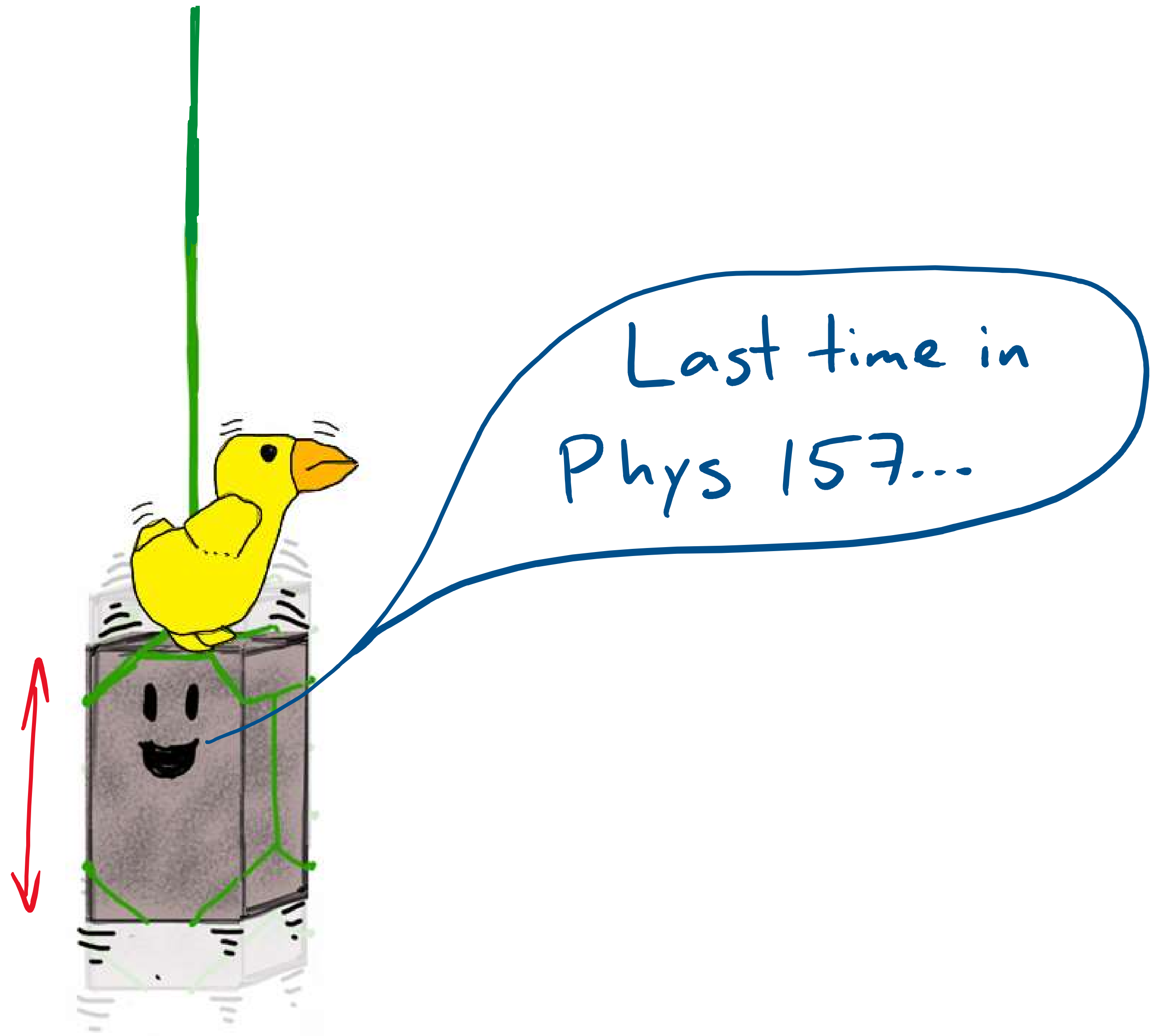
Office hours today: after class (Remo)
4-5pm, 8-9pm (Zoom)
Remo homework sessions: 5-8pm Monday and Tuesday

Learning goals for today:

- To qualitatively and quantitatively relate the position vs time, velocity vs time, and acceleration vs time for simple harmonic motion.
- To deduce the constant k characterizing the restoring forces based on the observations of simple harmonic motion given the mass.


Last time in
Phys 157...

