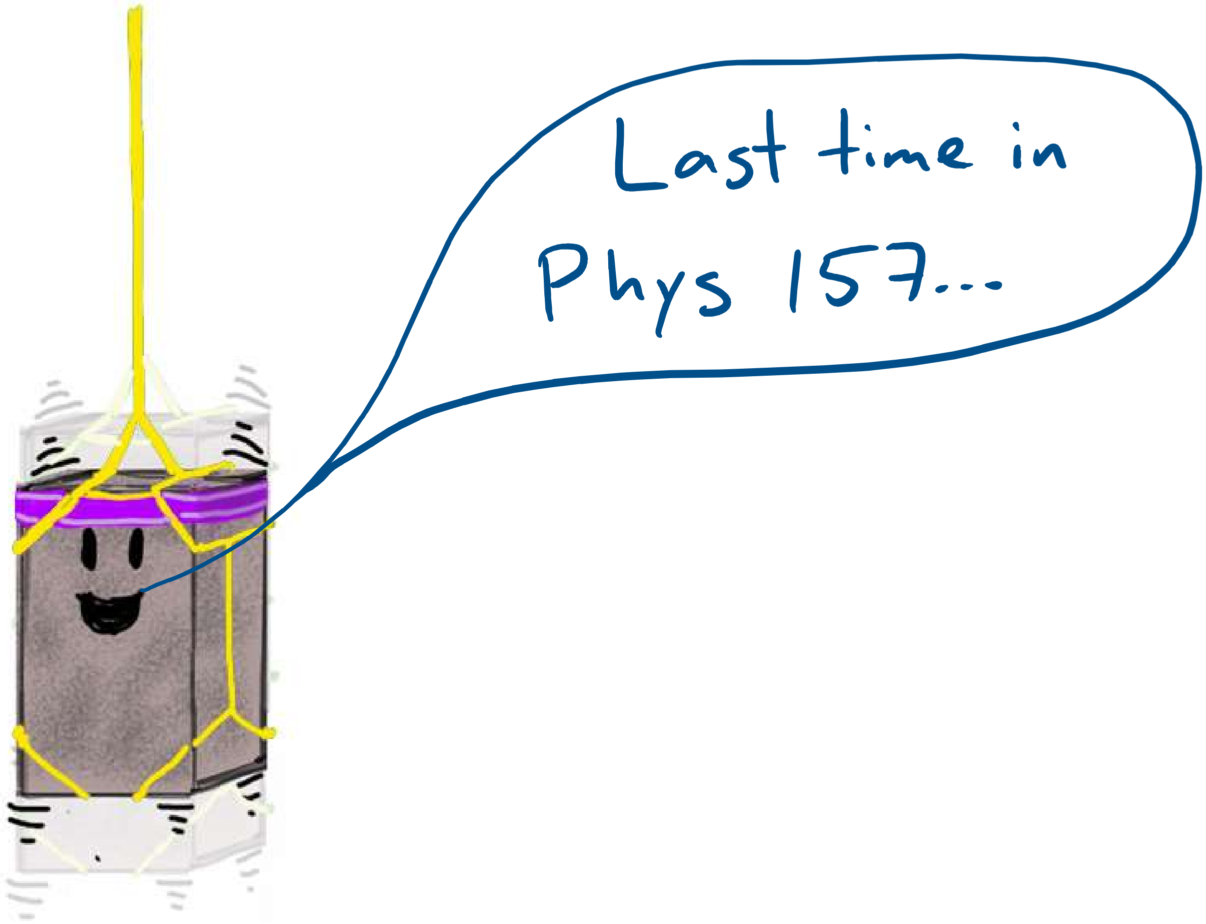
Office hours today:

- after class (Remo)
- 3:30-4:30pm (Zoom) – midterm 2 recap

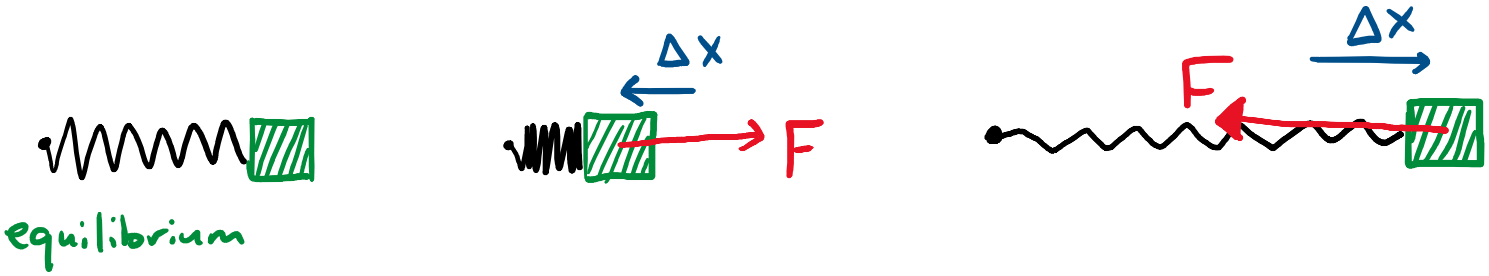
Learning goals for today:

- For simple harmonic motion, to relate the parameters appearing in the sinusoidal function describing an oscillation to the physical properties of the oscillation, including the period, frequency, amplitude, and phase
- To deduce the parameters describing simple harmonic motion from a graph of the motion.
- To describe how the amplitude and phase of a sinusoidal oscillation can be determined from the initial conditions at the start of the oscillation


Last time in Phys 157...

