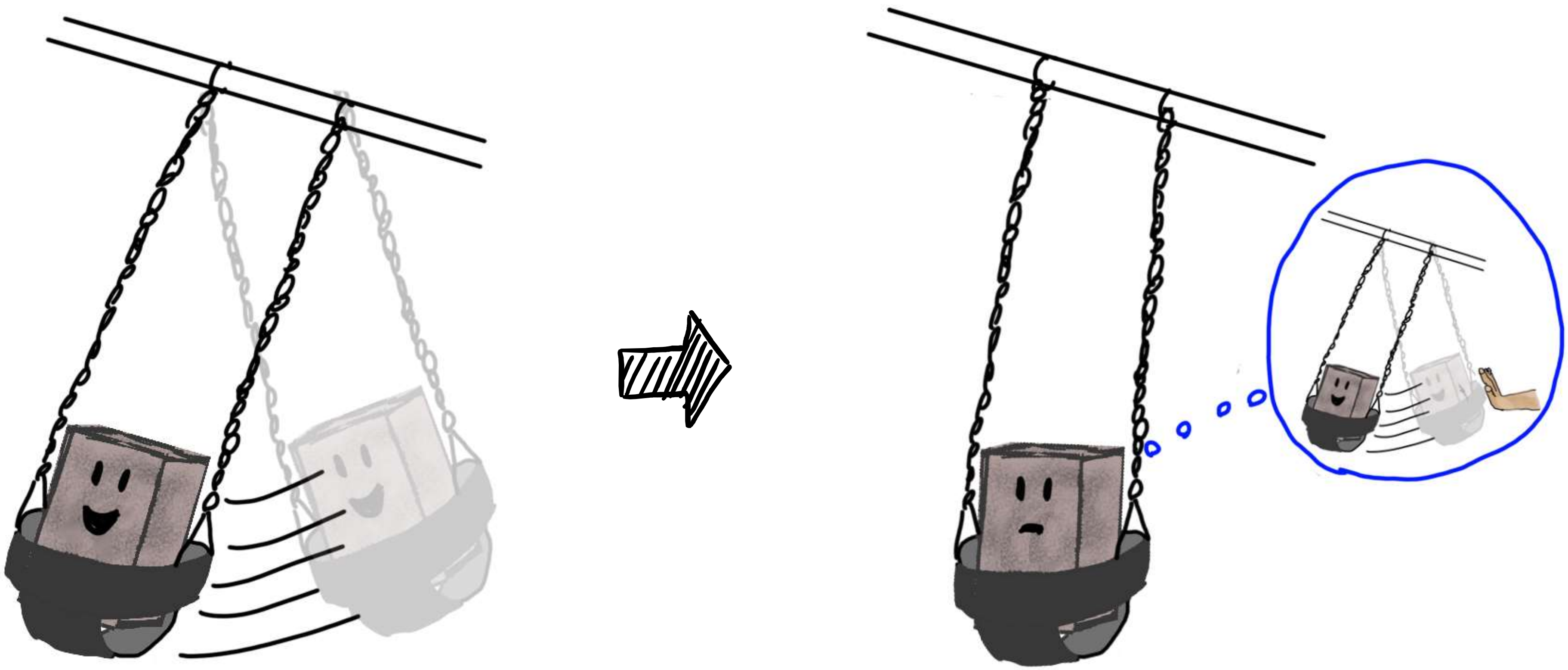
Office hours today: after class in Remo, 4-5pm, 8-9pm in Zoom

Homework sessions: M/Tu 5-8pm in Remo

Learning goals for today:
To determine the time constant and/or damping constant given a description of the motion of a damped oscillator.
To explain qualitatively the physical effects of different amounts damping, including critical damping
To explain how the amplitude of a driven oscillation depends on the driving frequency and to describe the phenomenon of resonance.

Last time in Phys 157...

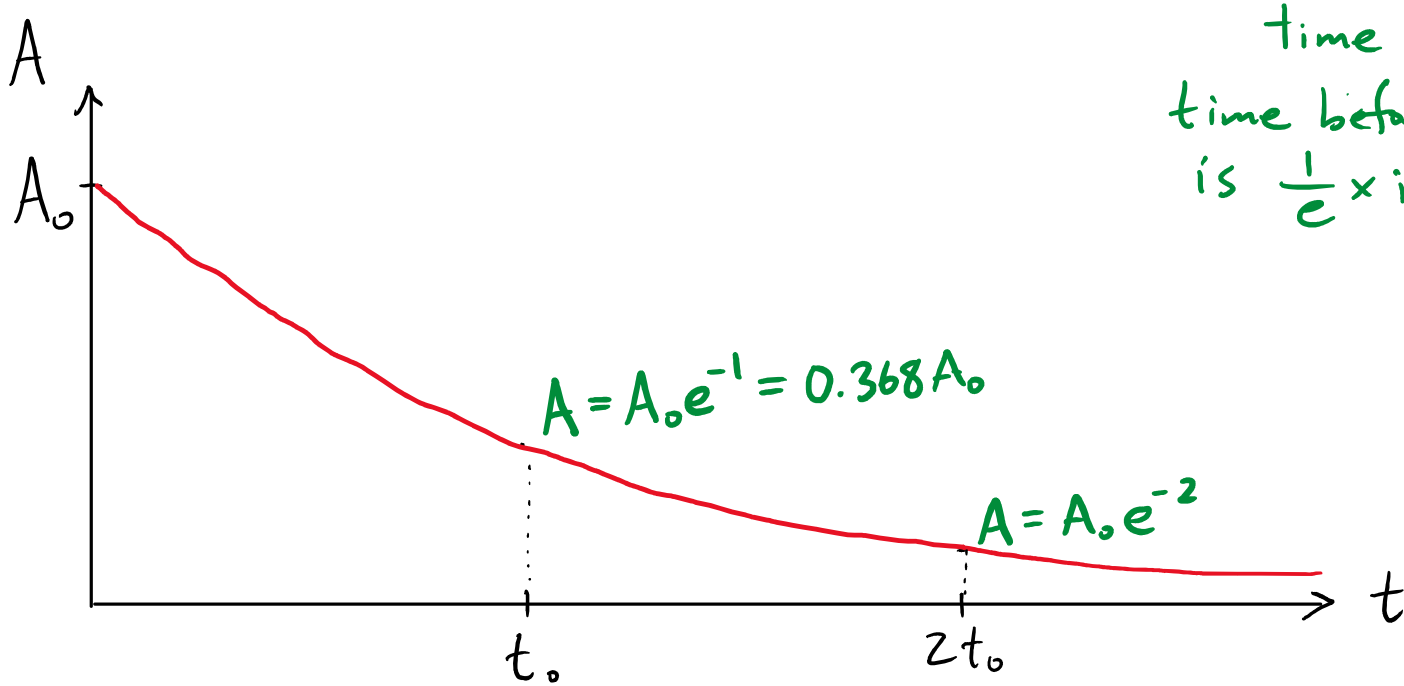


Exponential decay:

