approx \frac{2L}{C}\left(1 + \frac{v^2}{C^2}\right) \quad \text{if } v \ll C$$

If $\theta = 90^\circ : v = 0$

$$T_T = \frac{L}{C} + \frac{L}{C} = \frac{2L}{C}$$

Note $T_L - T_T \approx \frac{2L}{C}\frac{v^2}{C^2}$ for $v \ll C$

2. $L = L'\sqrt{1 - (v/c)^2} \quad J = 10\sqrt{1 - (v/c)^2}$

$\therefore v = 0.866c = \underline{\underline{2.6 \times 10^8 \text{ m/s}}}$

3. $v/c = 0.96 L = L_0\sqrt{1 - 0.96^2} = \underline{\underline{0.28 \text{ meter}}}$

$$T = \frac{d}{v} = \frac{0.28 \text{ meter}}{0.96C} = 9.7 \times 10^{-10} \text{ sec} = \underline{\underline{0.97 \text{ ns}}}$$

4. $T = T'/\sqrt{1 - \beta^2} \quad 3600a = 3595/\sqrt{1 - (v/c)^2}$

$\therefore v/c = 0.037 \quad v = 0.037c = \underline{\underline{1.1 \times 10^7 \text{ m/s}}}$

5. $v/c = 1 - 10^{-6}$ traveling at v the distance is contracted for particle

$L' = 175000 \text{ LY } \sqrt{1 - (1 - 10^{-6})^2} \approx 247; 487 \text{ LY}$

This length passes particle at speed $v/c = 1 - 10^{-6}$

$$\text{So } \Delta t = \frac{247.487 \text{ LY}}{v} = \frac{247.487 \text{ years}}{v/c} = \frac{247.487}{1 - 10^{-6}} \text{ yrs}$$

$= \underline{\underline{247.49}}$ years

6. $T = T'\gamma = \dfrac{1\,\mu\text{s}}{\sqrt{1 - (1/2)^2}} = \underline{\underline{1.15\,\mu\text{s}}}$

7. Distance traveled $= 2\pi R \times 10^6 = 3.7699 \times 10^8$ m. At v the Muon covers dist in 2.2μs so distance is countracted to $v(2.2 \times 10^{-6})$ meters

$3.7699 \times 10^8 (1 - v^2/c^2)^{1/2} = v(2.2 \times 10^{-6})$

Solved $v/c = 1 - 1.53 \times 10^{-12} \quad \underline{\underline{v = (1 - 1.5 \times 10^{-12})\,c}}$

8. Ship sees distance $(6 \times 10^{20} \text{ meters})\sqrt{1 - v^2/c^2} = v(25 \text{ yrs})$

thus $v/c = 1 - 7.7 \times 10^{-8} \qquad \underline{\underline{v = (1 - 7.7 \times 10^{-8})\,c}}$

Earth sees

$$T = \frac{D}{v} = \frac{6 \times 10^{20}\ \text{m}}{(1 - 7.7 \times 10^{-8})3 \times 10^{8}\ \text{m/s}} = 2.0 \times 10^{12}\ \text{s}$$

$$= \underline{\underline{6.3 \times 10^{4}\ \text{years}}}$$

9. $v = 1000$ km/hr $= (9.26 \times 10^{-7})$ c
Using earth radios 6.38×10^{3} km : $2\pi R = 40.1 \times 10^{6}$ m.
Time to circumnavigate $2\pi R/v = 1.44 \times 10^{5}$ s
Proper time on plane (T_0) is dilated and observed from earth.

$$1.443 \times 10^{5}\ \text{s} = T = \frac{T_0}{\sqrt{1 - v^2/c^2}} \approx T_0 \left(1 + \frac{1}{2} v^2/c^2\right)$$

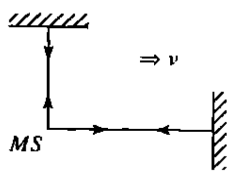
$$T - T_0 = T_0 \left(\frac{1}{2} v^2/c^2\right) = 6.2 \times 10^{-8}\ \text{s}$$

The clocks differ by 62 ns

5% of 62 ns = 3 ns accuracy needed

10. $$v = \frac{v' + u}{1 + \frac{v'u}{c^2}} = \frac{0.5c + 0.1c}{1 + 0.5(0.1)} = 0.571\ \text{c}$$

$$L = L'\sqrt{1 - v^2/c^2} = 1.00\sqrt{1 - 0.571^2} = 0.821\ \text{meter} = \underline{\underline{82\ \text{cm}}}$$

11. 