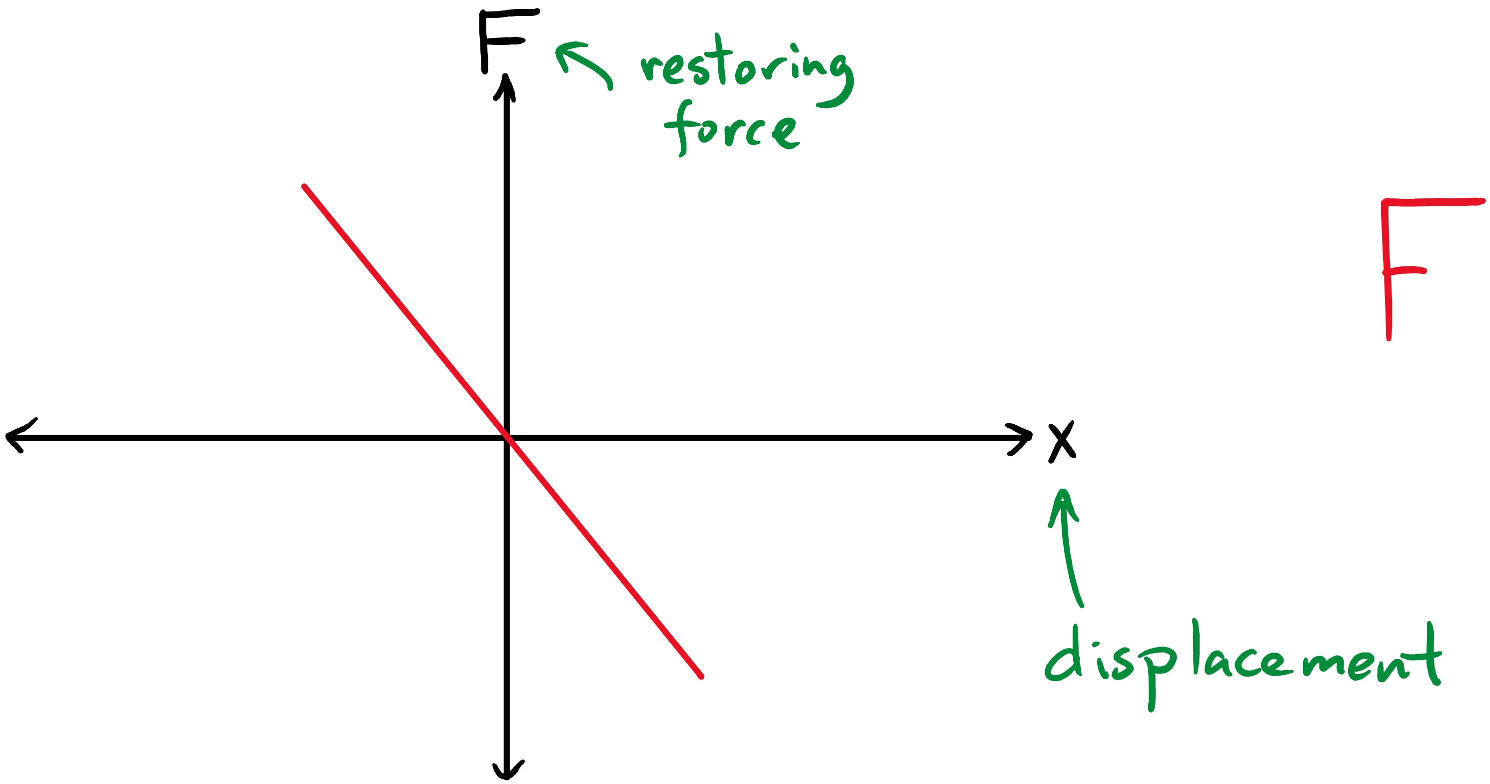
RESTORING FORCES: For an object in STABLE equilibrium, a displacement in one direction leads to a net force in the other direction.

e.g.



This leads to oscillations.

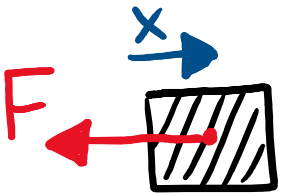
HOOKE'S LAW: Applies to almost any system perturbed a small amount from stable equilibrium



$$F = -kx$$

exact for "ideal spring"

Oscillations with Hooke's Law:



$F = -kx$

Newton: $a = \frac{F}{m} = -\frac{k}{m} \cdot x$

$$\downarrow \qquad \downarrow$$

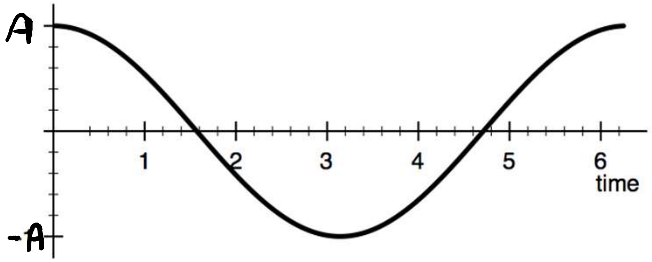
$$\frac{dv}{dt} = -\frac{k}{m} x$$

$$\frac{dx}{dt} = v$$

We can predict how velocity and position change with time.