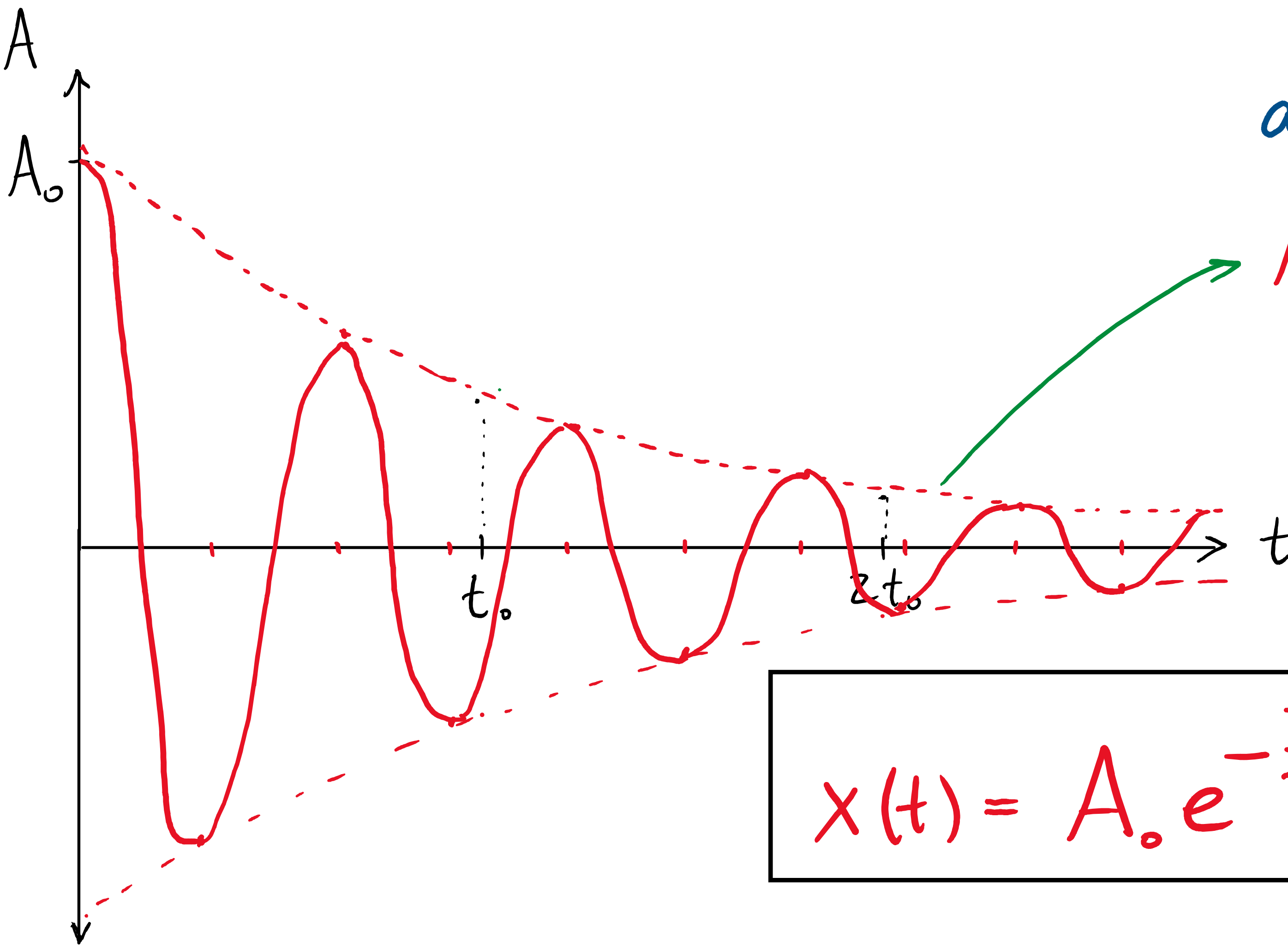
- when amplitude/energy decreases by fixed fraction each period

$$\boxed{A = A_0 e^{-t/t_0}}$$

time constant = time before amplitude is $\frac{1}{e} \times$ initial amplitude



Damped oscillations

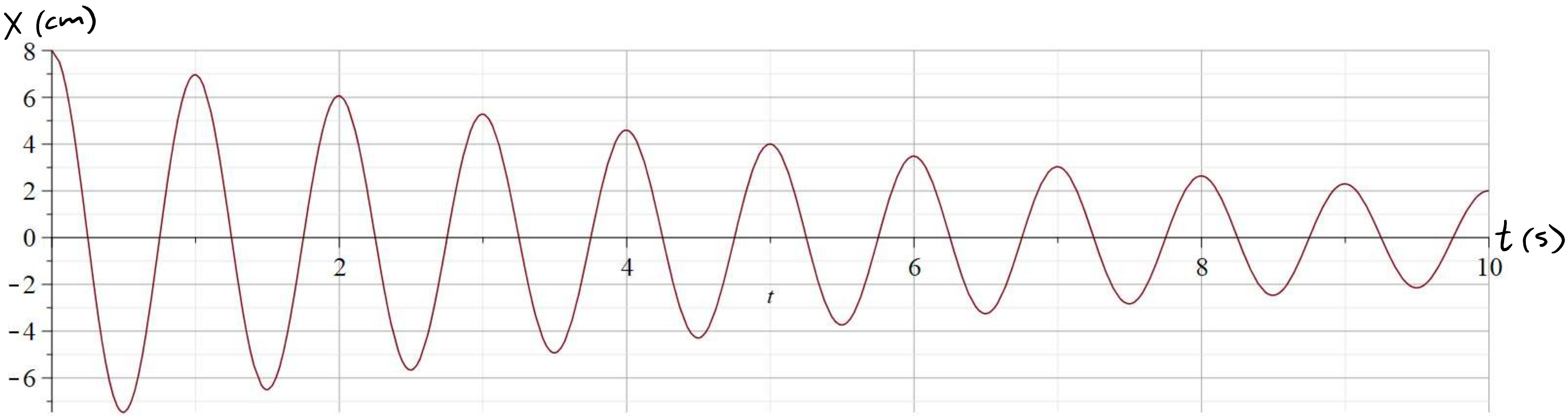


amplitude:

$$A = A_0 e^{-\frac{t}{t_0}}$$

multiplies cosine

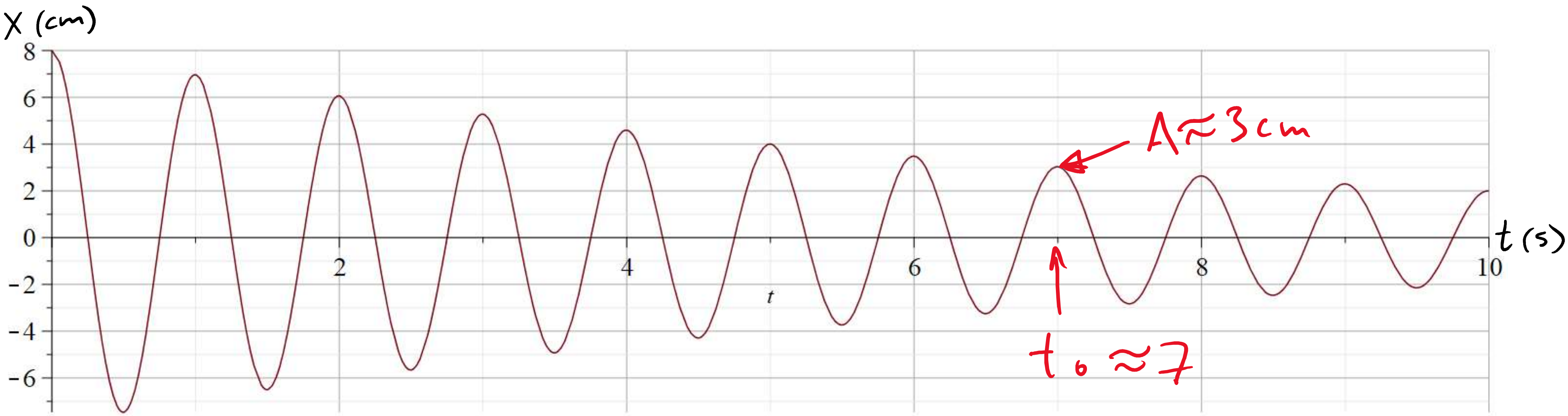
$$\boxed{x(t) = A_0 e^{-\frac{t}{t_0}} \cos(\omega t + \phi)}$$



The graph shows displacement vs time for a damped oscillation. The time constant t_0 in this case is nearest to

A) 1s   B) 3s   C) 5s   D) 7s   E)9s

EXTRA: Can you find t_0 exactly?

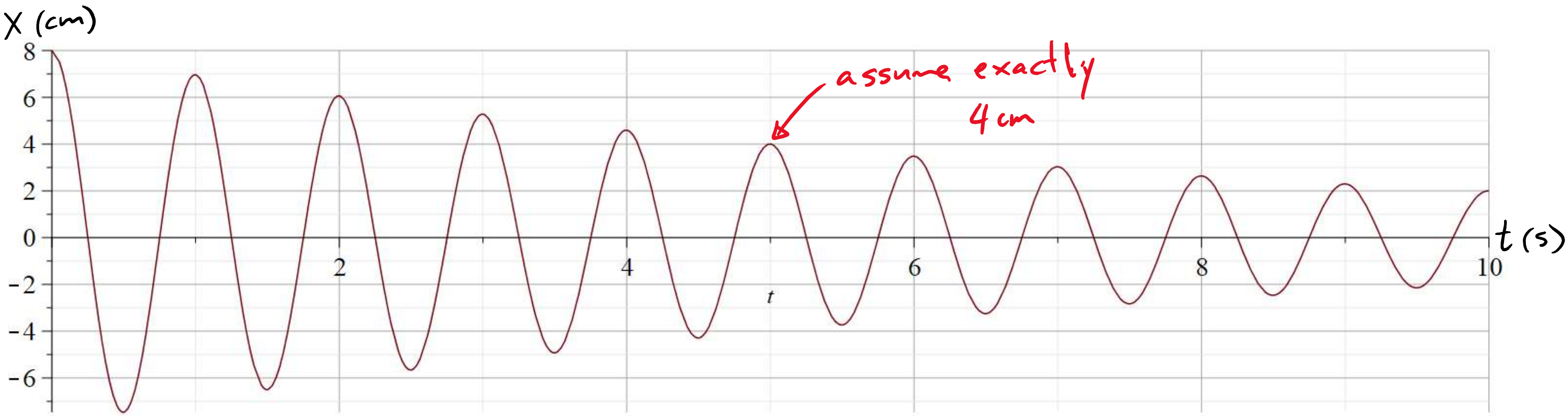


The graph shows displacement vs time for a damped oscillation. The time constant t_0 in this case is nearest to

At t_0, ampl. should be $\frac{8cm}{e} \approx \frac{8cm}{2.718} \approx 3cm$

A) 1s B) 3s C) 5s D) 7s E)9s

EXTRA: Can you find t_0 exactly?



The graph shows displacement vs time for a damped oscillation. The time constant t_0 in this case is nearest to

A) 1s B) 3s C) 5s D) 7s E)9s

EXTRA: Can you find t_0 exactly?

$A = 8 \cdot e^{-t/t_0}$

$4cm = 8 \cdot e^{-5s/t_0}$

$\Rightarrow e^{-5s/t_0} = \frac{1}{2}$

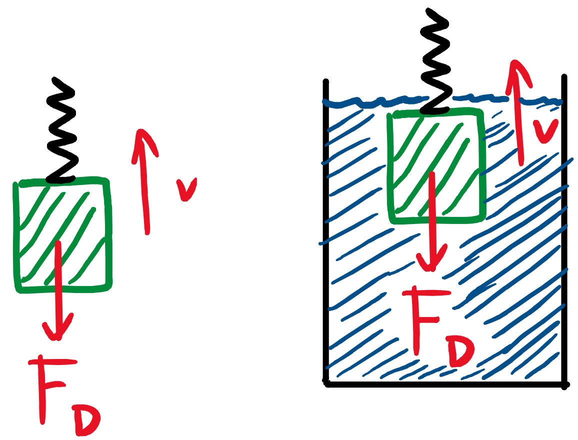
$\Rightarrow -\frac{5s}{t_0} = \ln(\frac{1}{2})$

$\Rightarrow t_0 =$

Forces that lead to damping are velocity dependent & opposite direction to velocity.

