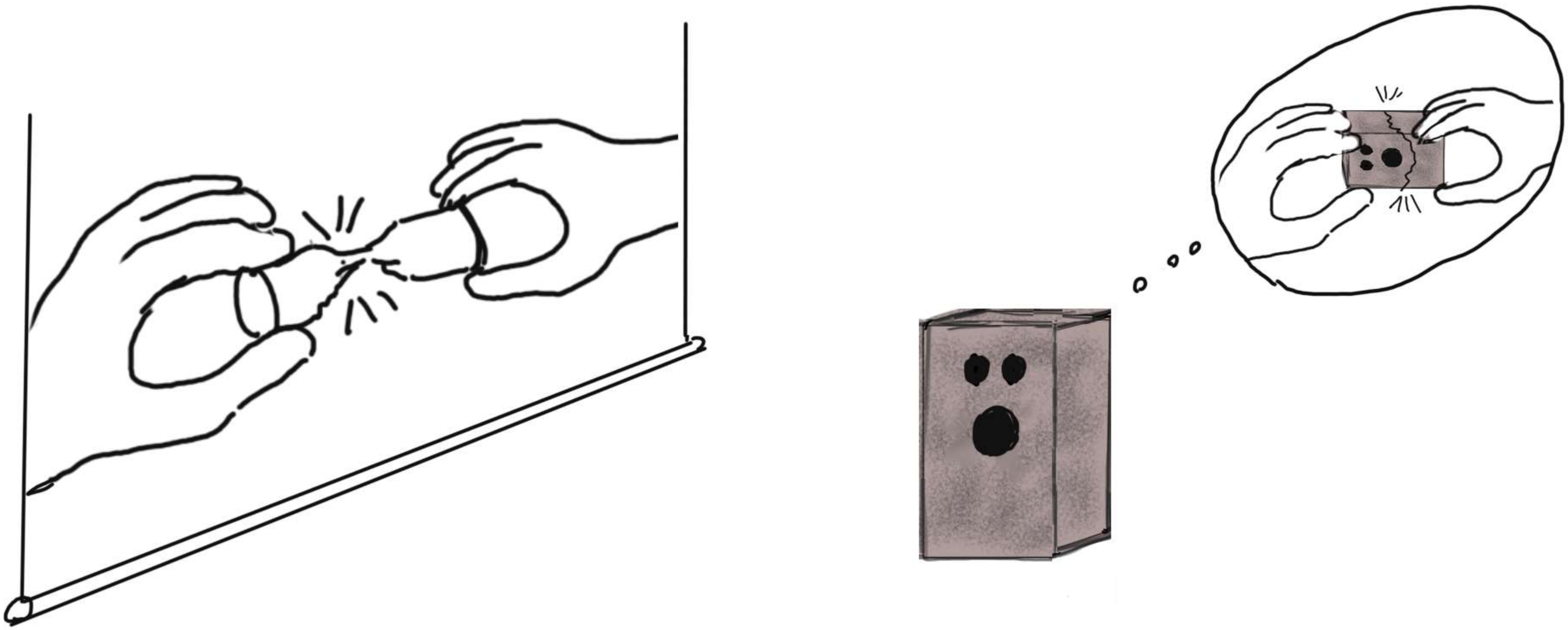
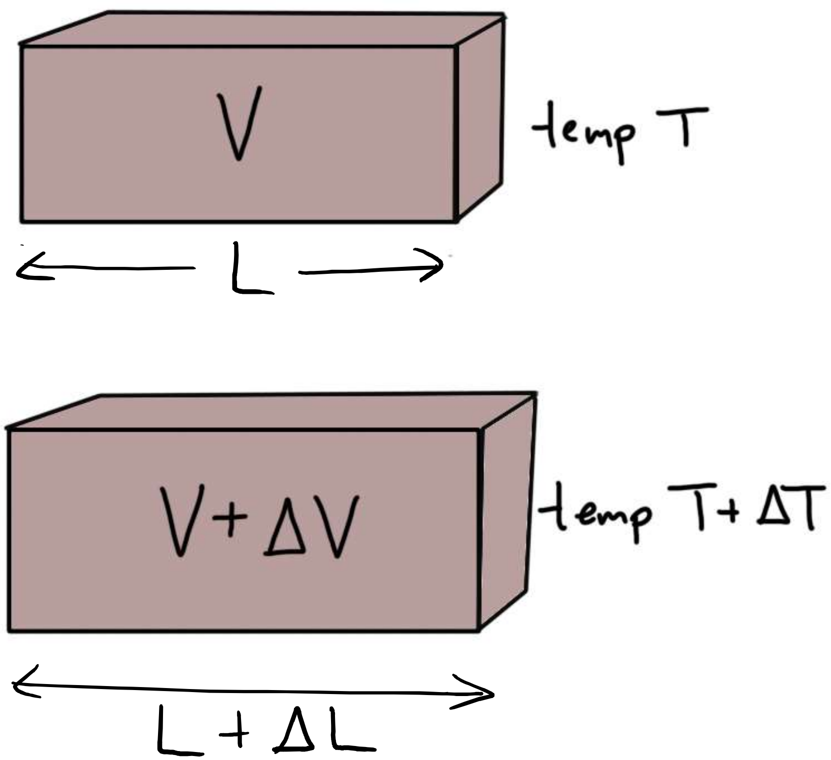
Learning Goals:

- For static systems with multiple parts, to use Newton’s second and third laws to related the various internal forces
- For a multipart system subject to temperature changes, to write an expression describing the change in length of each part due to the temperature change and the changing forces from adjacent parts
- For a multipart system subject to temperature changes to calculated the final lengths and forces for each part of the system



Last time in physics 157...

Thermal expansion:

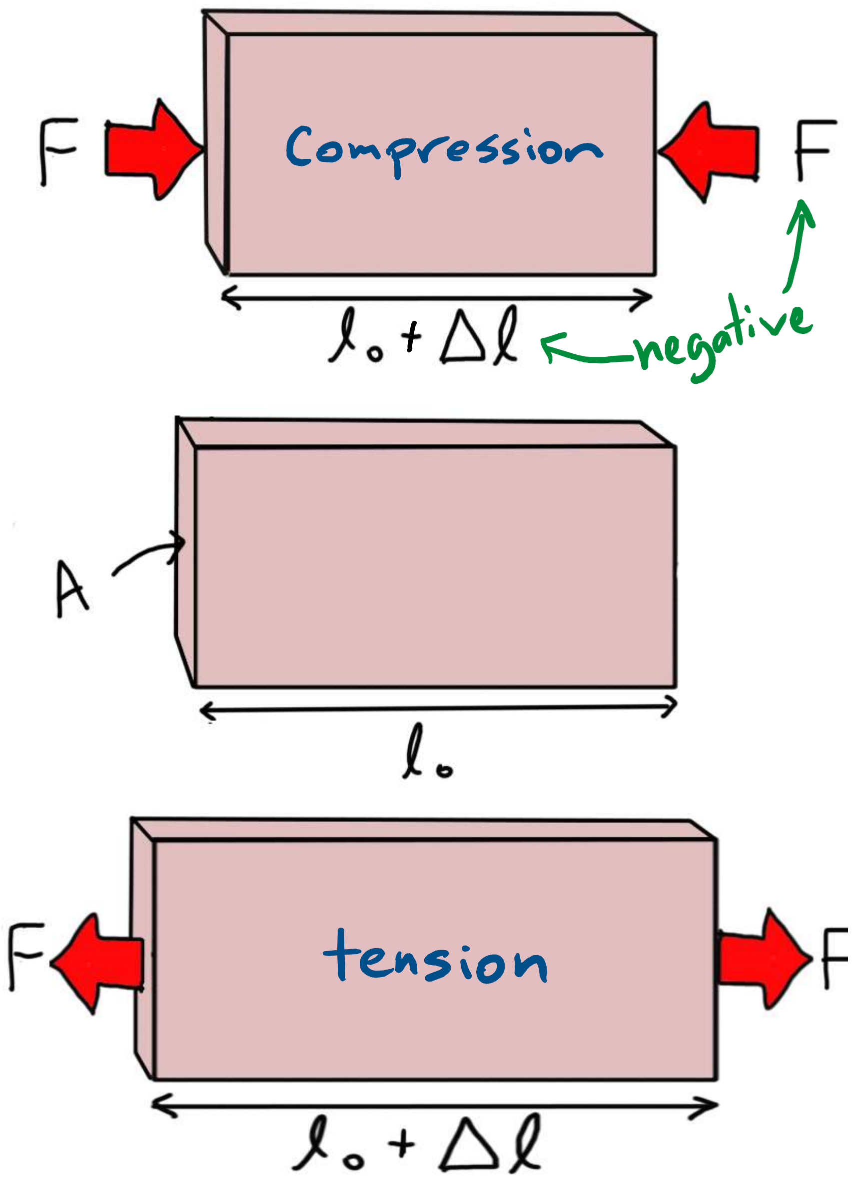


$$\Delta L = \alpha L_0 \Delta T$$

$$\Delta V = \beta V_0 \Delta T$$

$\beta = 3\alpha$ for solids

Stress & Strain



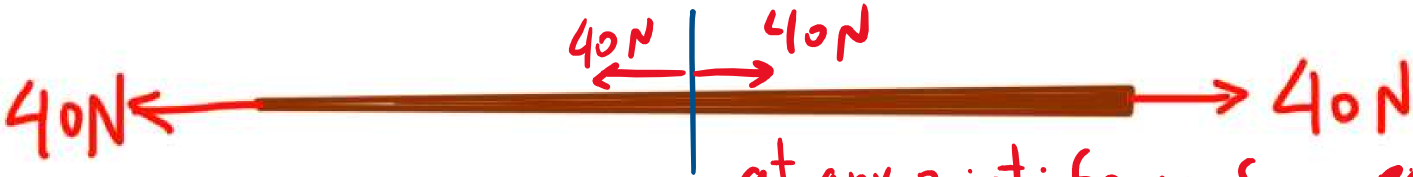
$$\frac{F}{A} = Y \frac{\Delta l}{l_0}$$

sign of F:

- positive for tension = pulling
- negative for compression = pushing

A copper wire of varying thickness is pulled at each end with a force of 40N. Which of the following statements is true?

A) The tension force is a constant 80N all along the wire, and the stress is also constant along the wire.