Simple Harmonic Motion

satisfies $\frac{d^2x}{dt^2} = -\omega^2 x$

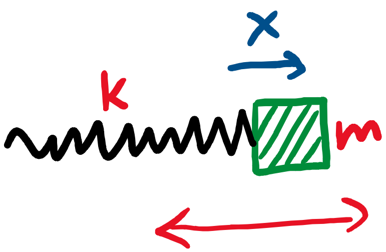
$\frac{k}{m}$

$$x(t) = A\cos(\omega t + \phi)$$

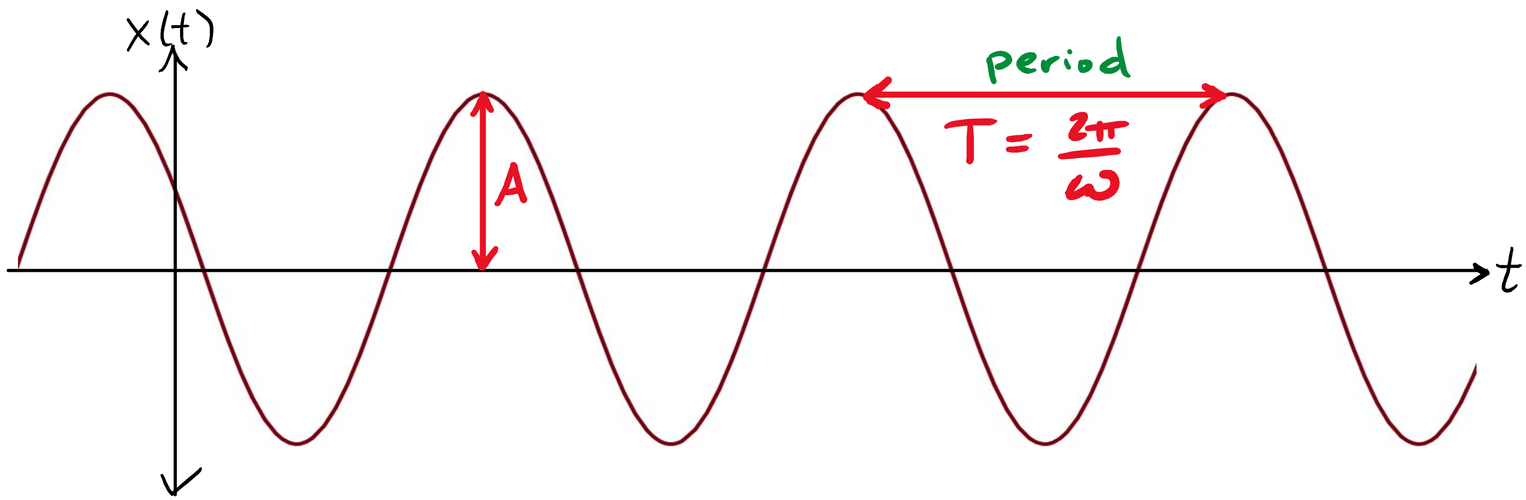
Amplitude

angular frequency

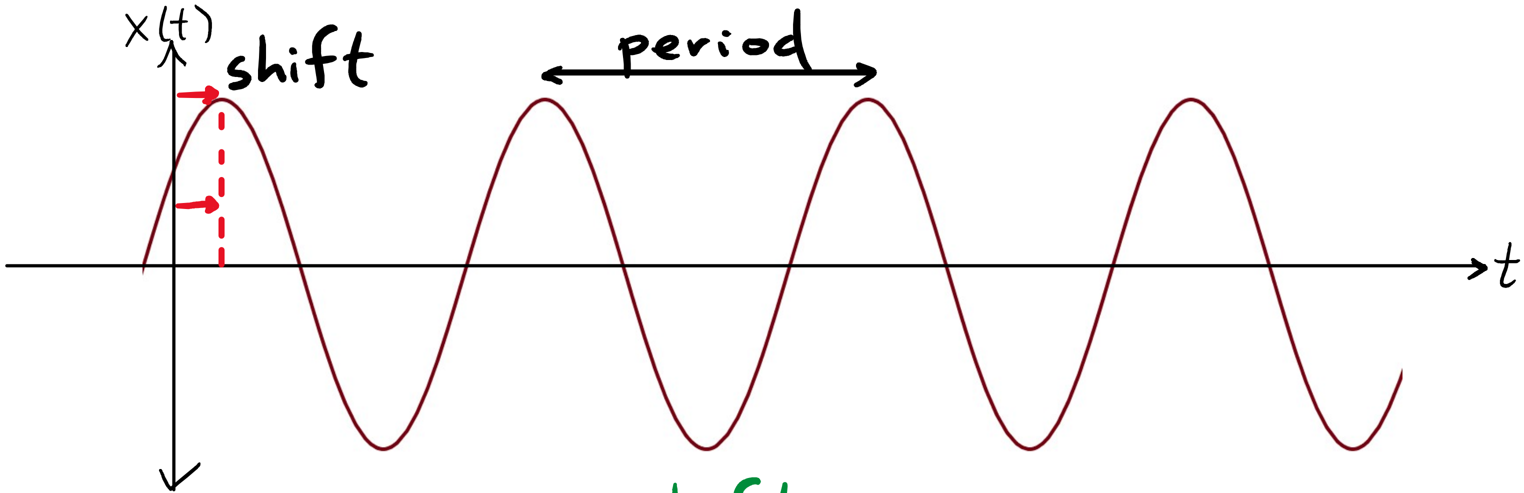
phase



$$\omega = \sqrt{\frac{k}{m}}$$



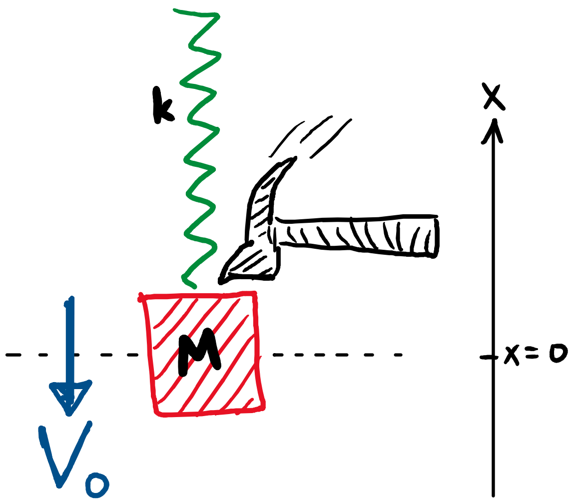
How to find ϕ



to the left

$$\phi = \pm\, 2\pi \times \frac{\text{shift}}{\text{period}}$$

to the right

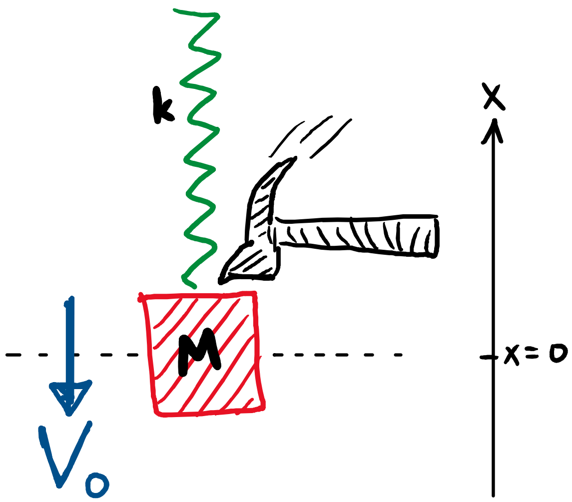


A mass on a spring is struck with a hammer, giving it an initial downward velocity when it is at its equilibrium position. Which of the following functions could describe the motion of the mass?