Solution is $x(t) = A\cos(\omega t + \phi)$ with $\omega = \sqrt{\frac{k}{m}}$

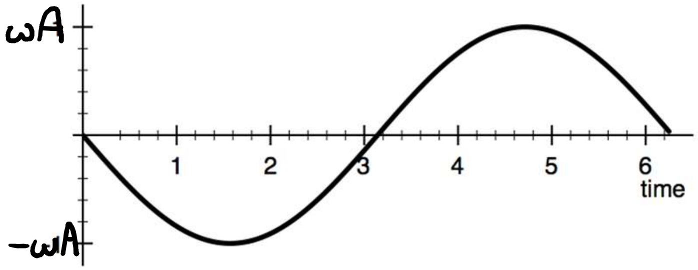
Position:



$$x(t) = A\cos(\omega t + \phi)$$

$$\Big\downarrow \frac{d}{dt} \quad (\text{slope})$$

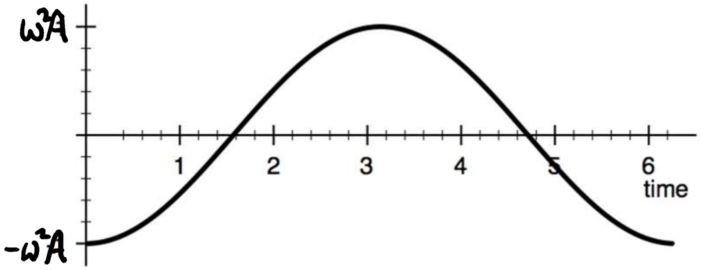
Velocity:



$$v(t) = -A\omega\sin(\omega t + \phi)$$

$$\Big\downarrow \frac{d}{dt} \quad (\text{slope})$$

Acceleration:



$$\begin{gathered} a(t) = -A\omega^2\cos(\omega t + \phi) \\ \shortparallel \\ -\omega^2 x(t) \end{gathered}$$

Newton's Law $a = -\frac{k}{m}x$ holds if $\omega = \sqrt{\frac{k}{m}}$

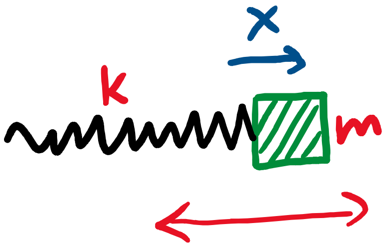
SIMPLE HARMONIC MOTION

$$x(t) = A\cos(\omega t + \phi)$$

Amplitude

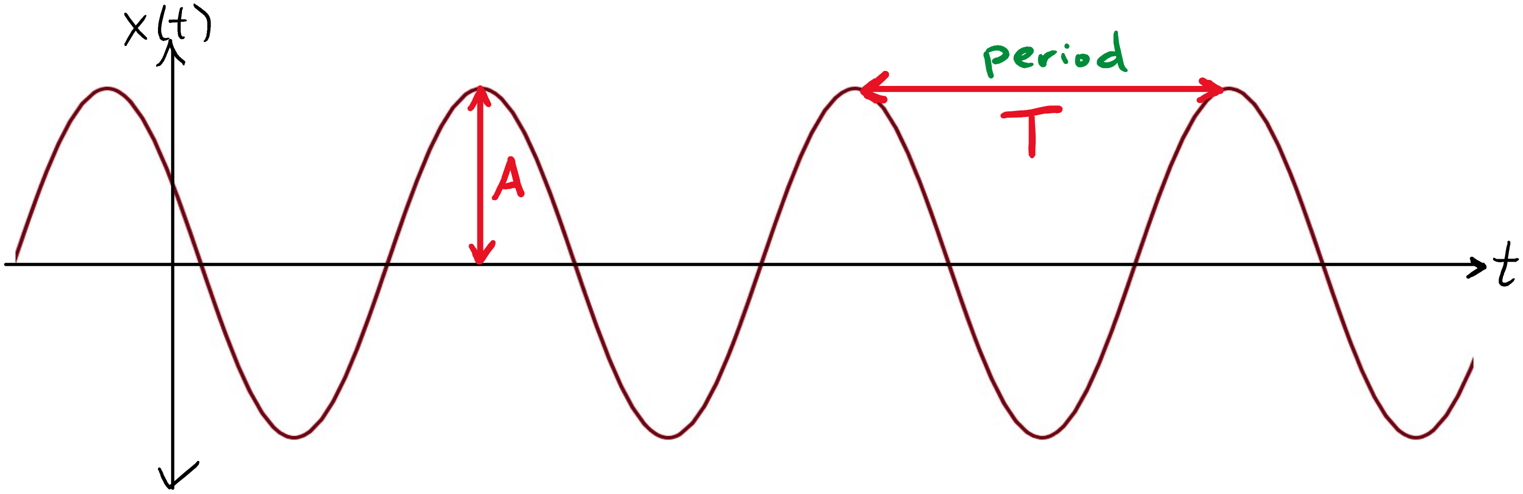
angular frequency

phase



$$\omega = \sqrt{\frac{k}{m}}$$

DISCUSSION:
What determines A & ϕ?