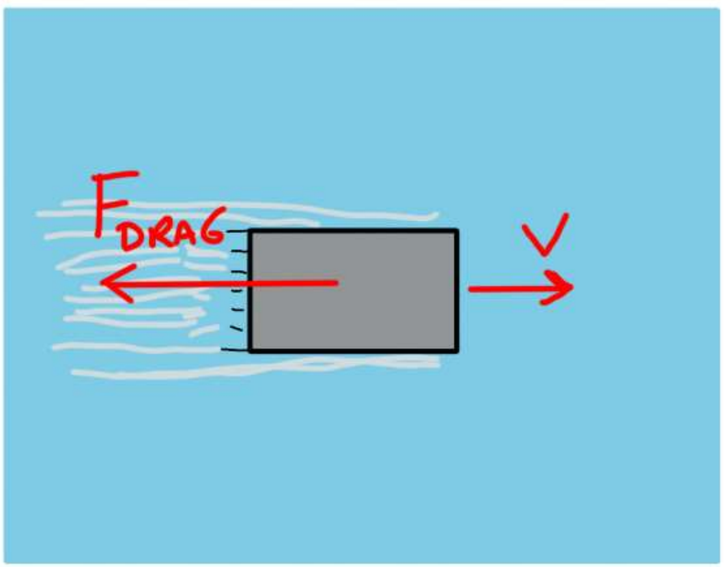
examples:

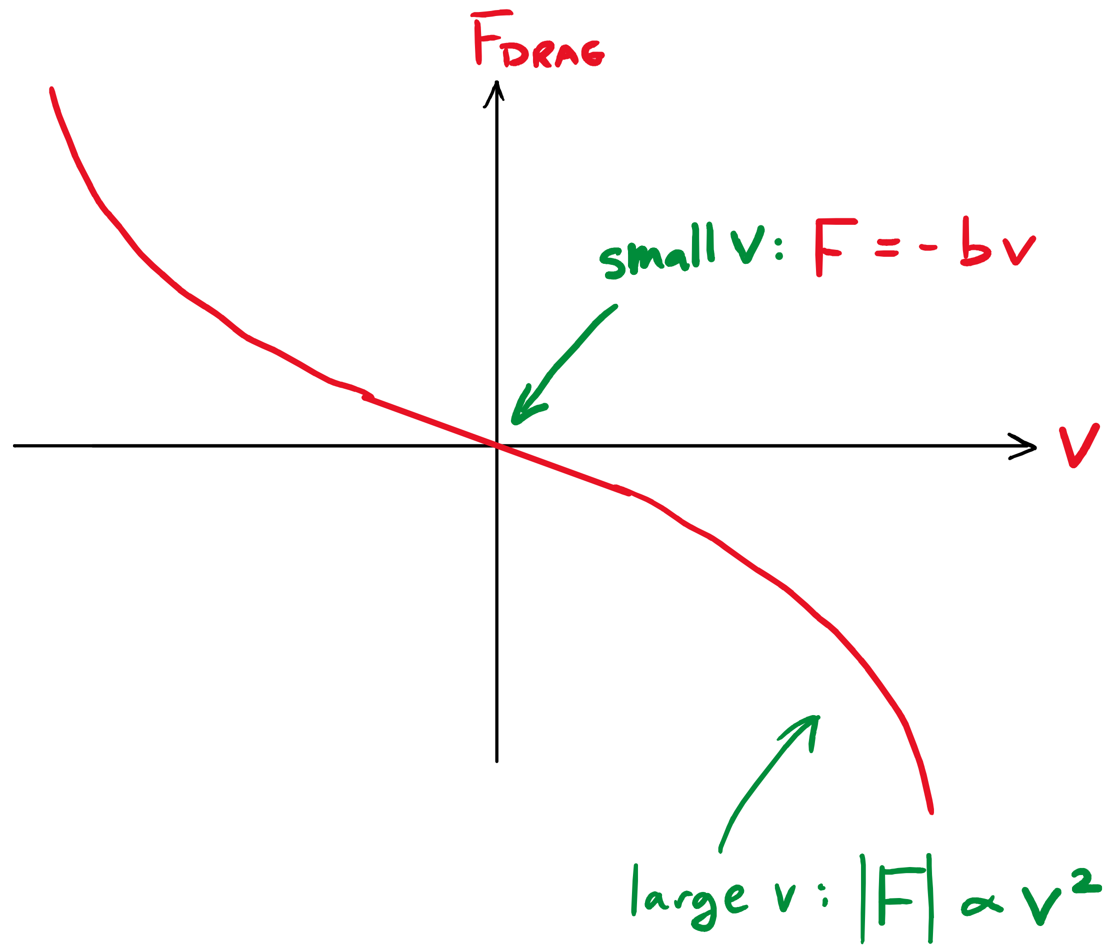
friction



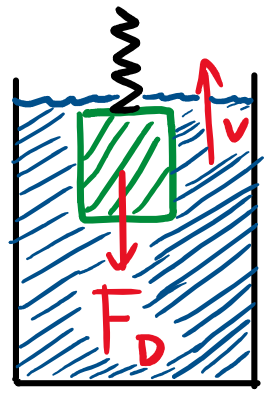
drag forces in air or fluids

Example: drag forces from air/fluids





Example: viscous fluid drag



damping constant

$$F_D = -bv$$

$$F_{NET} = -kx - bv$$

Equations of motion:

$$\frac{dx}{dt} = v$$

This is $a = \frac{F}{m}$

$$\frac{dv}{dt} = -\frac{k}{m}x - \frac{b}{m}v$$

use these to predict how x and v change with time

$$\frac{dx}{dt} = v \qquad \frac{dv}{dt} = -\frac{k}{m}x - \frac{b}{m}v$$

Solution is:

$$x(t) = A_0 e^{-\frac{t}{t_0}} \cos(\omega t + \phi)$$

check: calculate $v = \frac{dx}{dt}$ and then verify 2nd eqn.