A) $x(t) = A\cos(\omega t - \pi/2)$
B) $x(t) = A\cos(\omega t)$
C) $x(t) = A\cos(\omega t + \pi/2)$
D) $x(t) = A\cos(\omega t + \pi)$
E) None of the above

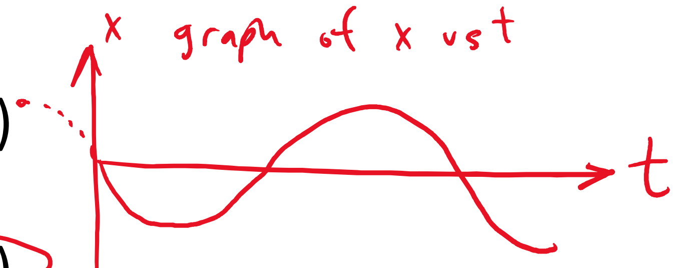
Hint: sketch the graph of $x(t)$

EXTRA: can you determine A in terms of v_0 and ω?



A mass on a spring is struck with a hammer, giving it an initial downward velocity when it is at its equilibrium position. Which of the following functions could describe the motion of the mass?

graph of x vs t



A) $x(t) = A\cos(\omega t - \pi/2)$
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C) $x(t) = A\cos(\omega t + \pi/2)$
D) $x(t) = A\cos(\omega t + \pi)$
E) None of the above

initial position: $x=0$

after $t=0$, x decreases until some min. value, then comes back up to $x=0$ but with +ve velocity. So x then increases above 0, etc..

Hint: sketch the graph of $x(t)$

like cosine, but shifted to left by $\frac{1}{4}$ period.

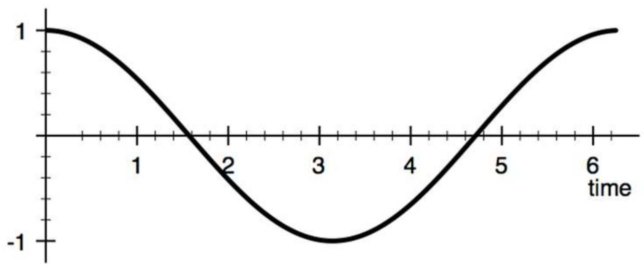
$\therefore \phi = +\frac{1}{4} \times 2\pi$

$= \frac{\pi}{2}$

EXTRA: can you determine A in terms of v_0 and ω?

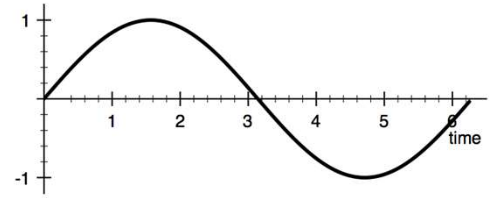
↳ velocity is $\frac{dx}{dt} = -A\omega \sin(\omega t + \frac{\pi}{2})$. At $t=0$, $v = -v_0$, so: $-v_0 = -A\omega\sin(\frac{\pi}{2}) \Rightarrow A = \frac{v_0}{\omega}$

Velocity vs displacement

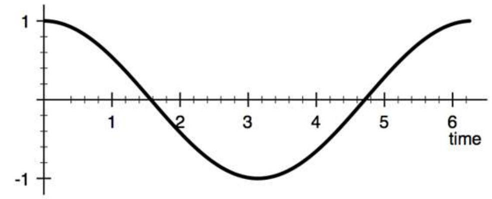


A plot of displacement as a function of time for an oscillator is shown above. Which of the diagrams to the right describes the ***velocity*** as a function of time for the same motion?

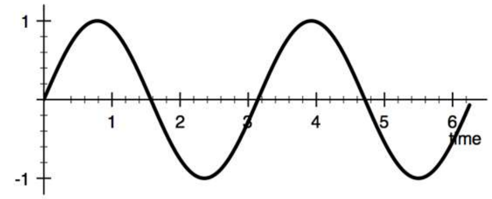
A.



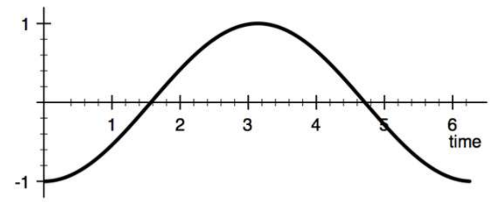
B.



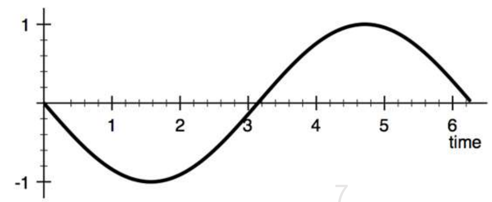
C.



D.



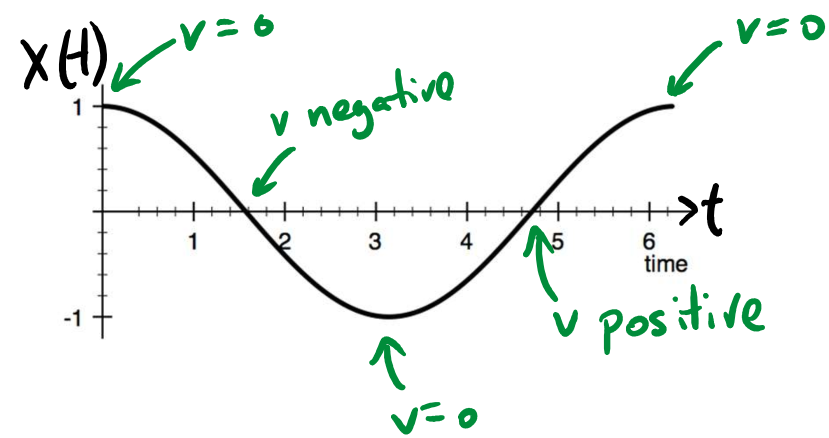
E.