Demo with duck:
https://youtu.be/_BOQtQFXDJk

For the function $x(t) = 5 \cos(3t + 5)$, what is the period?

A)3
B)1/3
C)6π
D)2π/3
E)5

For the function x(t) = 5 cos(3t + 5), what is the period?

A)3
B)1/3
C)6π
D)2π/3
E)5

cos repeats when 2π is added to the inside (i.e. the argument)

adding $T = \frac{2\pi}{3}$ to t adds 2π to $(3t+5)$

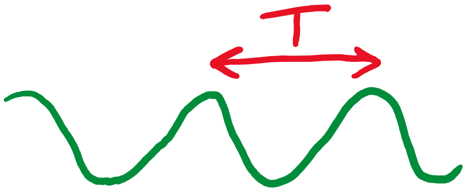
so $T = \frac{2\pi}{3}$ is the period

FREQUENCY & PERIOD

angular frequency

$$x(t) = A\cos(\omega t + \phi)$$

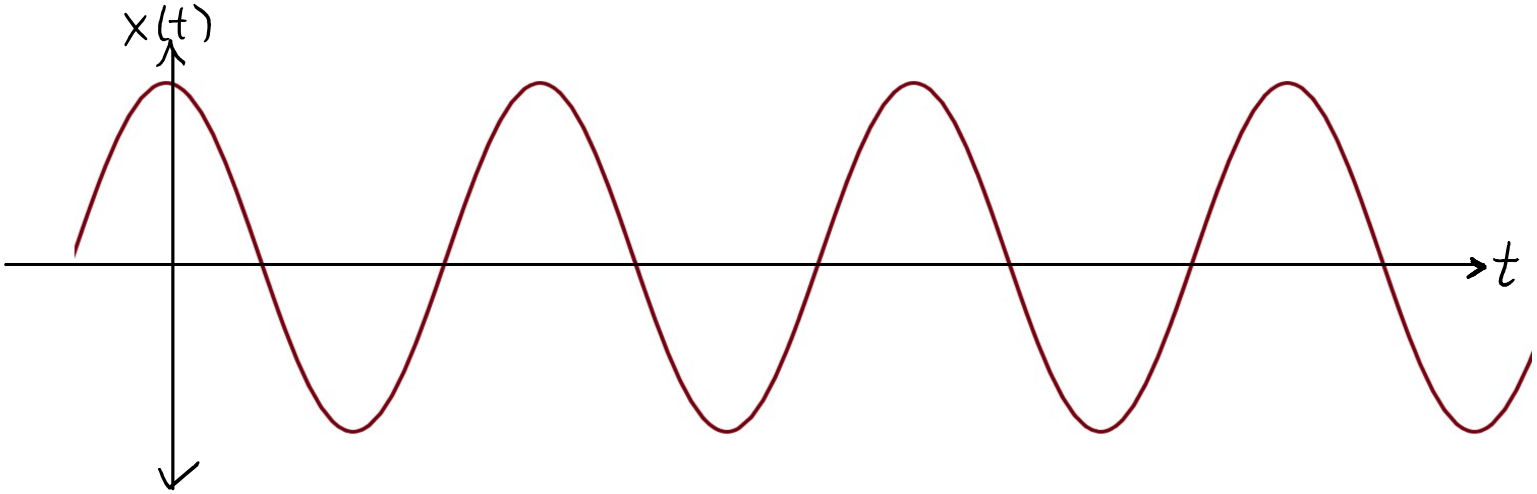
Period T : time from max → max



$T = \frac{2\pi}{\omega}$ since cos repeats every 2π.