B) The tension is a constant 40N all along the wire, and the stress is also constant along the wire.
C) The stress is constant throughout the wire but the tension varies.
D) The tension is a constant 80N all along the wire, but the stress varies along the wire.
E) The tension is a constant 40N all along the wire, but the stress varies along the wire.



at any point: force from each half on other is 40N, since net force on each side is zero.

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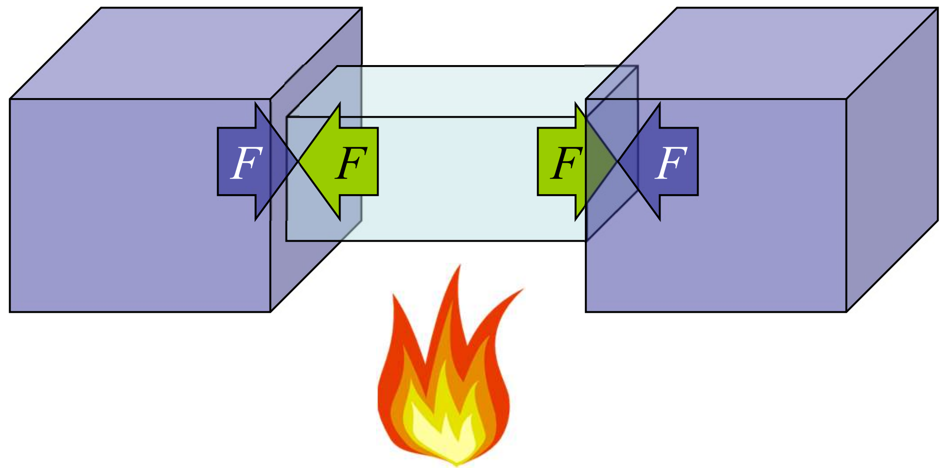
stress $= F/A$ varies since A varies

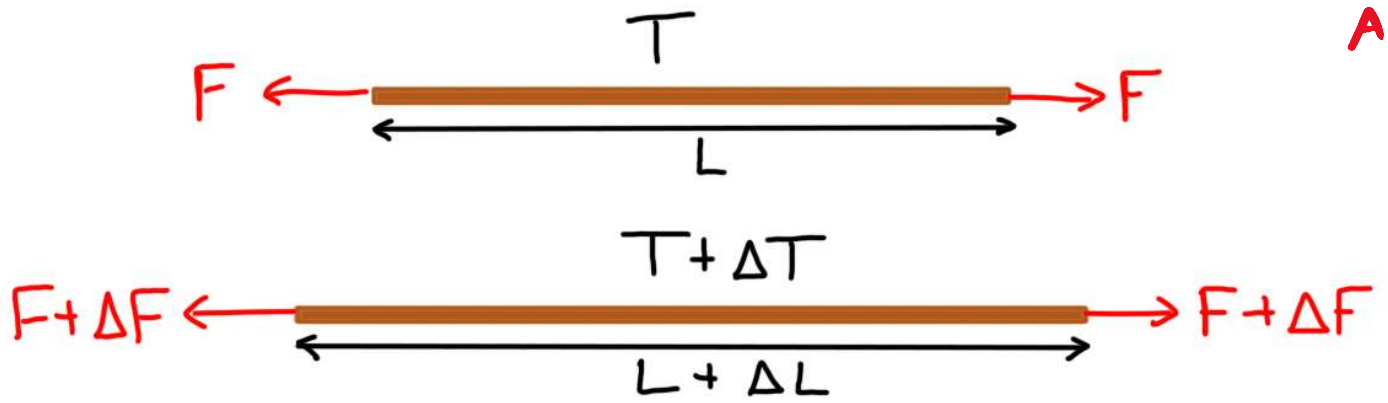
THERMAL STRESS : forces on a material due to surrounding materials preventing thermal expansion/contraction



* Ignore the "thermal stress" formula from the textbook *

$\Delta L = \alpha L \Delta T$

$\frac{F}{A} = Y \frac{\Delta L}{L}$



A copper wire under a tension force of F and at temperature T initially has a length L. If we heat up the wire by ΔT and also change the tension force by ΔF, by how much does the length of the wire change?