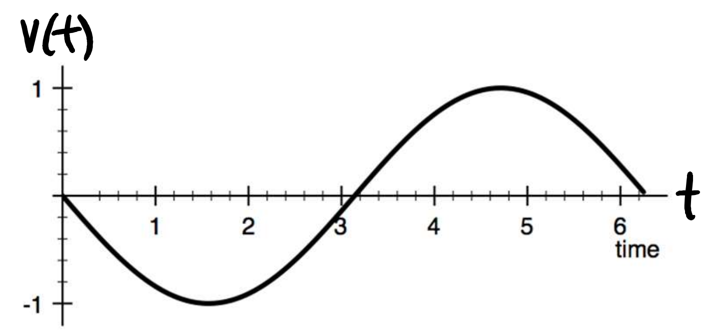


Velocity from displacement:

$$v = \frac{dx}{dt}$$

v(t) = slope of x(t) at time t

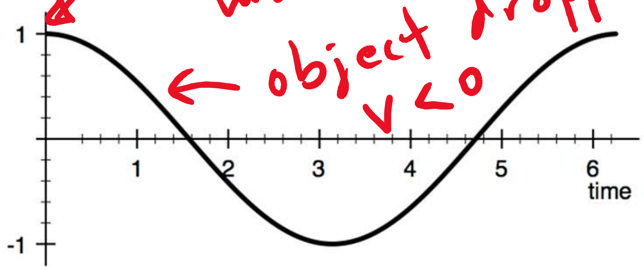




Velocity vs displacement

object at top of motion here, so v=0

object dropping, so v<0



A plot of displacement as a function of time is shown above. Which of the diagrams to the right describes the ***velocity*** as a function of time for the same motion?

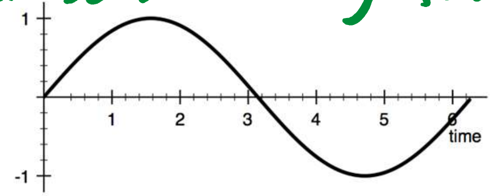
other method:

$$v = \frac{dx}{dt}$$

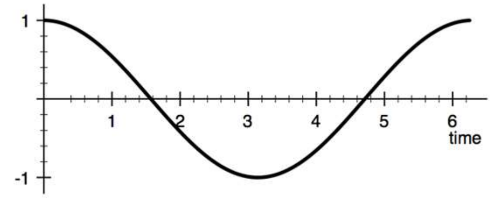
= slope of graph

because switching from v>0 to v<0

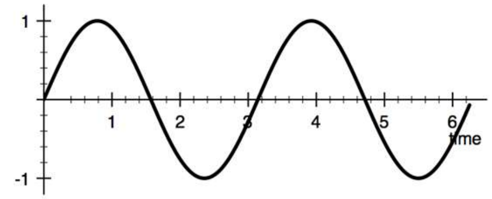
A.



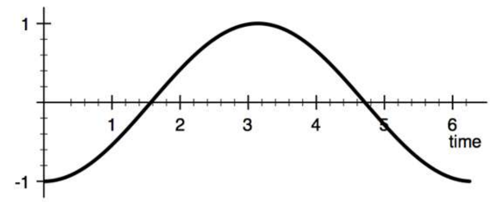
B.



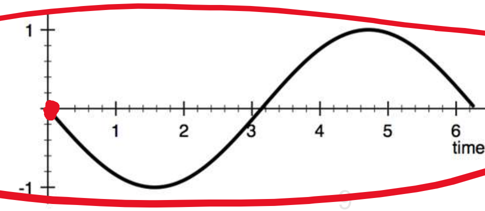
C.



D.

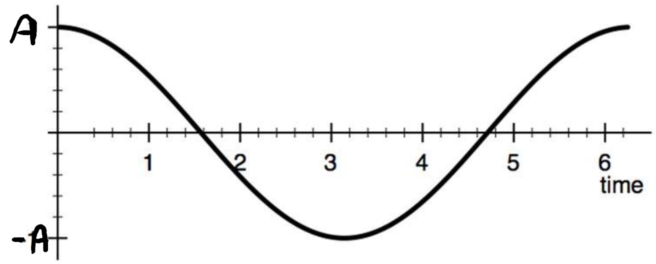


E.



v starts at 0 and goes negative

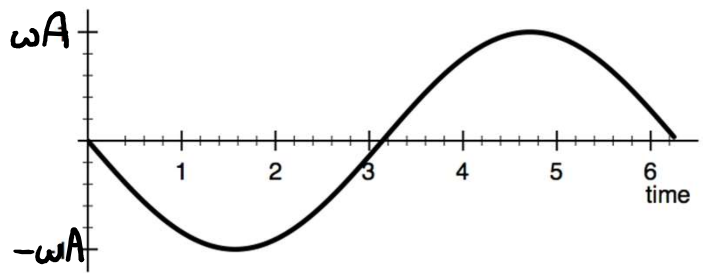
Position:



$x(t) = A\cos(\omega t + \phi)$

$\big\downarrow \frac{d}{dt}$ (slope)

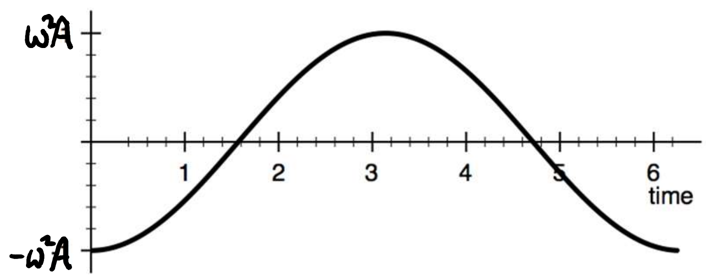
Velocity:



$v(t) = -A\omega\sin(\omega t + \phi)$

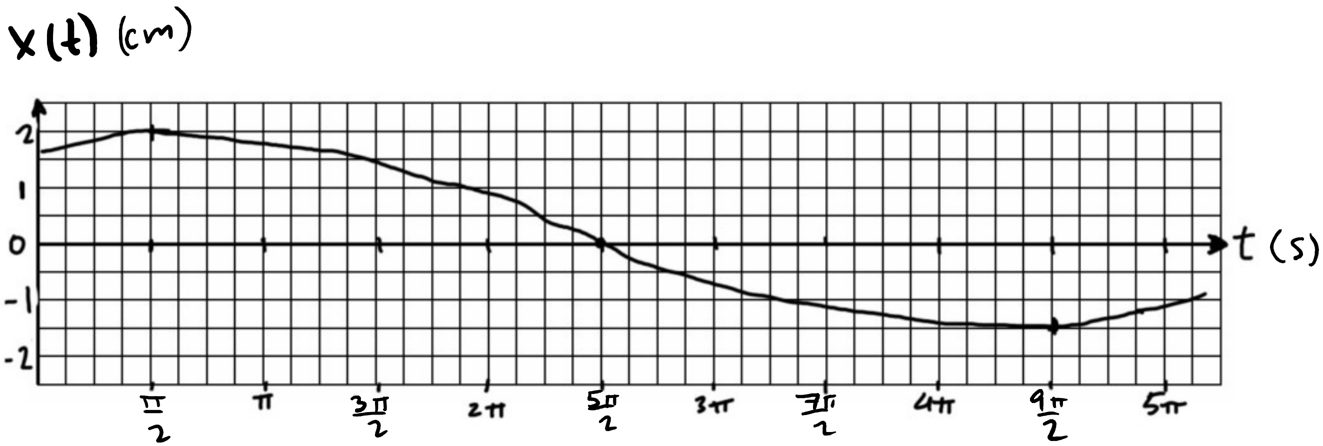
$\big\downarrow \frac{d}{dt}$ (slope)

Acceleration:



$$a(t) = -A\omega^2\cos(\omega t + \phi) = -\omega^2 x(t)$$

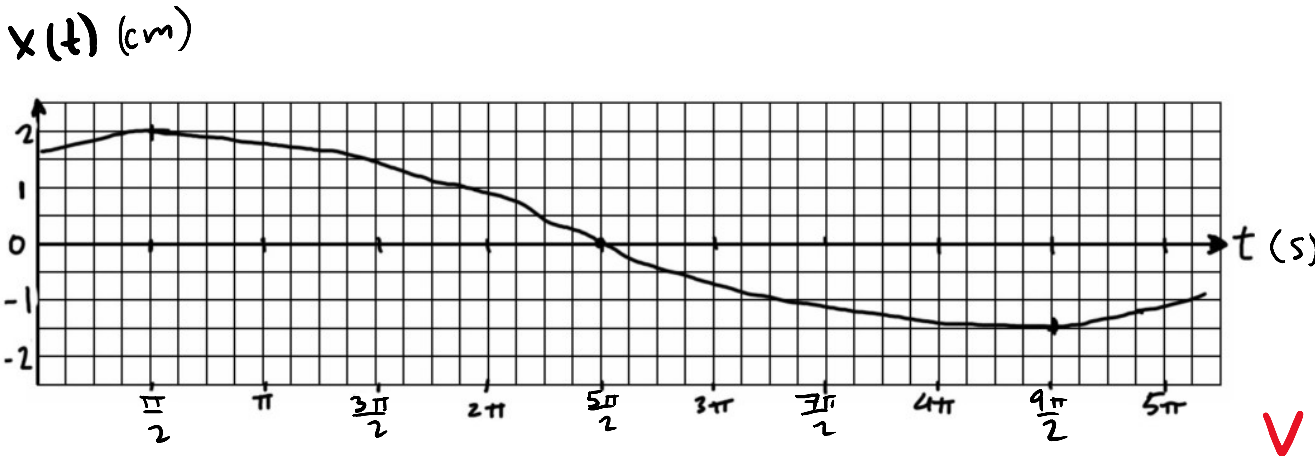
$$\omega = \frac{2\pi}{T}$$



$$x(t) = A\cos(\omega t + \phi)$$

For the displacement graph shown, what is the maximum magnitude of velocity, in cm/s?

A) 4 B) 2 C) 1 D) 1/2 E) 1/4



$$\omega = \frac{2\pi}{T}$$

$$v = \frac{dx}{dt}$$

$$x(t) = A\cos(\omega t + \phi)$$

$$= -A\omega \sin(\omega t + \phi)$$

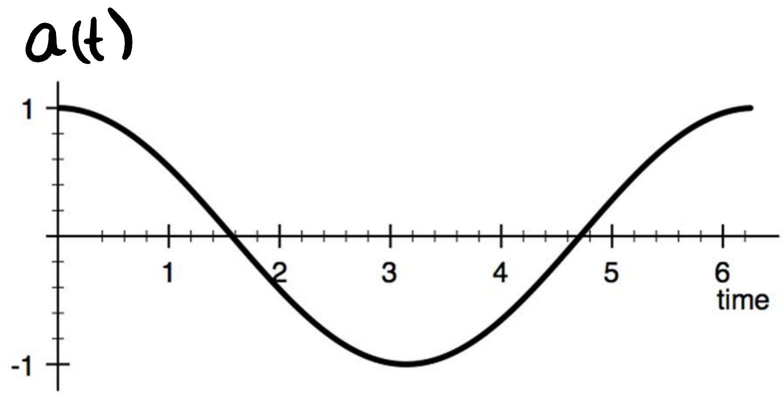
For the displacement graph shown, what is the maximum magnitude of velocity, in cm/s?

