can be used to solve for the remaining unknowns x_P and T_1 in the Lorentz transformations. The left side of the first transformation is zero, so $x_P = v_1 T_P$ is the solution to part (a) of the problem. Once we know x_P, then, with $\gamma(v_1) = 2.29$, we can simply compute the right side of the second Lorentz transformation:

$$t_1 = \gamma(v_1)(T_P - (v_1^2/c^2)T_P) = (2.29)[(735\text{ s}) - (0.9)^2(735\text{ s})] = 320\text{ s}.$$

For the time T_2 that the second rocket records, we can use the same reasoning, but must not forget that for the first 60 s the clock on the second rocket and the clock on the platform run in synchrony. Thus, employing the method we used to solve for T_1, we obtain

$$\begin{aligned} T_2 - (60\text{ s}) &= \frac{T - (60\text{ s})}{\gamma(v_2)} = [T - (60\text{ s})]\sqrt{1 - v_2^2/c^2} \\ &= [(735\text{ s}) - (60\text{ s})]\sqrt{1 - (0.98c)^2/c^2} \\ &= (675\text{ s})(0.20) = 134\text{ s}, \end{aligned}$$

and it follows that $T_2 = 134\text{ s} + 60\text{ s} = 194\text{ s}$.

QUESTIONS

1. Moving observers see the time of events such as the ticking of a clock differently from one another. The more rapidly the observer moves, the more slowly he or she sees the clock ticking. What kind of clock does this refer to? Is a person a clock? What properties of your body illustrate aspects of a clock?

2. Think about a moving vehicle that emits a light wave into a vacuum. Is there any way an observer in the vehicle could measure any property of that wave, other than the speed of light?

3. A person with a stopwatch stands on the platform of a train station marking the arrival of the head of the train at one end of the platform and the tail of the train at the other end of the platform. This is *not* a good operational way to measure the possible simultaneity of the arrivals of the head and tail in the rest frame of the platform. Why is that? What would be the right way to measure the possible simultaneity?

4. The velocity addition formula shows that one cannot "superpose" two velocities, each of which is greater than $c/2$, such that the result would be an object that one would see moving at a speed greater than that of light. But what about superposing *three* velocities, each of which is greater than $c/3$? Would that work?

5. We have noted that Maxwell suggested a method for measuring the speed of light on the moving Earth, namely, setting up parallel mirrors at a distance L from each other and measuring the time it takes for light to make a round-trip from one mirror to the other and back. Suppose you did this experiment first on Earth and then on a spaceship moving uniformly with respect to Earth. Would you find a difference? Explain.

6. You have been given a superwatch accurate to a nanosecond. You want to set the watch so that it tells the "correct" time as measured by a friend who lives a thousand miles away and has an identical watch. You want to do this remotely—say, by telephone. How would you go about completing your task, and what physical assumptions would be involved?

7. Consider a wire carrying a current. Physically, this means that there are positively charged ions at rest in the laboratory, while electrons move with what is known as the "drift speed." The wire is electrically neutral, which means

that the charge density of the ions is the same as that of the electrons. Will the wire still look electrically neutral to an observer that moves along it, at rest with respect to the electrons?

8. Imagine a person rotating with angular velocity ω while holding a flashlight directed outward. A sequence of light detectors on a circle of radius R is designed to flash upon the arrival of the light. If $R\omega$ were larger than c, something that could be easily arranged by making R large enough, the sequence of flashes would propagate with a speed exceeding that of light. Does this scenario violate special relativity?

9. A rapidly moving muon (see Example 2–2), or indeed any unstable particle, travels farther than one would expect because to an observer at rest in the laboratory, the unstable particle's "clock" is time dilated. How would an observer moving with the particle explain the same effect?

10. A skeptic might argue that, by moving a clock, you have somehow altered its workings, which is why it reads slow. How could you convince such a person by producing the same effect while the clock is safely anchored to, say, Earth?

PROBLEMS

1. ▮▮ Consider light propagating a distance L from point A to a mirror M and back. What would the travel time for the round-trip be if there were an ether wind of velocity v making an angle θ with the line AM? Show that your results reduce to the longitudinal and transverse results of the Michelson–Morley test when $\theta = 0°$ and $90°$, respectively.