Hint: treat the change in length from thermal expansion and the change in length from the force increase separately

Click A when you have an answer, B if you are stuck

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Thermal expansion:

$$(\Delta L)_T = \alpha L \Delta T$$

Mechanical stretching:

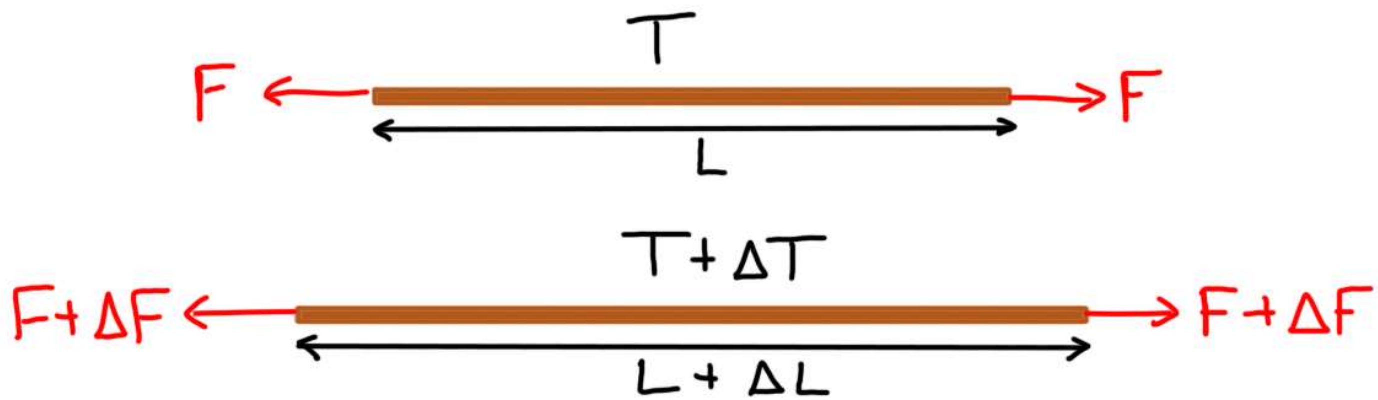
$$\frac{\Delta F}{A} = Y \frac{(\Delta L)_F}{L} \Rightarrow \Delta L_F = \frac{\Delta F}{A} \cdot \frac{L}{Y}$$

Total expansion:

$$\Delta L = (\Delta L)_T + (\Delta L)_F$$

$$\Rightarrow \Delta L = \alpha L \Delta T + \frac{L}{Y} \cdot \frac{\Delta F}{A}$$

NET CHANGE IN LENGTH



$$\Delta L = \Delta L_T + \Delta L_F$$

from stress/strain formula

$$\Delta L_T = \alpha L \Delta T \qquad \Delta L_F = \frac{1}{Y} L \frac{\Delta F}{A}$$

This applies to each part of a system

A harder problem:

Stressed Rods

A compound bar consisting of a copper rod with a length of 1 m and cross-section area of 2.00 cm^2 is placed end to end with a steel rod with length 1m and cross-section area 2.00 cm^2. The compound rod is placed between two rigid walls. Initially there is no stress in the bars at room temperature 20^o C.

Find the force on each wall at 40^o C.

$\alpha_{steel} = 12 \times 10^{-6}\ K^{-1}$, $\alpha_{copper} = 17 \times 10^{-6}\ K^{-1}$,
$Y_{steel} = 200 \times 10^{9}\ N\ m^{-2}$, $Y_{copper} = 110 \times 10^{9}\ N\ m^{-2}$

STEP 1 : visualize what will happen. Draw a before/after picture. Give names to known + unknown quantities + label diagram. Understand which quantities are changing and which quantities are fixed.