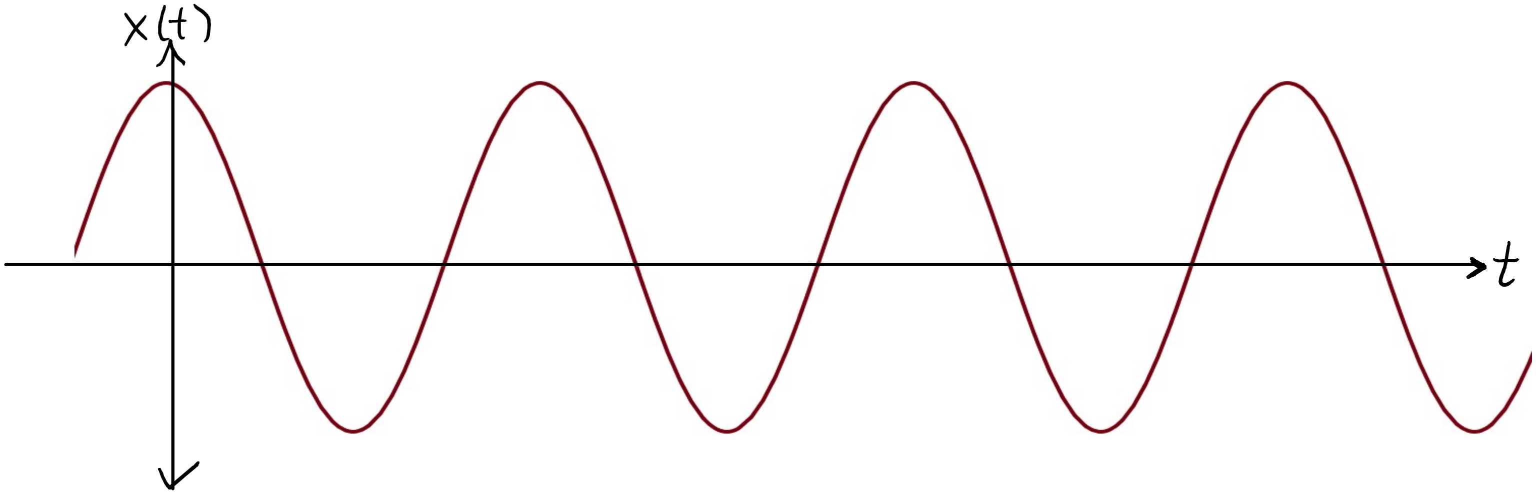
Frequency f : oscillations per time $f = \frac{1}{T}$

gives: $\omega = 2\pi f$



The graph shows a displacement $x(t) = A\cos(\omega t)$. Adding a small positive phase $x(t) = A\cos(\omega t + \phi)$ will

A) Shift the graph to the right B) Shift the graph to the left
C) Squish the graph so the peaks are closer together
D) Stretch the graph so the peaks are further apart
E) Both A and C

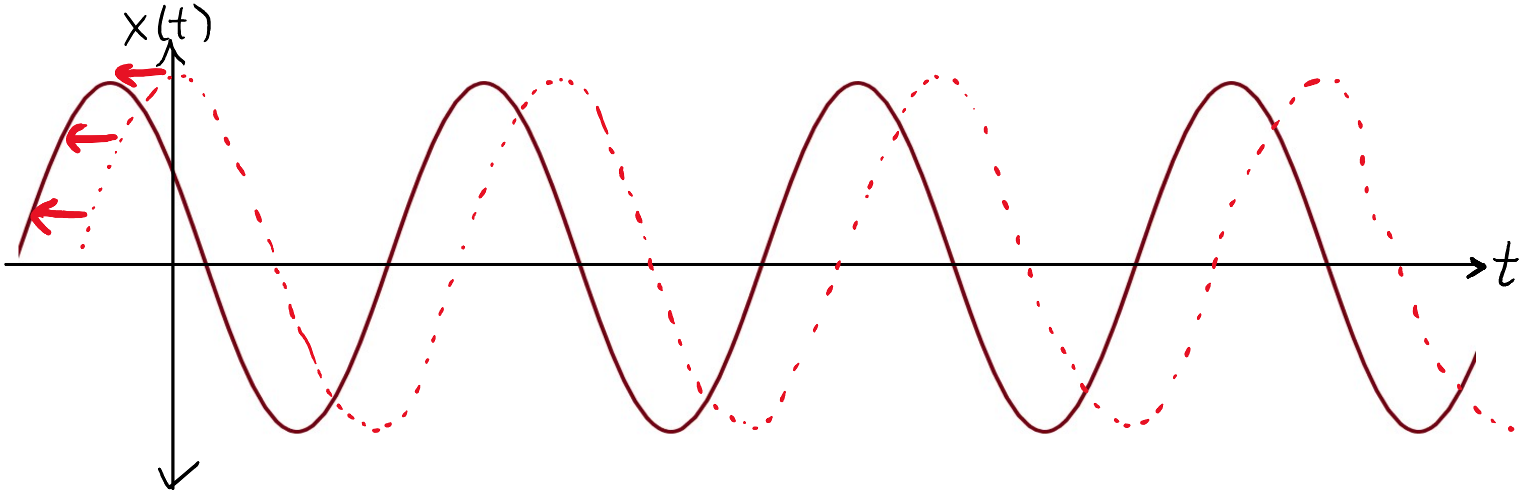


With positive ϕ, we are seeing a later part of the cosine graph than previously, so the graph is shifted to the left

The graph shows a displacement $x(t) = A\cos(\omega t)$. Adding a small positive phase $x(t) = A\cos(\omega t + \phi)$ will