$$t_0 = \frac{2m}{b} \qquad \omega = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

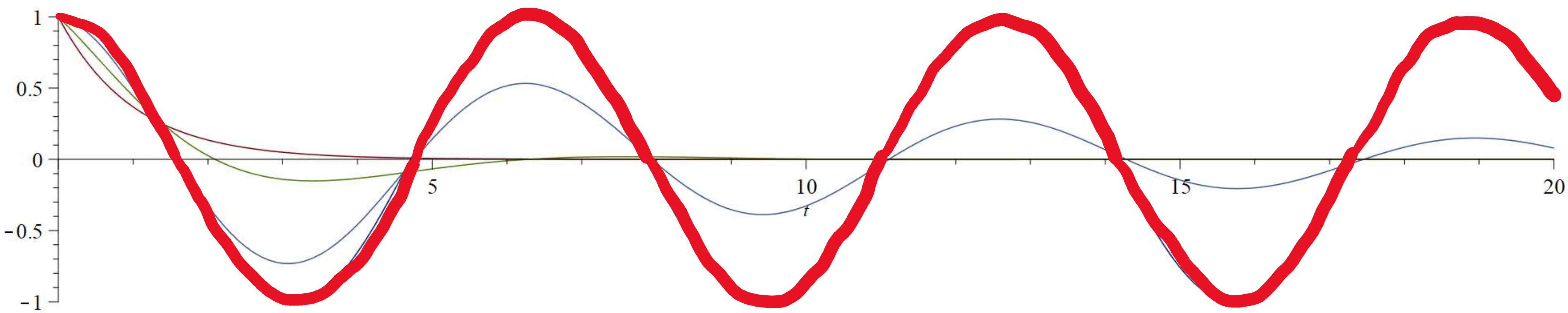
can usually ignore

valid for $b < 2\sqrt{km}$

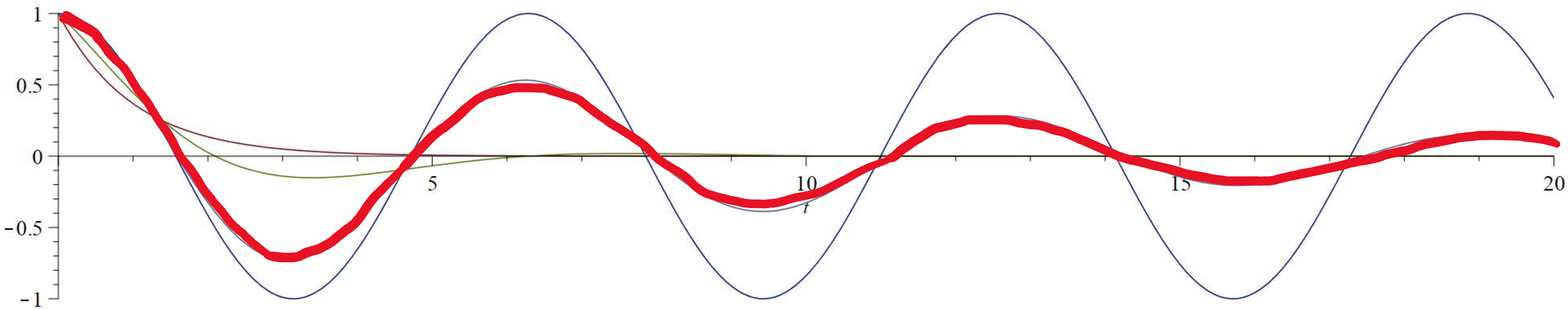
Simulation or demo of damped oscillation

https://youtu.be/-ALSLYnSOYE

no damping (idealized situation)