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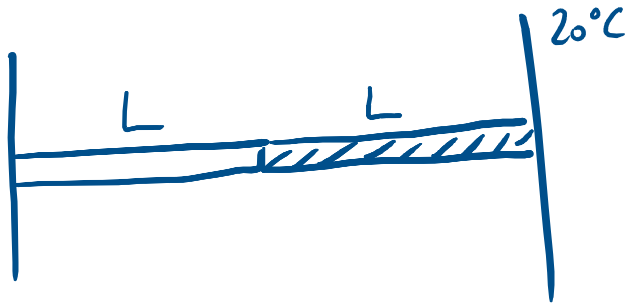
BEFORE:

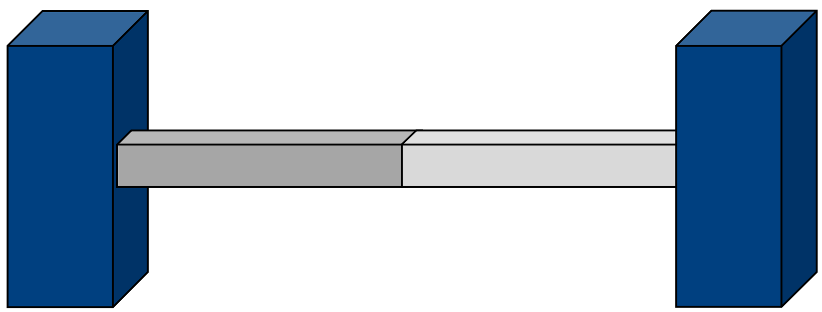


Clicker: As the system is heated, we expect that

A) Both rods will increase in length

B) Both rods will stay the same length

C) One rod will get longer and the other rod will get shorter



Clicker: As the system is heated, we expect that

A) Both rods will increase in length

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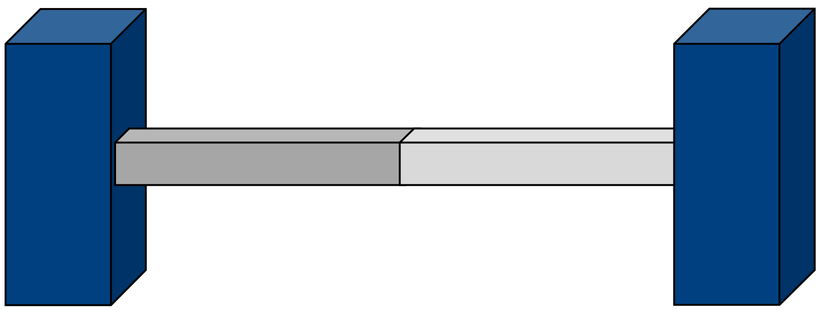
C) One rod will get longer and the other rod will get shorter

rigid walls, so B or C

but different materials so will likely change relative length



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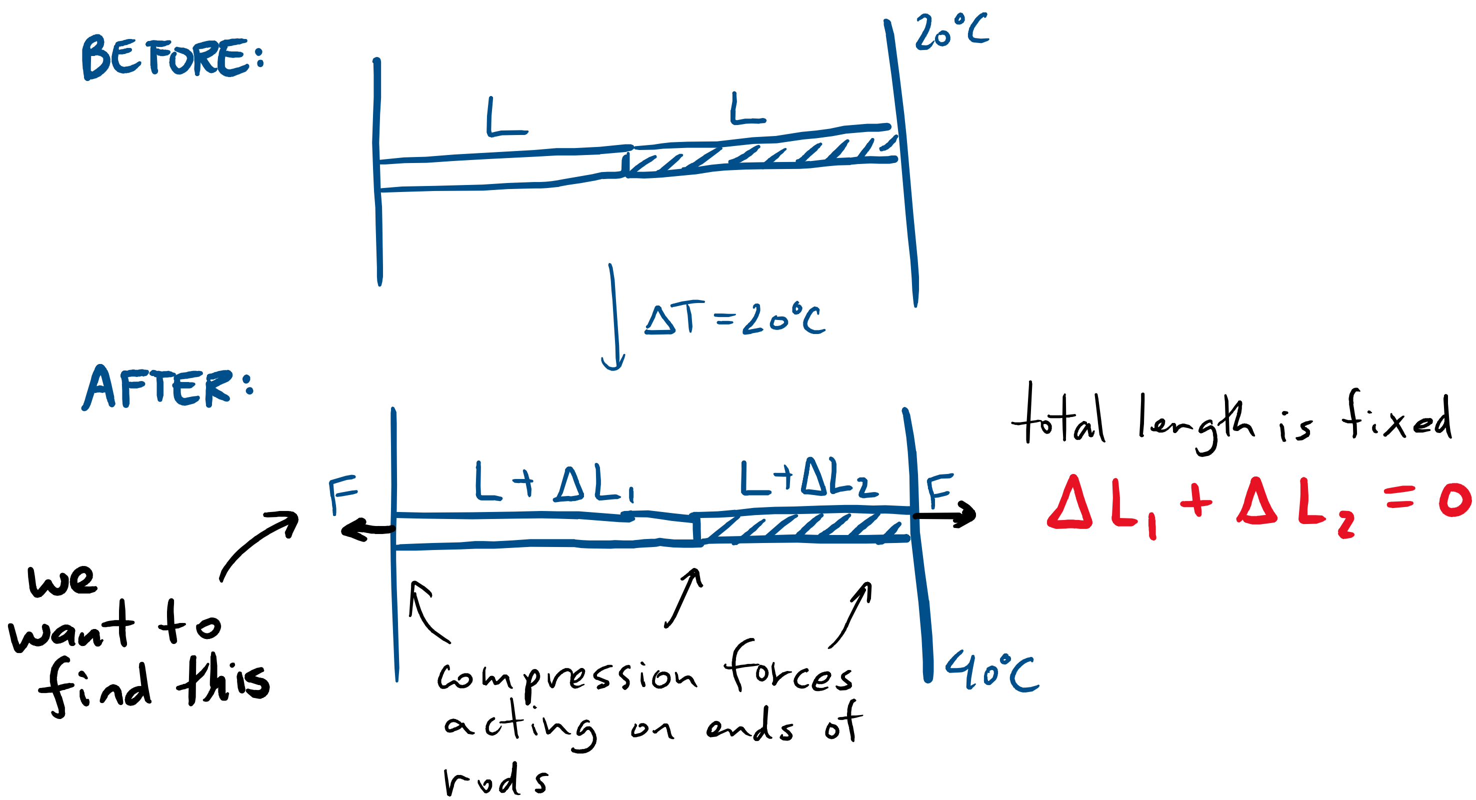