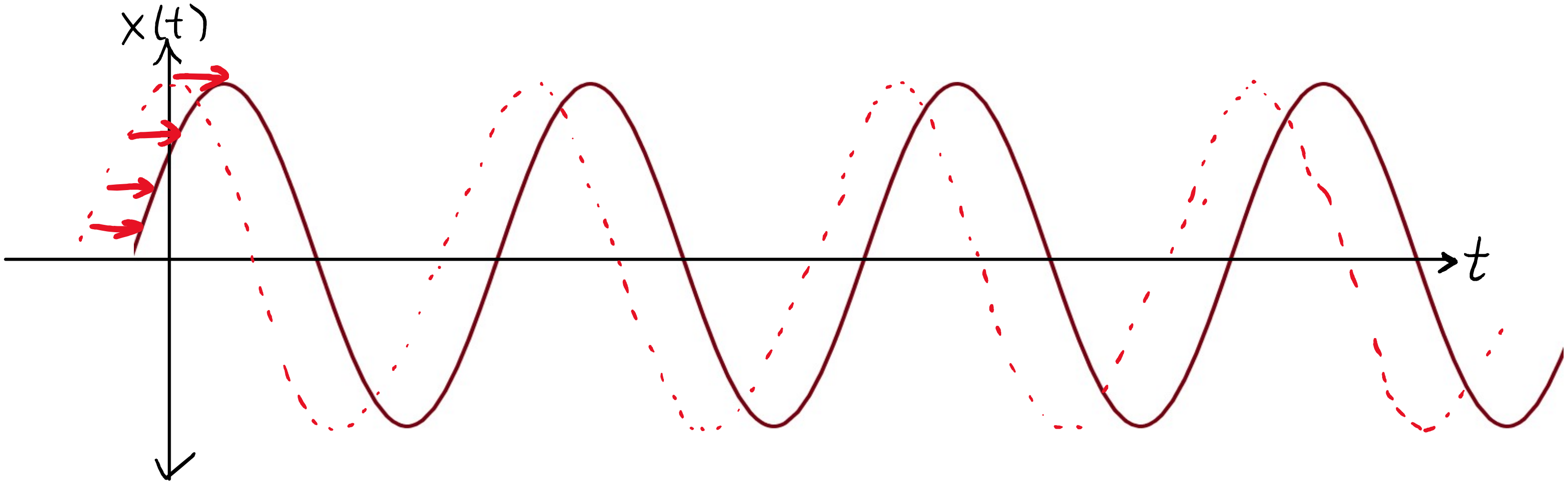
A) Shift the graph to the right
B) Shift the graph to the left
C) Squish the graph so the peaks are closer together
D) Stretch the graph so the peaks are further apart
E) Both A and C

– same as adding $\frac{\phi}{\omega}$ to our time positive ϕ shifts left



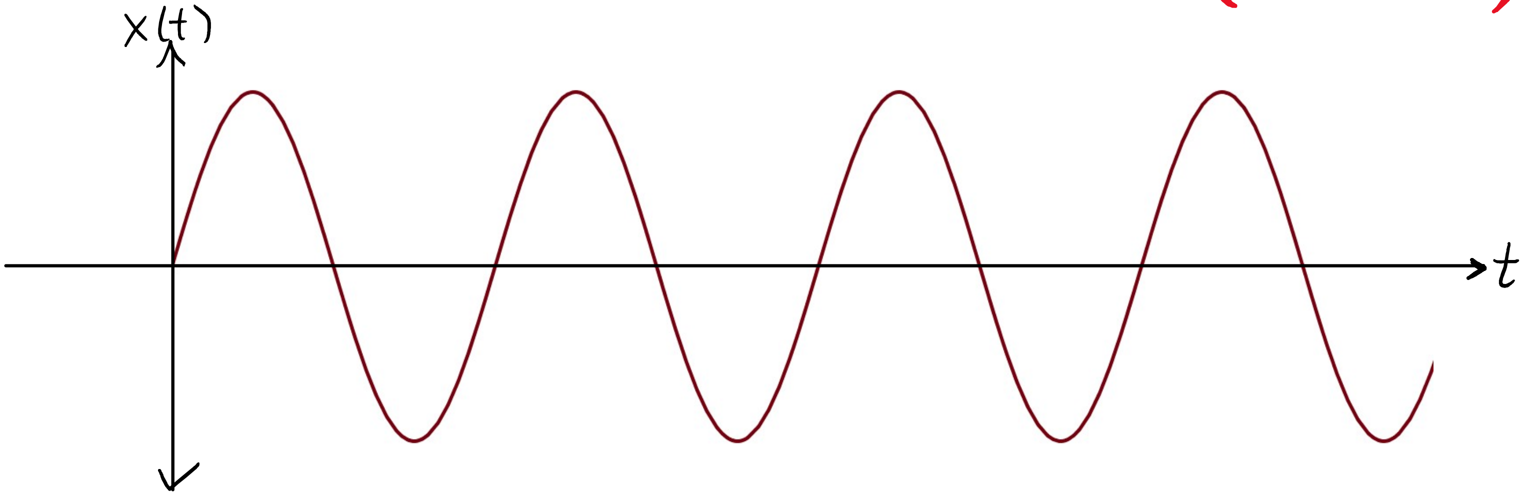
$A\cos(\omega t+\phi)$

negative ϕ shifts right



★ shift of 2π is a whole period ★

$$x(t) = A\cos(\omega t + \phi)$$



For the displacement graph shown, what is the phase ϕ?