sin goes from -1 to 1, so max value of v is $A\omega$

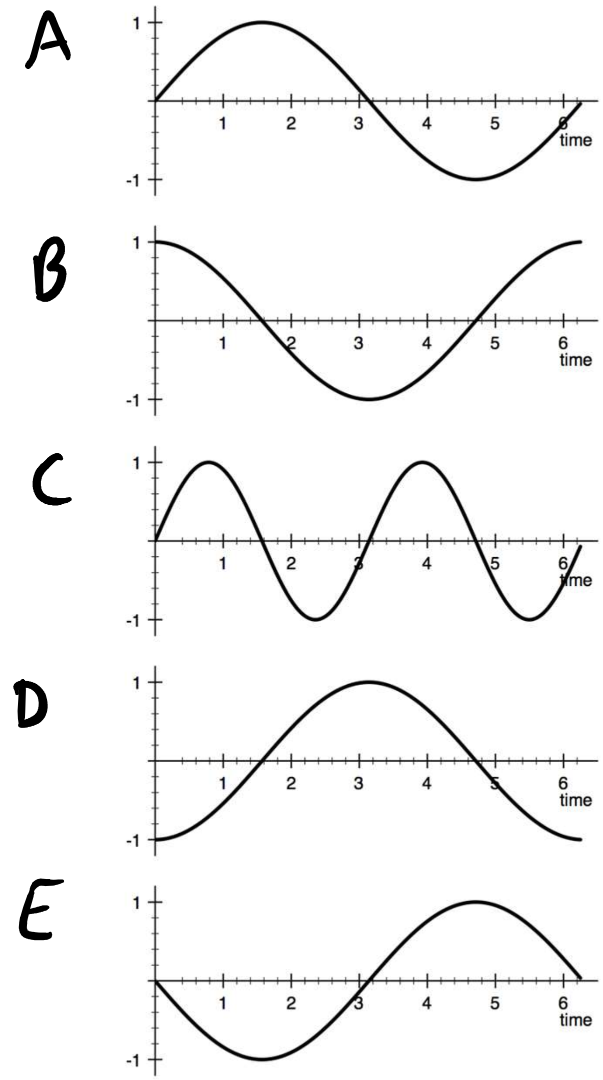
A) 4 B) 2 C) 1 D) 1/2 E) 1/4

$A = 2\,\text{cm}$, $\omega = \frac{2\pi}{T} = \frac{2\pi}{8\pi} = \frac{1}{4}$ $A\omega = \frac{1}{2}$

Acceleration vs displacement:

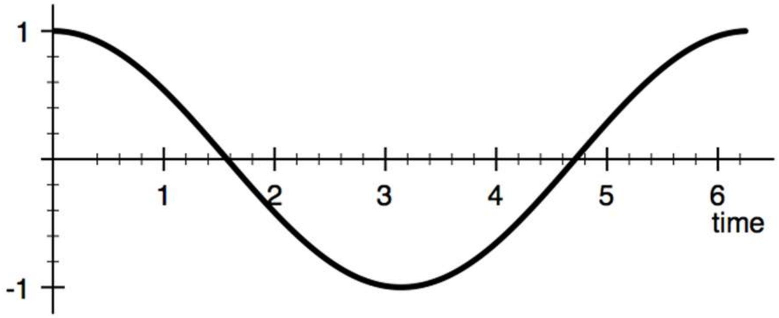


A plot of upward ***acceleration*** (in cm/s) as a function of time (in s) is shown above for a mass hanging from a spring. Which of the pictures to the right could represent x(t)?



Acceleration vs displacement:

a(t)



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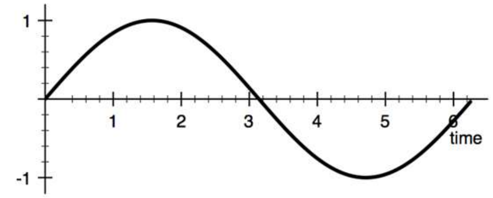
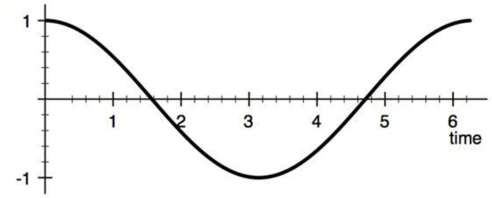
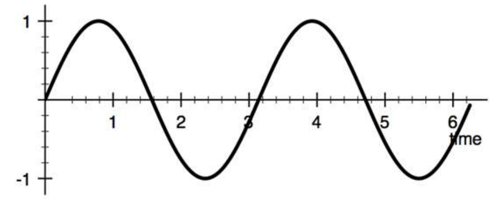


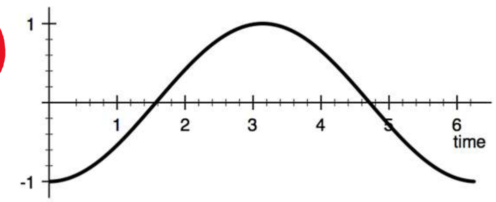
B



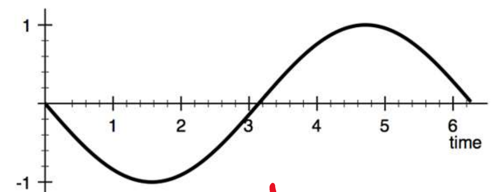
C



D

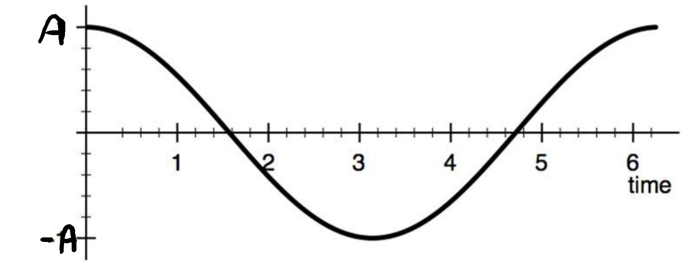


E



Have $a = -\omega^2 x$ for SHM, so x is maximum when a is minimum

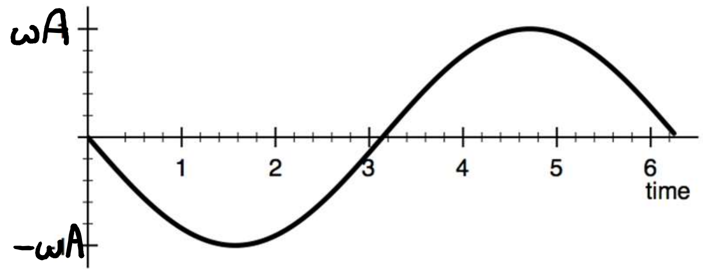
Position:



$$x(t) = A\cos(\omega t + \phi)$$

$$\Big\downarrow \frac{d}{dt} \quad \text{(slope)}$$

Velocity:



$$v(t) = -A\omega \sin(\omega t + \phi)$$

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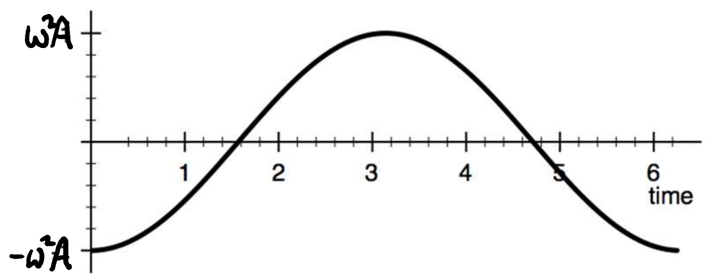
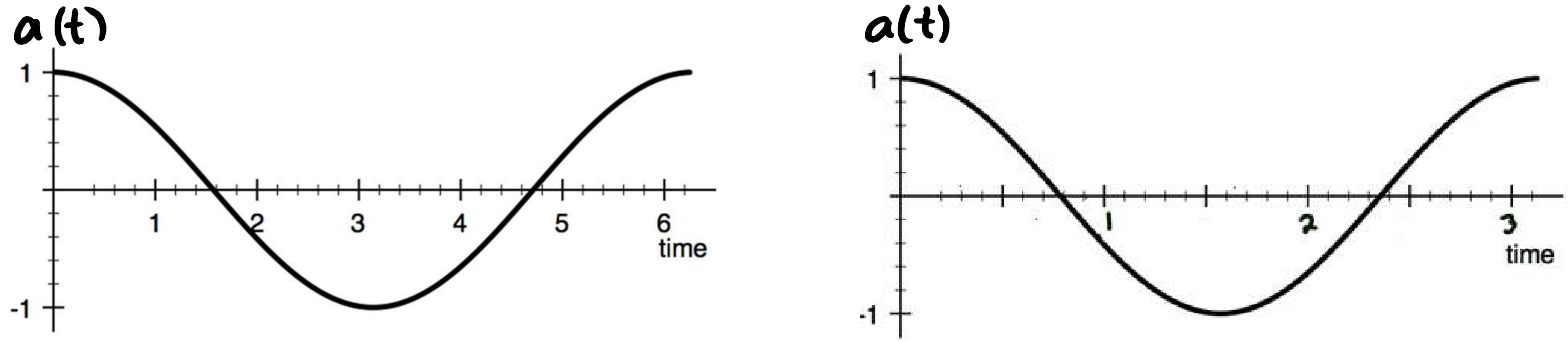


$$a(t) = -A\omega^2 \cos(\omega t + \phi)$$

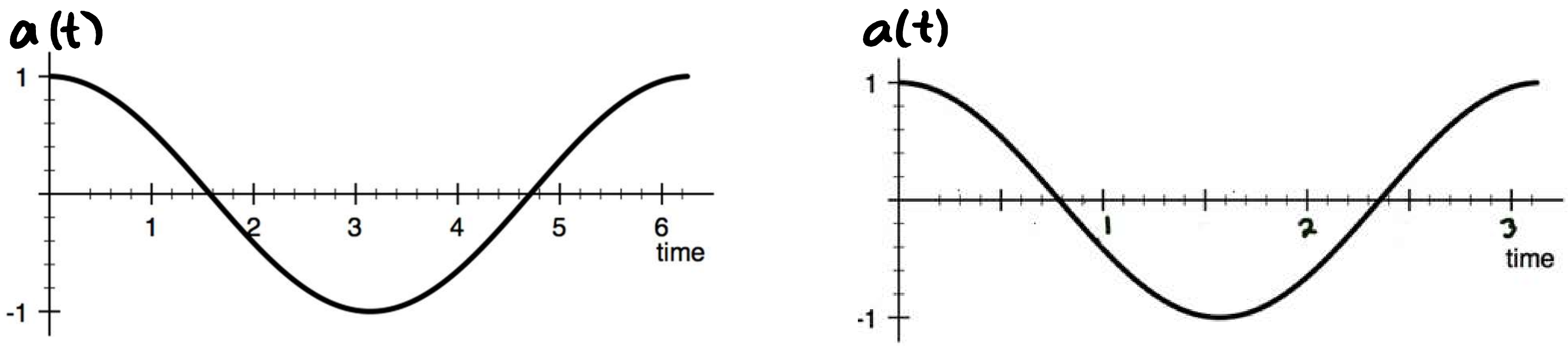
$$\parallel$$

$$-\omega^2 x(t)$$



The graphs show **acceleration** as a function of time for two different harmonic oscillators. The amplitude of the **displacement** in the first case is 1cm. For the second oscillator, the amplitude of the **displacement** is

A) 4cm B) 2cm C) 1cm D) 0.5 cm E) 0.25 cm



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A) 4cm B) 2cm C) 1cm D) 0.5 cm E) 0.25 cm

Have $a = -\omega^2 x$, so $x = -\frac{a}{\omega^2}$. T is half in 2nd case so ω is double, so amplitude of x is $\frac{1}{4}$

$\phi = \pm 2\pi \frac{t_{max}}{T}$ $\omega = \sqrt{\frac{k}{m}}$ $x(t) = A\cos(\omega t + \phi)$ $\omega = \frac{2\pi}{T}$

Approximately what is the spring constant of the spring in the simulation?

see: https://youtu.be/PD30ieYknac

A) 1 N/m

B) 2 N/m

C) 4 N/m

D) 8N/m

E)16N/m

$\phi = \pm 2\pi \frac{t_{max}}{T}$ $\omega = \sqrt{\frac{k}{m}}$ $x(t) = A\cos(\omega t + \phi)$ $\omega = \frac{2\pi}{T}$

Approximately what is the spring constant of the spring in the simulation?