Assume microphone & speaker at center (ms) $f_0 = 3600$ Hz "C" $= 330$ m/s $v = 8$ m/s

Longitudinal path; WRT (With Respect To) moving air

$$f_1 = 3600\gamma \left(1 - \frac{8}{330}\right) \quad \text{where } \gamma \equiv \left(1 - \frac{8^2}{330^2}\right)^{-1/2}$$

Mirror observes $f_2 = f_1 \gamma \left(1 + \frac{8}{330}\right)$

reflection conserves frequency (WRT mirror)

Leaves mirror WRT moving air $f_3 = f_2 \gamma \left(1 + \frac{8}{330}\right)$

Received at microphone

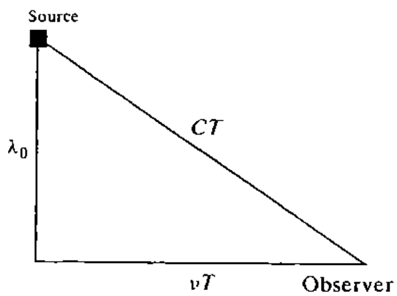
$$f_4^2 = f_3 \gamma \left(1 - \frac{8}{330}\right) = 3600\gamma^7 \left(1 + \frac{8}{330}\right)^2 \left(1 - \frac{8}{330}\right)^2$$

Transverse path: WRT moving air $f_1^T = 3600\gamma$
Mirror observes $f_2^T = f_1 \gamma$
Leaves mirror, WRT air $f_3^T = f_2^T \gamma$
Detector recevies $f_4^T = f_3^T \gamma = 3600(\gamma^4)$

$$\Delta f = f_4^2 - f_4^T = 3600\gamma^4 \left[1 - \left(\frac{8}{370}\right)^2 - 1\right] = 3600 \left(1 - \frac{8^2}{330^-}\right)^{-2} \left(\frac{8}{330}\right)^2$$

$$= \underline{\underline{2.12\ \text{Hz}}}$$

12.



Take the distance from source to horizontal path of observer equal to the wavelength (λ_0) emitted by source. $T_0 = \frac{\lambda}{C}$ is period if observer were at rest.

To arrive at moving observer, light must travel the hypotenuse a time T:

$$\lambda^2 = C^2T^2 - v^2T^2 \Rightarrow T = \frac{\lambda}{C}\frac{1}{\sqrt{1 - v^2/c^2}} = T_0\gamma$$

This is the transverse Doppler effect

$T = T_0\gamma$ or $f = f_0/\gamma$

13. $$x'^2 + y'^2 + z'^2 - c^2t'^2 = \gamma^2(x - vt)^2 + y^2 + z^2 - c^2\gamma^2\left(t - \frac{v}{c}\frac{x}{c}\right)^2$$

$$= y^2 + z^2 + \frac{1}{1 - v^2/c^2}(x^2 + v^2t^2 - 2xvt) - \frac{C^2}{1 - v^2/c^2}\left(t^2 + \frac{v^2}{c^2}\frac{x^2}{c^2} - 2\frac{v}{c^2}xt\right)$$

$$= x^2 + y^2 + z^2 - c^2t^2 \qquad \text{QED}$$

14. $x' = \gamma(x - vt) \quad t' = \gamma\left(t - \frac{v}{c^2}x\right) \quad \gamma \equiv (1 - v^2/c^2)^{-1/2}$

$\therefore x'\sqrt{1 - v^2/c^2} = x - vt$ and $t'\sqrt{1 - v^2/c^2} = t - \frac{vx}{c^2}$

$$x = \sqrt{1 - v^2/c^2}x' + v\left(\sqrt{1 - v^2/c^2}t' + \frac{v}{c^2}x\right)$$

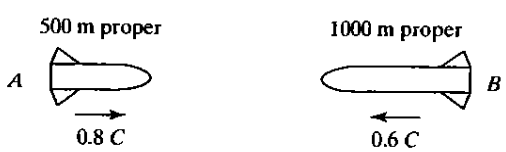
$$x(1 - v^2/c^2) = \sqrt{1 - v^2/c^2}x' + \sqrt{1 - v^2/c^2}vt'$$

$$x = \frac{x' + vt'}{\sqrt{1 - v^2/c^2}}$$

Likewise $t'\sqrt{1 - v^2/c^2} = t - \frac{v}{c^2}\left(x'\sqrt{1 - v^2/c^2} + vt\right)$

$$\therefore t = \frac{t + v/c^2x}{\sqrt{1 - v^2/c^2}} \qquad \text{QED}$$

15.