2. ▮ At what speed would you have to move past a 10-cm ruler so that you would observe its length to be 5 cm?

3. ▮ A meter stick moves parallel to its axis with speed $0.96c$ relative to you. What would you measure for the length of the stick? How long does it take for the stick to pass you?

4. ▮ Find the speed relative to Earth of a uniformly moving spaceship whose clock runs 5 s slow per hour compared with an Earth-based clock.

5. ▮ A supernova at a distance of 175,000 light years from Earth emits particles that travel in a straight line to Earth. If these particles travel with a speed v such that $v/c = 1 - 10^{-6}$, how long will the trip last, as measured by an observer traveling with the particle?

6. ▮ If a particle decays with a lifetime of a microsecond as measured in its rest frame, what lifetime would an observer moving past it with half the speed of light observe the particle to have? (For this problem, the lifetime of a particle can be considered to be the tick of a clock, as in the discussion preceding Example 2–2.)

7. ▮▮ A beam of muons—see Example 2–2—is injected into a storage ring, a device that uses electromagnetic fields to maintain the muons in uniform circular motion. The ring's radius is 60 m. Find the speed of the muons, as a multiple of c, that is needed so that 10^6 revolutions are possible before the muons decay.

8. ▮▮ The radius of our galaxy is approximately 3×10^{20} m. A spaceship sets out to cross the galaxy in 25 years, as measured on board the ship. With what uniform speed does the spaceship need to travel? How long would the trip take, as measured by a timepiece stationed on Earth?

9. ▮▮ Suppose that you wanted to test time dilation by taking a clock around the world on a commercial airliner. Assuming some reasonable speed for the

airplane—1,000 km/hr, say—and some reasonable route, how accurate does your clock have to be to check the dilation formula to an accuracy of 5 percent? (In actuality, the effects of gravity are important here, as we shall learn in Chapter 17.)

10. ▮ Sally and Shelly are given identical meter sticks for their birthdays. Shelly gets on a spaceship that leaves Sally behind, moving at a speed of $0.5c$ relative to Sally. Shelly can ride her jet motorcycle at a speed of $0.1c$ relative to the ship, and does so, in a direction away from Sally. Shelly carries his meter stick with him, aligned with the motion. What, according to Sally, is the length of Shelly's meter stick?

11. ▮▮ Consider an apparatus for performing a Michelson–Morley experiment to measure the speed of sound in the laboratory. A sound wave of frequency 3,600 Hz replaces light. The speed of sound in air is 330 m/s. The arms of the interferometer are 2 m long, and the apparatus is placed in front of a large fan, which blows air along one of the arms at 8 m/s. Estimate the frequency of the beats that occur because of the interference of the waves reflected along the two arms of the interferometer. (*Hint*: Be careful! You are measuring the Doppler shift for sound for the case of a moving medium, and both the speed and the wavelength change.)

12. ▮▮▮ It is possible to derive the "transverse" Doppler shift in relativity theory by using nothing more complicated than the Pythagorean theorem, along with some of the basic principles of relativity. Consider a source separated from a receiver by a distance d. The time required for a signal to reach the receiver is d/c. Suppose now that the receiver is in motion at speed v at right angles to the line between source and receiver. How much time does it take for a signal to reach you this time, and how does the motion of the receiver change the frequency it receives? The answer to this question will give you the transverse Doppler shift. What principles of relativity did you use?

13. ▮▮ Start with the expression $x'^2 + y'^2 + z'^2 - c^2t'^2$ and show, with the aid of the Lorentz transformations, that this quantity is equal to $x^2 + y^2 + z^2 - c^2t^2$. This result establishes the invariance of s^2 defined by Eq. (2–32).

14. ▮▮ Invert Eqs. (2–25) and (2–27) directly to find x and t in terms of x' and t'.

15. ▮▮ Two relativistic rockets move toward each other. As seen by an observer on Earth, rocket A, of proper length 500 m, travels with a speed of $0.8c$, while rocket B, of proper length 1,000 m, travels with a speed of $0.6c$. **(a)** What is the speed of the rockets relative to each other? **(b)** The earthbound observer sets her clock to $t = 0$ when the two noses of the rockets just pass each other. What will the observer's clock read when the tails of the rockets just pass each other?