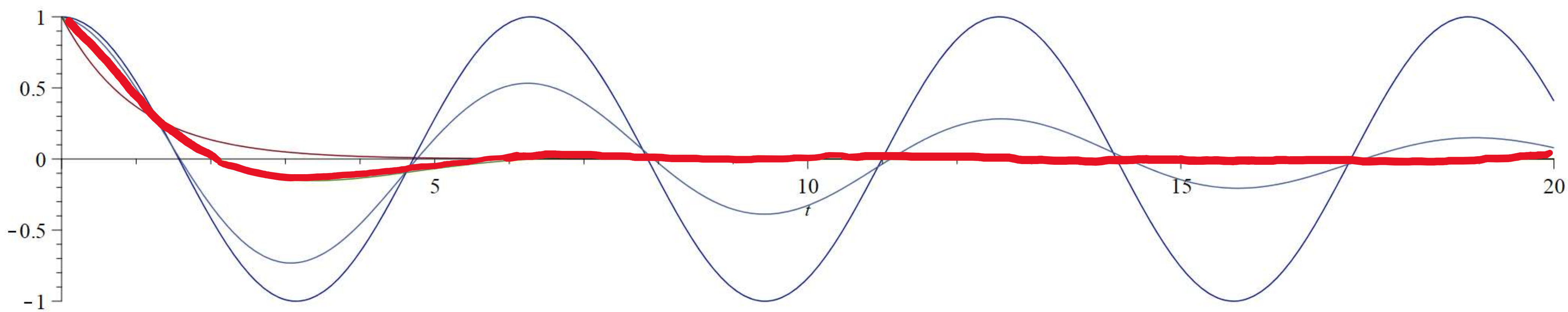


b=0



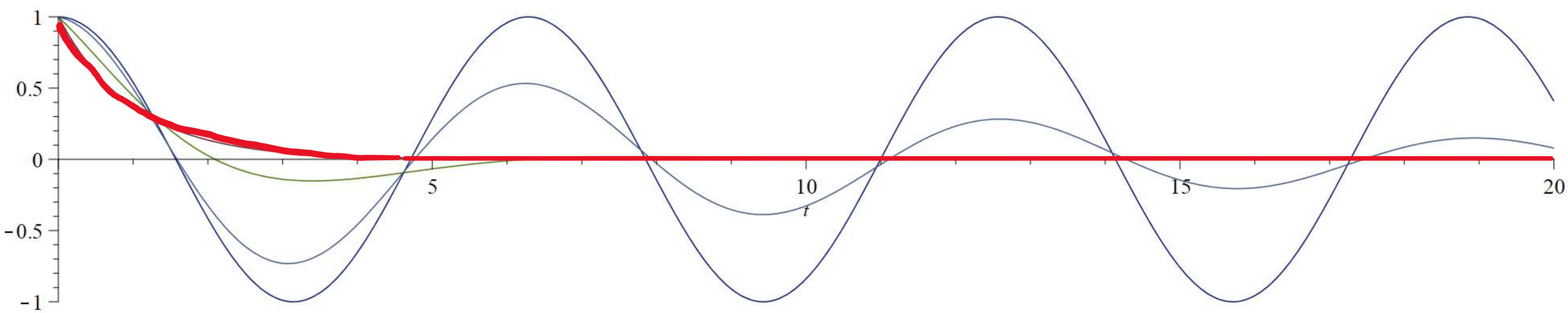
$b = 0.1 \times 2\sqrt{km}$

still have $\omega \approx \omega_{b=0} = \sqrt{\frac{k}{m}}$



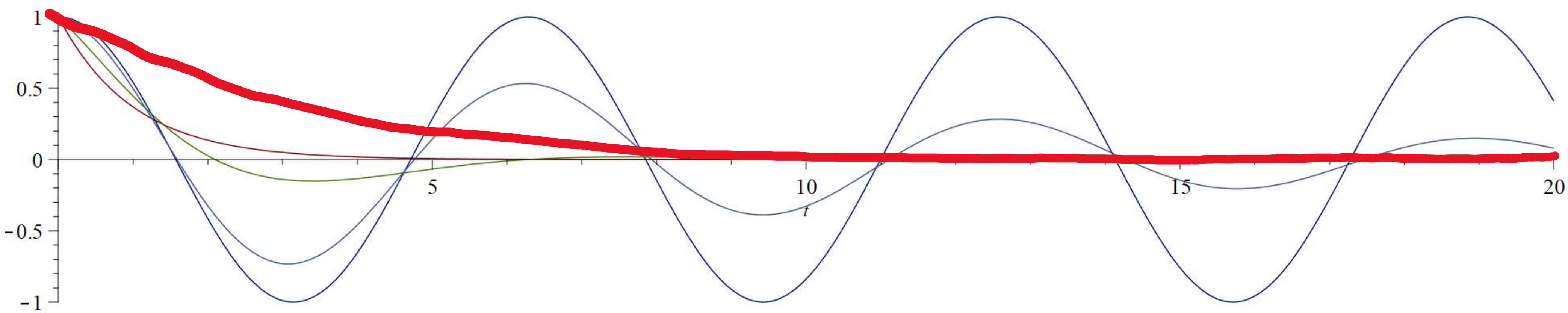
$b = 0.5 \times 2\sqrt{km}$

Critical damping

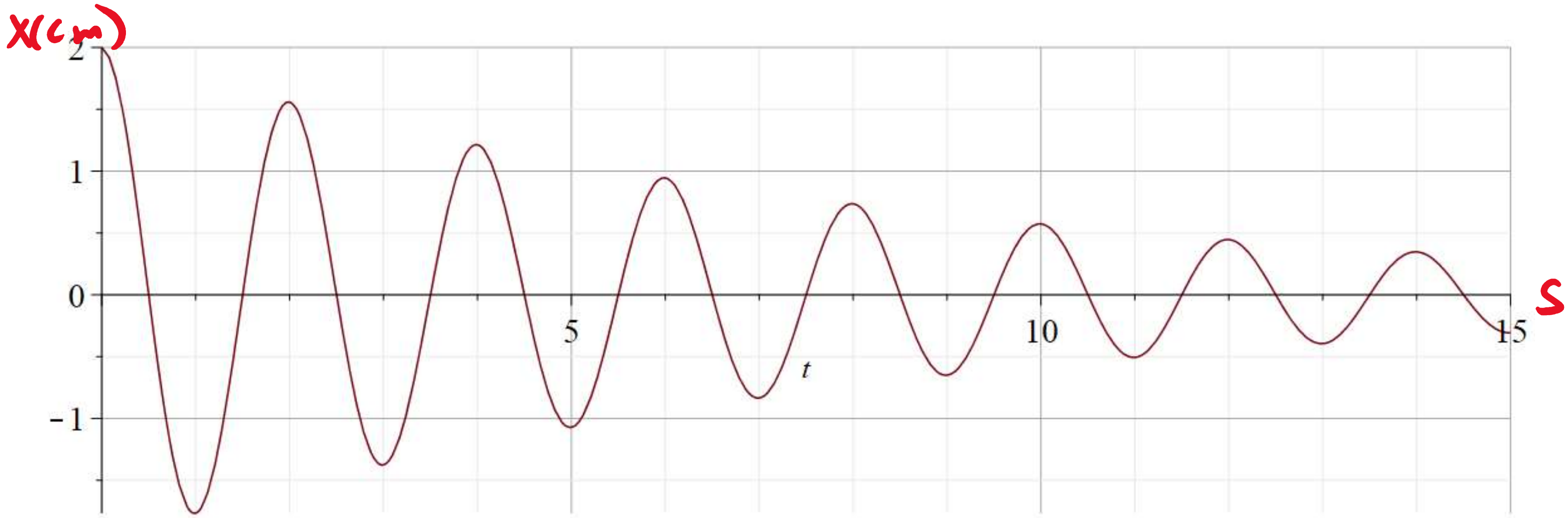


$b = 2\sqrt{km} \Rightarrow \omega = 0$ pure decay, no oscillations

Overdamping: $b > 2\sqrt{km}$



also exponential decay, but slower to reach equilibrium than critical damping



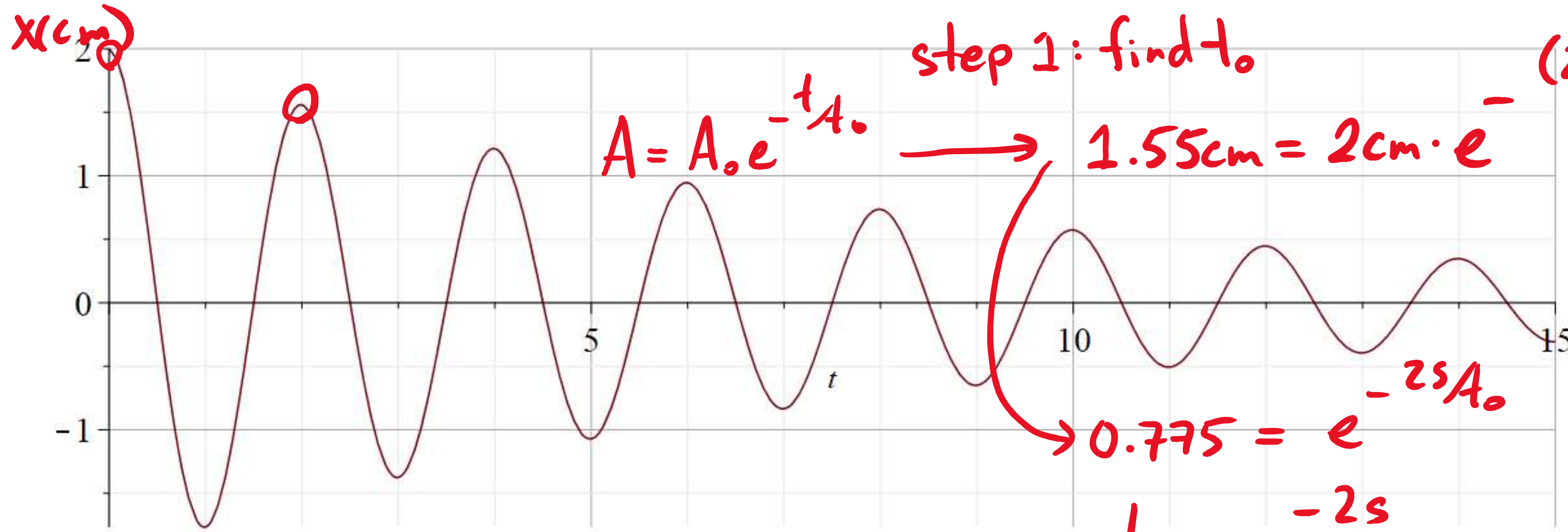
An object with mass 2kg oscillates on a spring with a small amount of damping.

a) What is the damping constant b?

EXTRA: What is the spring constant k?

$$t_0 = \frac{2m}{b}$$

$$\omega = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$



step 1: find t_0

$$A = A_0 e^{-t/t_0} \rightarrow 1.55\text{cm} = 2\text{cm} \cdot e^{-(2s)/t_0}$$

$$\rightarrow 0.775 = e^{-2s/t_0}$$