A) 1 N/m

B) 2 N/m

C) 4 N/m

D) 8N/m

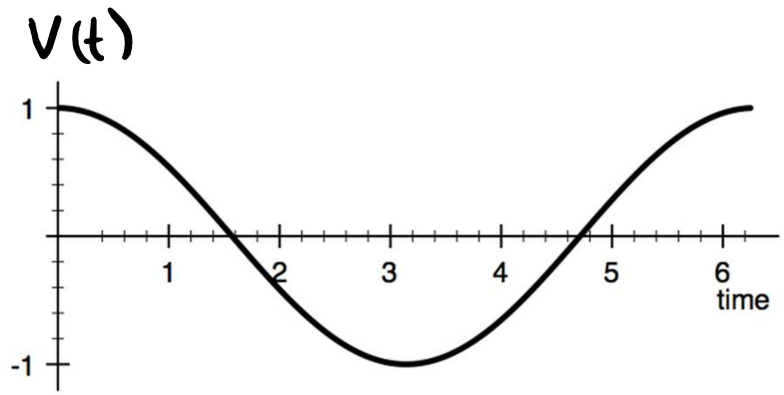
E)16N/m

have: $T = 1.5s$ so $\omega = \frac{2\pi}{T} \approx 4.2s^{-1}$

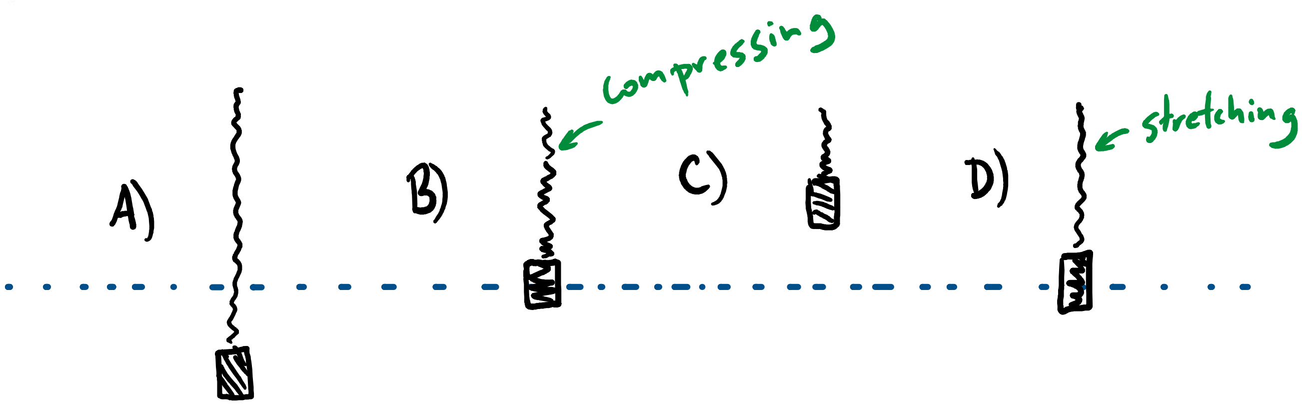
Using $\omega = \sqrt{\frac{k}{m}}$ have: $k = m\omega^2 = 0.25 \times (4.2)^2 \, N/m \approx 4 \, N/m$

EXTRA:

Simple Harmonic Motion:

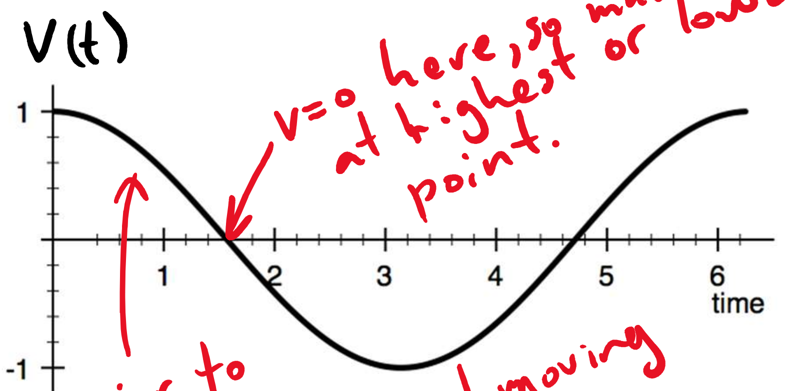


A plot of upward ***velocity*** (in cm/s) as a function of time (in s) is shown above for a mass hanging from a spring. Which of the pictures best represents the situation at t=1.6s?



EXTRA: what does x(t) look like?

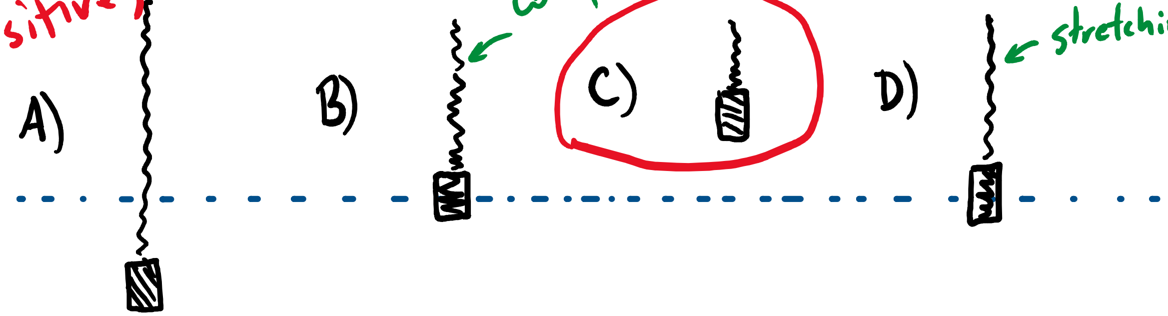
Simple Harmonic Motion:



v=0 here, so must be at highest or lowest point.

prior to t=1.6, v was positive, so object moving up

A plot of upward ***velocity*** (in cm/s) as a function of time (in s) is shown above for a mass hanging from a spring. Which of the pictures best represents the situation at t=1.6s?



compressing

stretching

EXTRA: what does x(t) look like?