(a) Let v be the speed (relative) of B seen from A:

$$-0.6c = \frac{0.8c + v}{1 + \frac{0.8v}{c}} \quad \therefore \frac{v}{c} = -0.946$$

Relative speed: $0.946c$ towards each other

(b) From the Earth B appears $1000\sqrt{1 - .6^2} = 800$ m long

A appears $500\sqrt{1 - 0.8^2} = 300$ m long

$800 + 300 = (0.8\text{ c})t + (0.6\text{ c})t$

$t = \frac{1100}{1.4\text{ c}} = 2.619 \times 10^{-6}\text{ s} = \underline{\underline{2.619\ \mu\text{s}}}$ seen from earth.

16. A see the length of B: $1000\sqrt{1 - 0.946^2} = 324$ meter

A sees B travel that 324 m in $\frac{324\text{ m}}{0.946(3 \times 10^8\text{ m/s})} = \underline{\underline{1.14\mu\text{s}}}$

17. B see the length of A: $500\sqrt{1 - 0.946^2} = 162$ meter

B see A travel that 162 m in $\frac{162\text{ m}}{0.946(3 \times 10^8)\text{ m/s}} = \underline{\underline{0.57\mu\text{s}}}$

18. He should not worry. The simultaneous closing of the doors in the shed frame is not simultaneous in the moving (vaulter) frame. The vaulter would observe the door closing at the front, then it would open before the rear door close. $t_2 - t_1 = 0$ but to the moving vaulter $t_2' - t_1' = \underbrace{\gamma(t_2 - t_1)}_{0} - \gamma\frac{v}{c_2}\underbrace{(x_2 - x_1)}_{8\text{ foot}} \neq 0$

19. $f' = f\gamma\left(1 - \frac{v}{c}\cos\theta\right)$ where $f = \frac{c}{\lambda} = \frac{3 \times 10^8}{580 \times 10^{-9}} = 5.17 \times 10^{14}$ Hz

$$f' = \frac{c}{(580\text{ nm})}\frac{1}{\sqrt{1 - (1/2)^2}}(1 + 0.5) = \frac{1.73\text{ c}}{580\text{ nm}} = \frac{c}{\lambda'}$$

$\lambda' = \frac{580\text{ nm}}{1.73} = \underline{\underline{335\text{ nm}}}$, *same* if emitter moved toward you.

20. $f' = f\gamma\left(1 - \frac{v}{c}\cos\theta\right) = (1.2 \times 10^{15}\text{ Hz})\,1.667(1 - 0.8)$

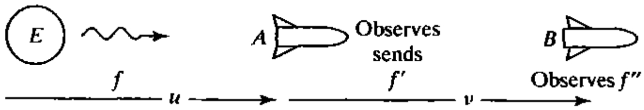
$= \underline{\underline{0.40 \times 10^{15}\text{ Hz}}}$ since $\gamma = \frac{1}{\sqrt{1 - 0.8^2}} = \underline{\underline{1.667}}$

21. $f' = f\gamma\left(1 - \frac{v}{c}\cos\theta\right)$ where $\gamma = (1 - 0.6^2)^{-1/2} = 1.25$

$= (0.40 \times 10^{15})(1.25)(1 - 0.6)\text{ Hz} = \underline{\underline{2.0 \times 10^{14}\text{ Hz}}}$

22. Speed of B seen from Earth: $\frac{0.8c + 0.6c}{1 + 0.6(0.8)} = \underline{\underline{0.946\text{ c}}}$

23.



(a) $f' = f\frac{1}{\sqrt{1 - (v/c)^2}}(1 - u/c) = f\sqrt{\frac{1 - u/c}{1 + u/c}}$

(b) $f'' = f'\frac{1}{\sqrt{1 - (v/c)^2}}(1 - v/c) = f'\sqrt{\frac{1 - u/c}{1 + u/c}}$

(c) Let w = velocity of B with respect to Earth

$$f'' = f\frac{1}{\sqrt{1 - (w/c)^2}}(1 - w/c)$$ Now equate this to result of (a) $\notin$ (b)

$$f\frac{1 - w/c}{\sqrt{1 - w^2/c^2}} = f\sqrt{\frac{1 - (u/c)}{1 + u/c}}\sqrt{\frac{1 - v/c}{1 + v/c}}$$

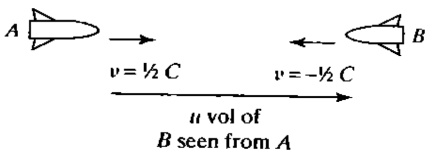
$$\frac{1 - w/c}{1 + w/c} = \frac{1 - u/c}{1 + u/c}\sqrt{\frac{1 - v/c}{1 + v/c}}$$

After Algebraic Manipulation Solving for W:

$$w = \frac{u+v}{1+\frac{uv}{c^2}}$$

Thus $v_{\text{relative to Earth}} = \dfrac{u+v}{1+\frac{uv}{c^2}}$

4.