$$t_0 = \frac{-2s}{\ln(0.775)} = 7.85s$$

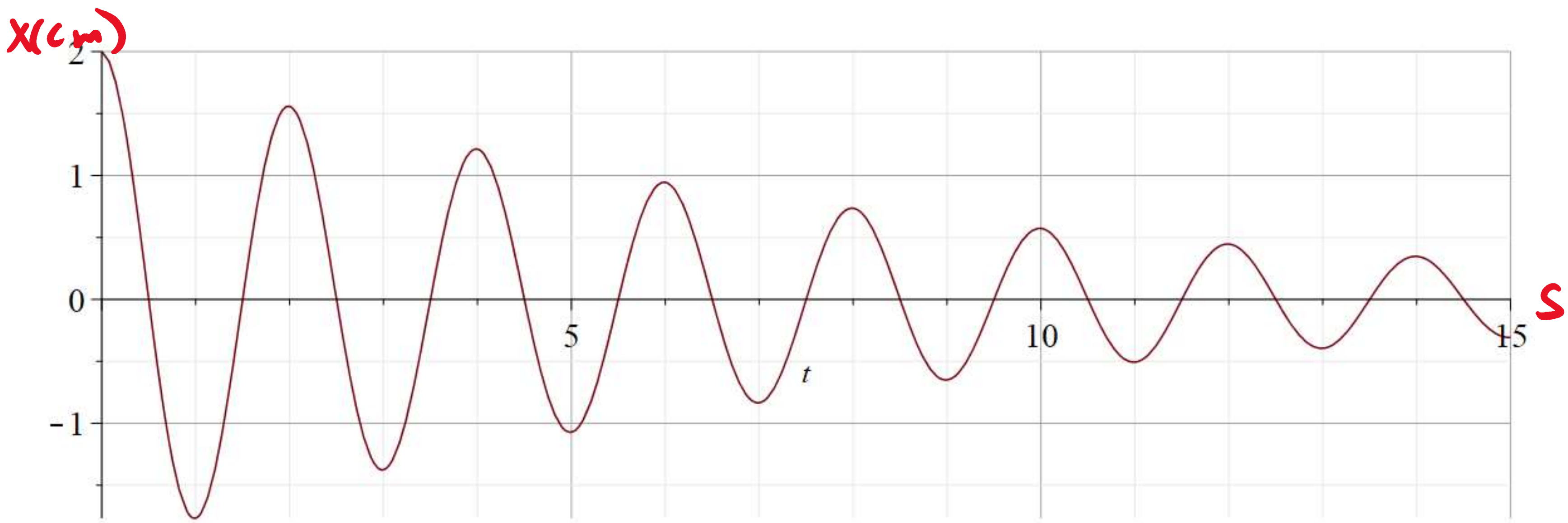
An object with mass 2kg oscillates on a spring with a small amount of damping.

a) What is the damping constant b?

step 2: find b:

$$t_0 = \frac{2m}{b} \Rightarrow b = \frac{2m}{t_0} = \frac{4kg}{7.85s} = 0.51\,\frac{kg}{s}$$

EXTRA: What is the spring constant k?



An object with mass 2kg oscillates on a spring with a small amount of damping.

a) What is the damping constant b?

EXTRA: What is the spring constant k?

step 1: find ω $\quad T = 2\,s$ so $\omega = \frac{2\pi}{T} = 3.1\,s^{-1}$

step 2: use $\omega = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$

$\Rightarrow k = m\omega^2 + \frac{b^2}{4m}$

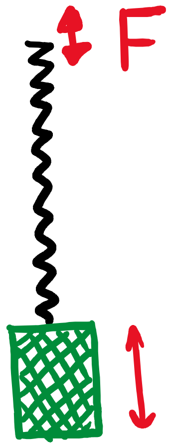
$= 19.2\,\frac{N}{m} + 0.03\,\frac{N}{m} \approx 19.2\,\frac{N}{m}$

we can ignore these

★ accurate to just use $\omega = \sqrt{\frac{k}{m}}$ unless highly damped.