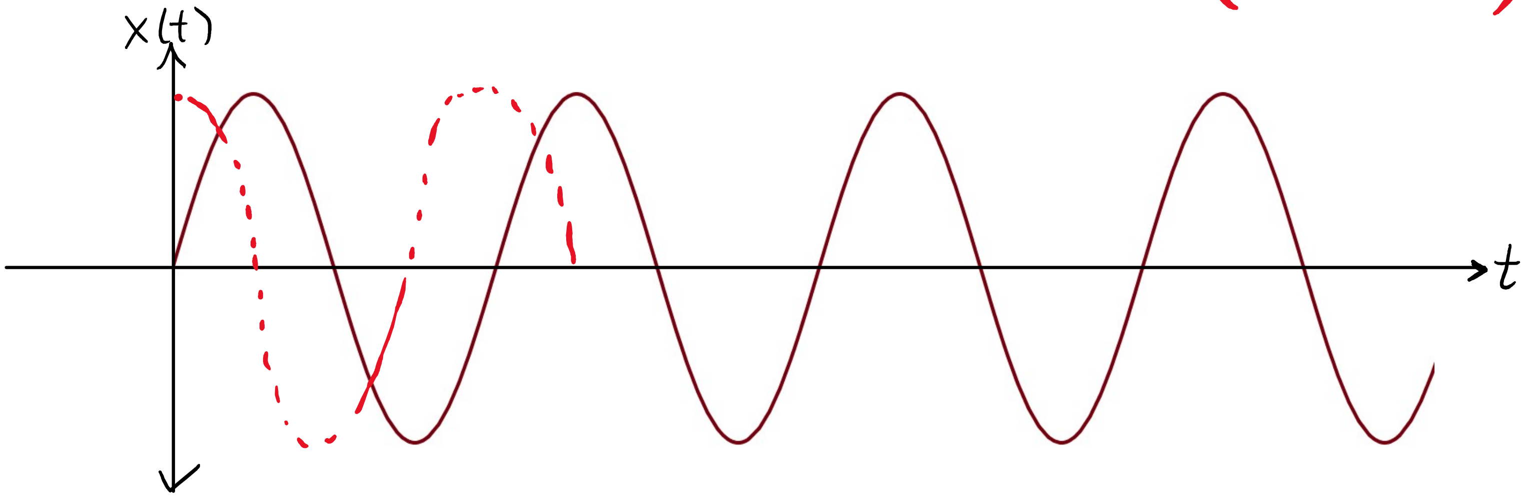
A) 0

B) π/2

C) π

D) - π/2

$x(t) = A\cos(\omega t + \phi)$



For the displacement graph shown, what is the phase ϕ?

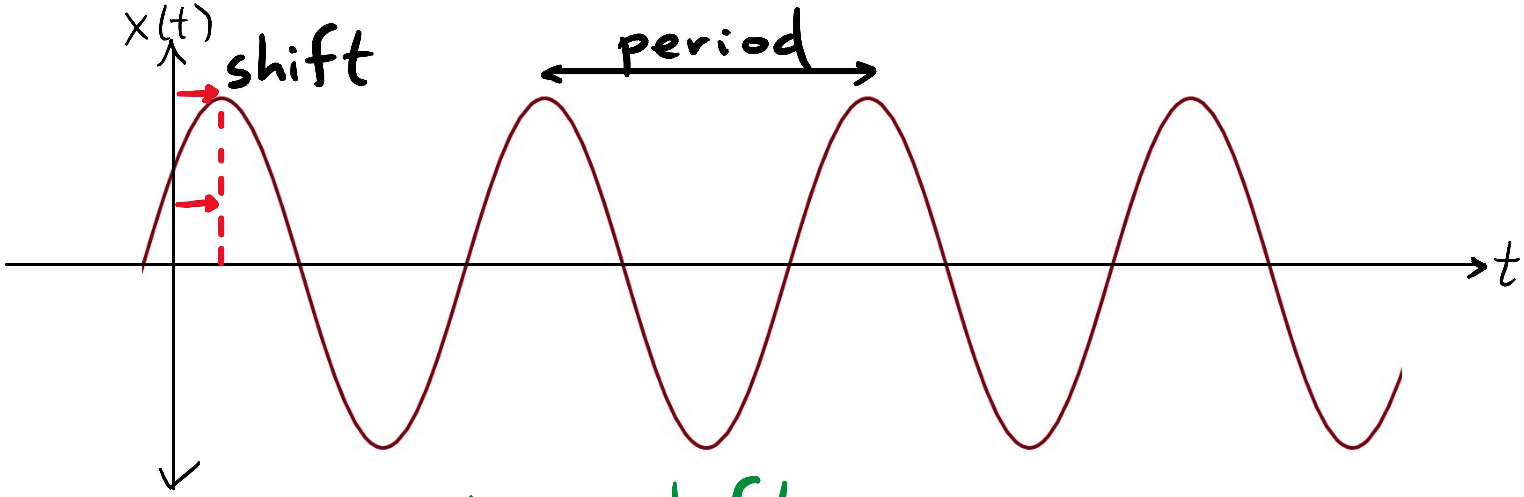
A) 0 B) π/2 C) π [✓ D) - π/2]