16. ▮▮ Consider the two relativistic rockets described in the previous problem. If the captain of rocket A, sitting near the nose of his rocket, sets his clock to $t = 0$ when the two noses pass each other, what will his clock read when he passes the tail of rocket B?

17. ▮▮ Consider the situation described in the previous problem. If the captain of rocket B, sitting near the nose of her rocket, sets her clock to $t = 0$ when the two noses pass each other, what will her clock read when the tail of rocket A passes the tail of rocket B?

18. ▮▮▮ A relativistic pole-vaulter holds a pole that is 16 ft long in his rest frame. He runs with the pole aligned in the direction of his motion with a speed such that $\sqrt{1 - v^2/c^2} = 1/2$. He approaches a shed that, in its rest frame, is 8 ft long. An observer at rest relative to the shed sees the pole as being only 8 ft long and arranges for gates at the two ends of the shed to slam shut as soon as the front of the pole reaches the far interior end of the shed. This observer sees the entire pole within the shed. On the other hand, the runner sees the shed as having a length of 4 ft and is worried that 12 ft of his pole will be amputated when the gates shut. Should he be worried? Answer this question by thinking carefully about just when each of the pole's ends arrives at the front and rear end of the shed in each of the two frames. Don't worry about what happens to the unfortunate pole-vaulter immediately after the gates shut; remember, this is just a thought experiment!

19. ▮ If you move toward an emitter of yellow light ($\lambda = 580$ nm) at half the speed of light, what wavelength would you observe? What would be the answer if the emitter moved toward you?

20. ▮ An observer on Earth sends light with frequency 1.2×10^{15} Hz to a spaceship traveling with speed $0.8c$ away from Earth. What will be the frequency of the light observed on the spaceship?

21. ▮ The spaceship in the previous problem transmits the light received from Earth, at the frequency that is observed, to a spaceship traveling ahead of it, away from Earth, with speed $0.6c$ relative to it. What is the frequency of the light as seen at the second spaceship?

22. ▮▮ If the second spaceship in the previous problem were unaware of the existence of the transmitting "middleman" spaceship, its crew would interpret the frequency of the emitted standard frequency of 1.2×10^{15} Hz as Doppler-shifted with a shift determined by the speed of the ship relative to Earth. How large would this speed have to be in order for it to agree with the observed frequency, as calculated in the previous problem?

23. ▮▮▮ Generalize the result of the previous set of problems. That is, consider the emission of radiation with frequency f from Earth. Rocket A traveling with speed u observes a frequency f' and transmits light with that frequency to rocket B, which is traveling with speed v relative to rocket A. The frequency observed by rocket B is f''. **(a)** What is f'? **(b)** What is f''? **(c)** Use the relation between f and f'' to calculate the velocity of B relative to Earth, and confirm that your result agrees with the formula for the addition of velocities.

24. ▮ Two spaceships approach each other. They are each viewed from Earth as having a speed half that of light. What is their speed relative to each other?

25. ▮▮ In Section 2–6, we found an expression for dx'/dt' [Eq. (2–41)]. Show that, to an observer traveling with speed v, the transverse velocity dy/dt will also be seen to be altered. (*Hint*: Calculate $dy'/dt' = (dy'/dt)/(dt'/dt)$, and use the Lorentz transformation law.)

APPENDIX

This appendix contains a more conventional (and more formal) derivation of the Lorentz transformations than that given in the text, which assumed the Lorentz contraction. When Einstein wrote his paper on special relativity in 1905, he had not seen Lorentz's paper of 1904 in which the transformations first appeared. Lorentz's derivation was done in the specific context of Maxwell's equations, while Einstein's derivation was of a very general character. We shall reproduce here not the 1905 derivation, but a simpler one that Einstein gave in his more popular writing. As we shall see, the algebra is very simple, but the assumptions must be stated with care.

We start with the usual situation: a system S and a second system S' that is moving uniformly with a speed v in the positive x-direction with respect to S.