$$v_B = \left.\frac{v_A+u}{1+\frac{v_A u}{c^2}}\right\} - \frac{c}{2} = \frac{\frac{c}{2}+u}{1+\frac{u\frac{c}{2}}{c^2}} \quad \left\{ \begin{array}{c}\text{Solve}\\ \text{for}\\ u\end{array}\right.$$

$\therefore\ u = -\frac{4}{5}$ c : Approach each other at $\frac{4}{5}$ c

5. $\dfrac{dy'}{dt'} = \dfrac{dy'}{dt}\dfrac{dt}{dt'}$ but $y = y'$

$$x' = \gamma(x - vt)$$

$$t' = \gamma\left(t - \frac{v}{c^2}x\right)$$

$$\frac{dy'}{dt'} = \frac{dy'/dt}{dt'/dt} = \frac{dy/dt}{\gamma\left(1-\frac{vu_x}{c^2}\right)}$$

because $\dfrac{dt'}{dt} = \gamma t - \gamma\dfrac{v}{c^2}\dfrac{dx}{dt} = \gamma\left(t - \dfrac{v}{c^2}u_x\right) \quad \left\{u_x = \dfrac{dx}{dt}\right.$

Thus $U'_y = \dfrac{u_y}{\gamma\left(1-\frac{v}{c^2}u_x\right)}$

CHAPTER 3

PROBLEMS

1. $t_{\text{Lab}} = \dfrac{0.80 \times 10^{-10}5}{\sqrt{1-v^2/c^2}} = \left.\dfrac{30 \text{ meters}}{v}\right\}$

   solved for v:

   $v/c = 1 - 3.2 \times 10^{-7}$

   $= 0.999\,9996$

   $v = 0.999\,9996\, c = \underline{\underline{c - 96 \text{ m/s}}}$

2. *Away from us* (lower frequency, longer wavelength)

   $$\frac{c}{f'} = \frac{c}{f}\sqrt{\frac{1+v/c}{1-v/c}} \to \lambda' = \lambda\sqrt{\frac{1+v/c}{1-v/c}}$$

   $$\frac{\lambda'-\lambda}{\lambda} = \frac{\lambda'}{\lambda} - 1 = \sqrt{\frac{1+v/c}{1-v/c}} - 1 = 1.95$$

   $$\frac{1+v/c}{1-v/c} = (1.95+1)^2 \to \frac{v}{c} = \frac{7.70}{9.70} = \underline{\underline{0.794}}$$

3. For $v = 10$ km/s $\quad \gamma = \dfrac{1}{\sqrt{1-\left(\dfrac{10}{3\times 10^5}\right)^2}} = 1 + 5.56 \times 10^{-10}$

   Mirror "sees" Doppler shifted

   $$f' = f\gamma\left(1 - \frac{v}{c}\cos\theta\right) = 10^{15}(1+5.6\times 10^{-10})(1-3.33\times 10^{-5})$$

   Mirror reflects same freq. (w.r.t. mirror)
   then source observes it arriving Doppler shifted

   $$f'' = f'\gamma\left(1-\frac{v}{c}\right) = f\gamma^2\left(1-\frac{v}{c}\right)^2$$

   $$= 10^{15}(1-5.56\times 10^{-10})^2(1-3.33\times 10^{-5})^2$$

   $$= 0.999\,933 \times 10^{15} \text{ Hz} = \underline{\underline{10^{15} \text{ Hz} - 6.7 \times 10^{10} \text{ Hz}}}$$

4. Suppose asteroid moving away at speed v w.r.t spaceship Asteroid received $f_1 = F_0\sqrt{\dfrac{1-v/c}{1+v/c}}$, then reflecte some $f_2 = f_1$ received by spaceship Doppler shifted $f_3 = f_2\sqrt{\dfrac{1-v/c}{1+v/c}} = f_0\left(\dfrac{1-v/c}{1+v/c}\right)$

   $$6.5\times 10^9 - 5\times 10^4 = 6.5\times 10^9\left(\frac{1-\beta}{1+\beta}\right) \to \beta = 3.85\times 10^{-6}$$

   $v = \beta c = \underline{\underline{1.15 \times 10^3 \text{ m/s}}}$

5. $\vec{p} = \gamma m\vec{v} \quad E = \gamma mc^2$

   $$p'x' - E't' = \gamma\left(\rho - \frac{v}{c^2}E\right)\gamma(x-vt) - \gamma\left(E - \frac{v}{c}pc\right)\gamma\left(t - \frac{v}{c^2}x\right)$$