shifts to the right by $\frac{1}{4}$ period

so $\phi = -\frac{1}{4} \times 2\pi$

$= -\frac{\pi}{2}$

How to find ϕ



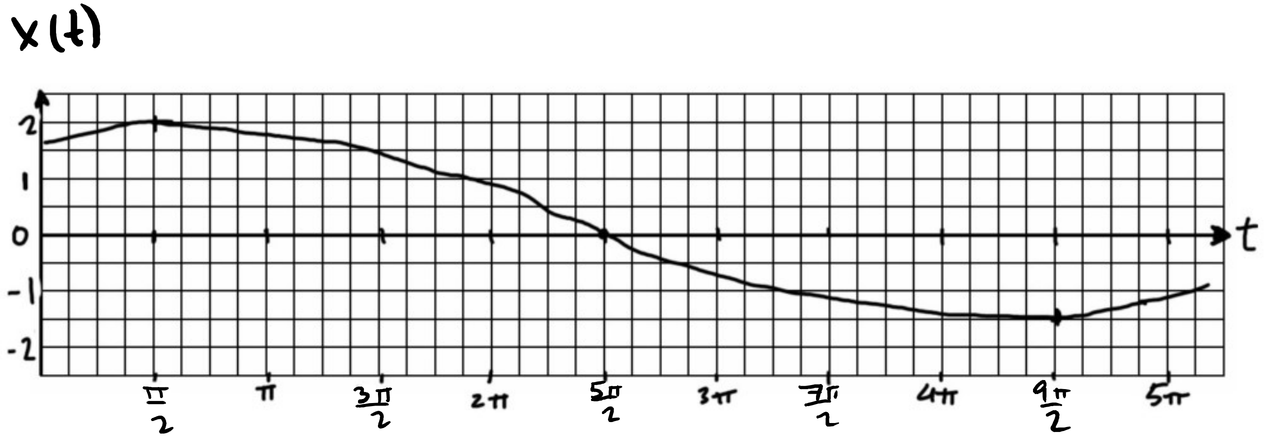
to the left

$$\phi = \pm 2\pi \times \frac{\text{shift}}{\text{period}}$$

to the right

EXTRA:

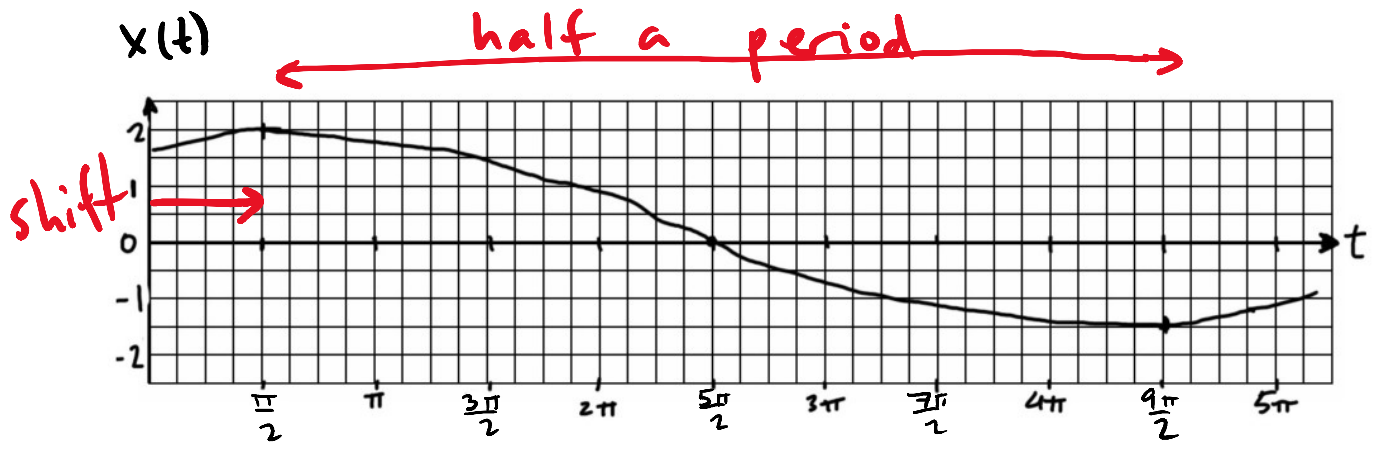
$\phi = \pm 2\pi \cdot \frac{\text{shift}}{\text{period}}$



$$x(t) = A\cos(\omega t + \phi)$$

For the displacement graph shown, what is the phase ϕ?

A) -π/8 B) -π/4 C) –π/2 D) π/4 E) π/8



$\phi = \pm 2\pi \cdot \frac{\text{shift}}{\text{period}}$

period is

$T = 2\times\left(\frac{9\pi}{2} - \frac{\pi}{2}\right)$

$\Rightarrow T = 8\pi$

$x(t) = A\cos(\omega t + \phi)$

For the displacement graph shown, what is the phase ϕ?

shift is by $\frac{\pi}{2}$

[✓ A) -π/8] B) -π/4 C) –π/2 D) π/4 E) π/8

phase is : $\phi = -2\pi \times \frac{\text{shift}}{\text{period}} = -2\pi \times \frac{\pi/2}{8\pi} = -\frac{\pi}{8}$