STEP 2: Isolate the parts of the system. For each part, draw before/after free body diagrams making use of Newton's Laws to relate forces.

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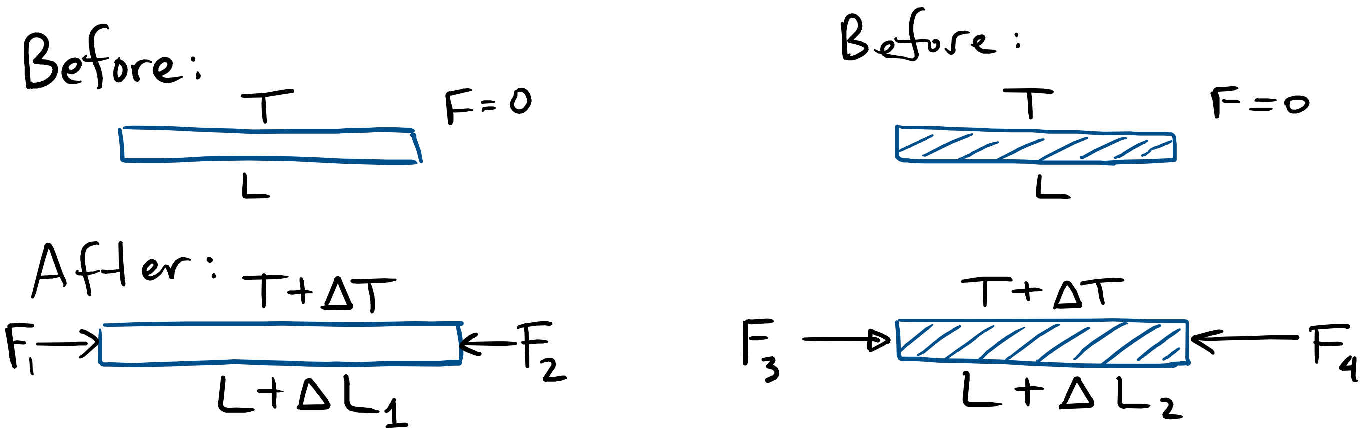
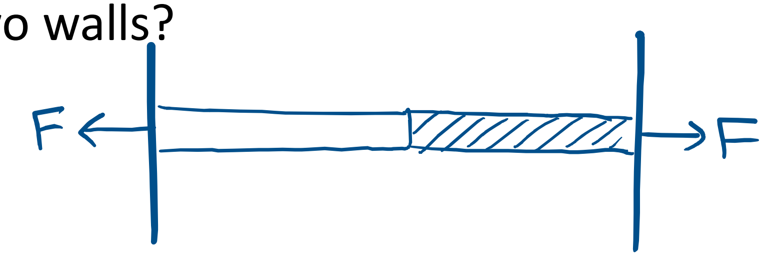


Question: What are F_1, F_2, F_3 and F_4 in terms of F, the magnitude of the forces on the two walls?