FORCED OSCILLATIONS:

We can add in an oscillating force by hand:

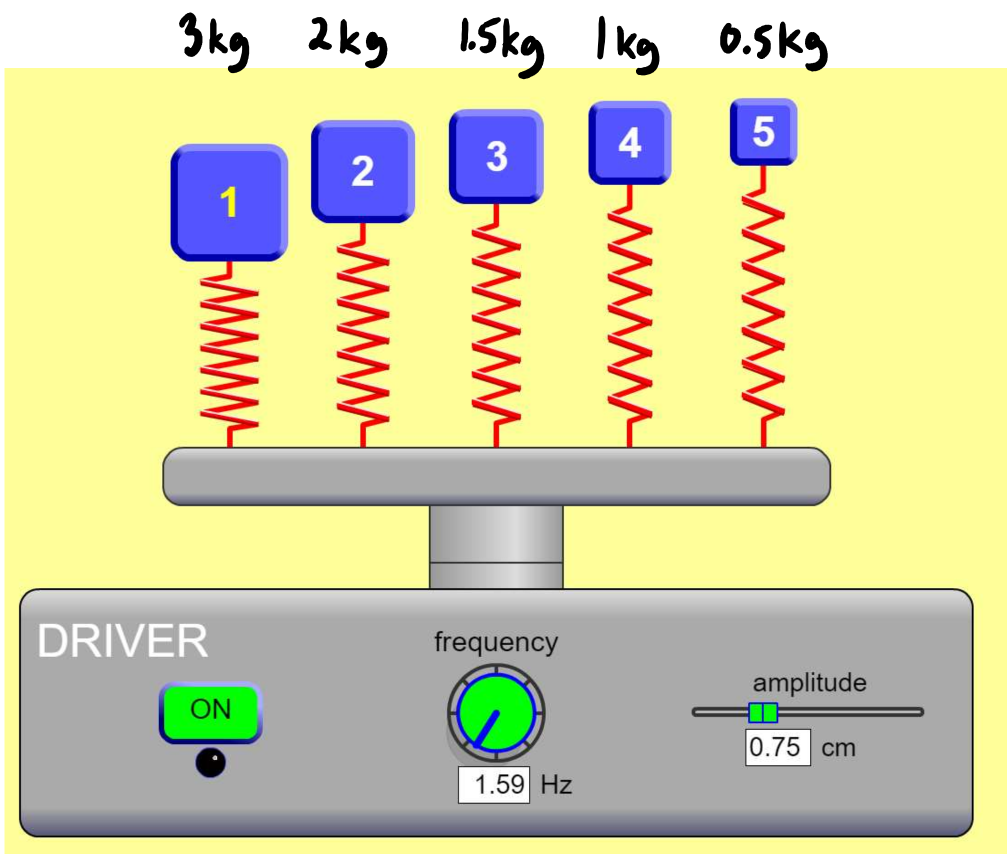


$F = F_0 \cos(\omega_D t)$

driving frequency: we choose this

- object will end up oscillating at driving frequency but ★ amplitude largest if ω_D matches $\omega_0 = \sqrt{\frac{k}{m}}$

- This is RESONANCE



Objects with the masses shown each sit in equilibrium on different springs, all with spring constant 200 N/m and damping constant 1 kg/s. If we turn on a driving force with frequency f = 1.59 s^{-1}, which mass will oscillate with the largest amplitude?

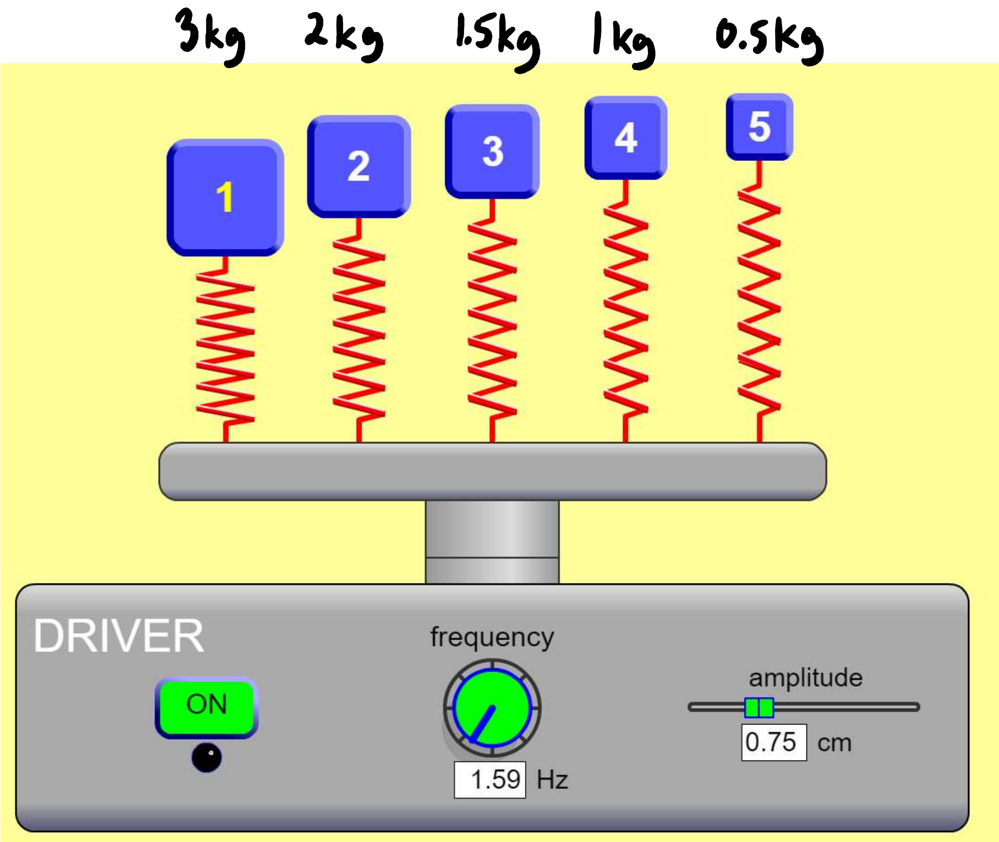
A) 1

B) 2

C) 3

D) 4

E) 5



Objects with the masses shown each sit in equilibrium on different springs, all with spring constant 200 N/m and damping constant 1 kg/s. If we turn on a driving force with frequency f = 1.59 s^{-1}, which mass will oscillate with the largest amplitude?

A) 1

B) 2

C) 3

D) 4

E) 5

Have $f_D = 1.59\ s^{-1}$

so $\omega_D = 2\pi f_D \approx 10\ s^{-1}$

Want this to match

$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{200\ N/m}{m}}$

so $m = 2$ kg works.