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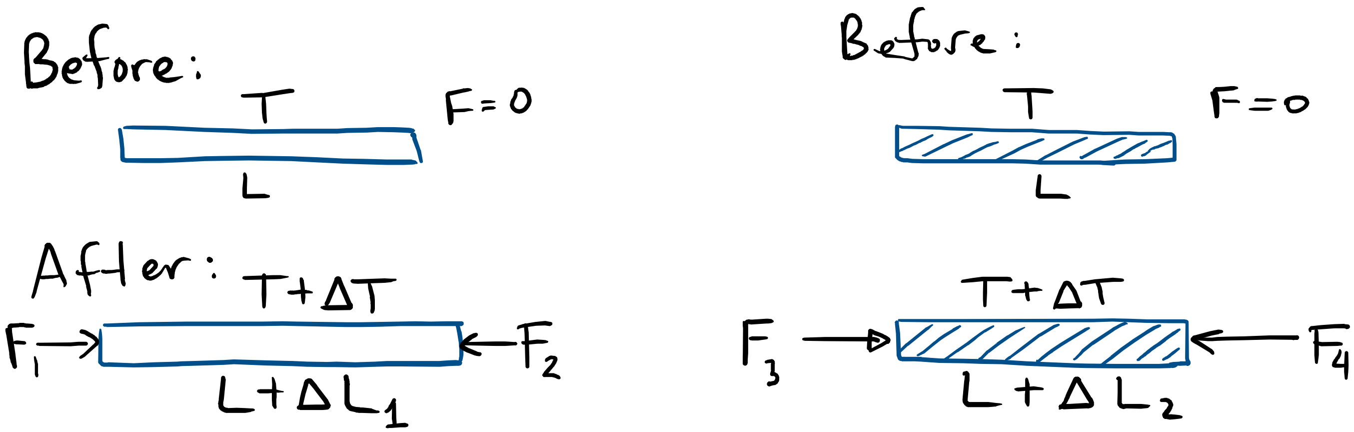

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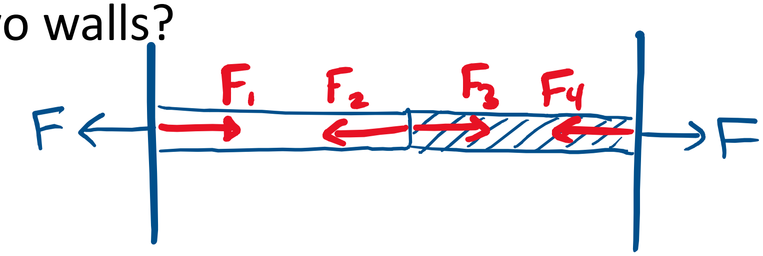


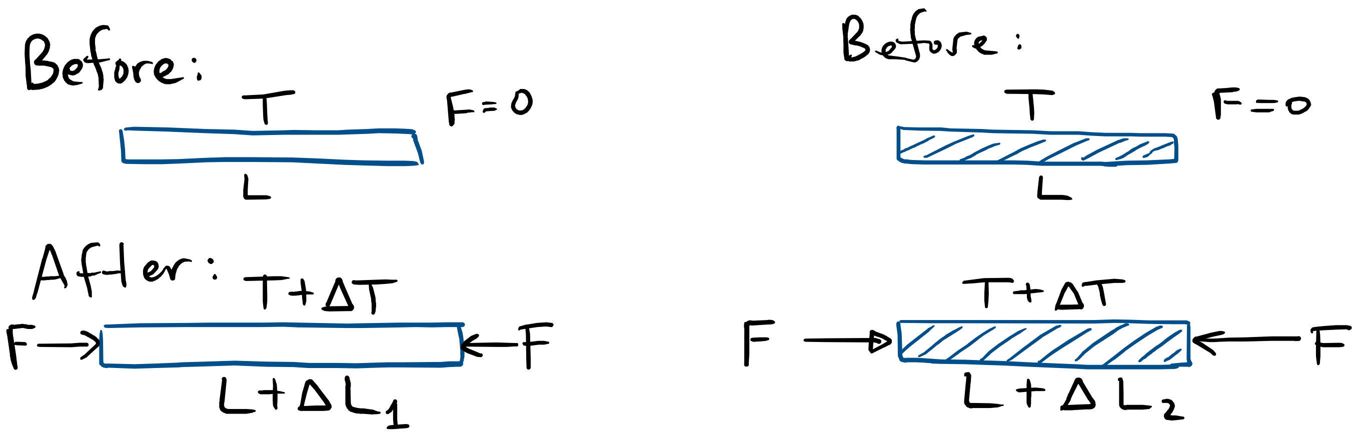
$F_1 = F$: Newton's 3rd.

$F_2 = F_1$: left bar not accelerating

$F_3 = F_2$: Newton's 3rd

$F_4 = F_3$: right bar not accelerating

STEP 3: For each part, write an equation relating the change in length to the changes in temperature and forces.



$$\Delta L_1 = \alpha_1 L \Delta T - \frac{F}{A} \cdot \frac{L}{Y_1}$$

$$\Delta L_2 = \alpha_1 L \Delta T - \frac{F}{A} \cdot \frac{L}{Y_2}$$

check that signs make sense with our interpretation of F as a positive number

STEP 4: collect equations & solve for unknowns

$$\Delta L_1 + \Delta L_2 = 0 \qquad (1)$$

$$\Delta L_1 = \alpha_1 L \Delta T - \frac{F}{A} \cdot \frac{L}{Y_1} \qquad (2)$$

$$\Delta L_2 = \alpha_2 L \Delta T - \frac{F}{A} \cdot \frac{L}{Y_2} \qquad (3)$$

STEP 4: collect equations + solve for unknowns

$$\Delta L_1 + \Delta L_2 = 0 \quad (1)$$

$$\Delta L_1 = \alpha_1 L \Delta T - \frac{F}{A} \cdot \frac{L}{Y_1} \quad (2)$$

$$\Delta L_2 = \alpha_2 L \Delta T - \frac{F}{A} \cdot \frac{L}{Y_2} \quad (3)$$

Plug (2) and (3) into (1):

$$\alpha_1 L \Delta T + \alpha_2 L \Delta T - \frac{F}{A}\frac{L}{Y_1} - \frac{F}{A}\frac{L}{Y_2} = 0$$

Isolate terms with F:

$$\frac{F \cdot L}{A}\left(\frac{1}{Y_1} + \frac{1}{Y_2}\right) = (\alpha_1 + \alpha_2) L \Delta T$$

Solve:

$$F = \frac{(\alpha_1 + \alpha_2)\Delta T \cdot A}{\left(\frac{1}{Y_1} + \frac{1}{Y_2}\right)} = 8.2 \times 10^3 \, N$$

Extra Clicker: If 0.2mm diameter nylon fishing line is good for catching fish up to 2kg, what thickness of line would you need to catch a 50kg fish?

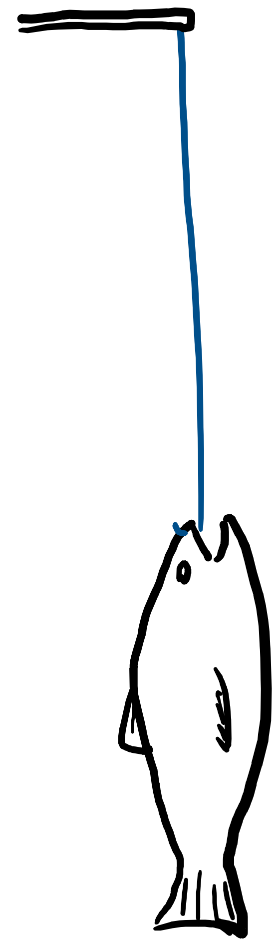
A) 0.5mm

B) 1mm

C) 2mm

D) 5mm

E) 300mm



EXTRA: By roughly how much would 1m of 0.2mm diameter line be stretched by a 2kg fish?

(Y_{nylon} = 3 GPa)

G = giga = 10^9

$$\frac{F}{A} = Y \frac{\Delta L}{L_0}$$

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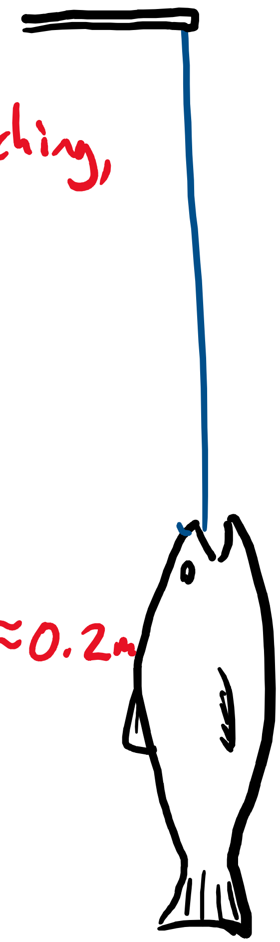
F is 25x before to get equivalent "safe" stretching, need A 25x before so diameter 5x before.



$$\Delta L \approx \frac{1m}{3\times10^9} \cdot \frac{20N}{\pi \cdot (10^{-4}m)^2} \approx 0.2m$$

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