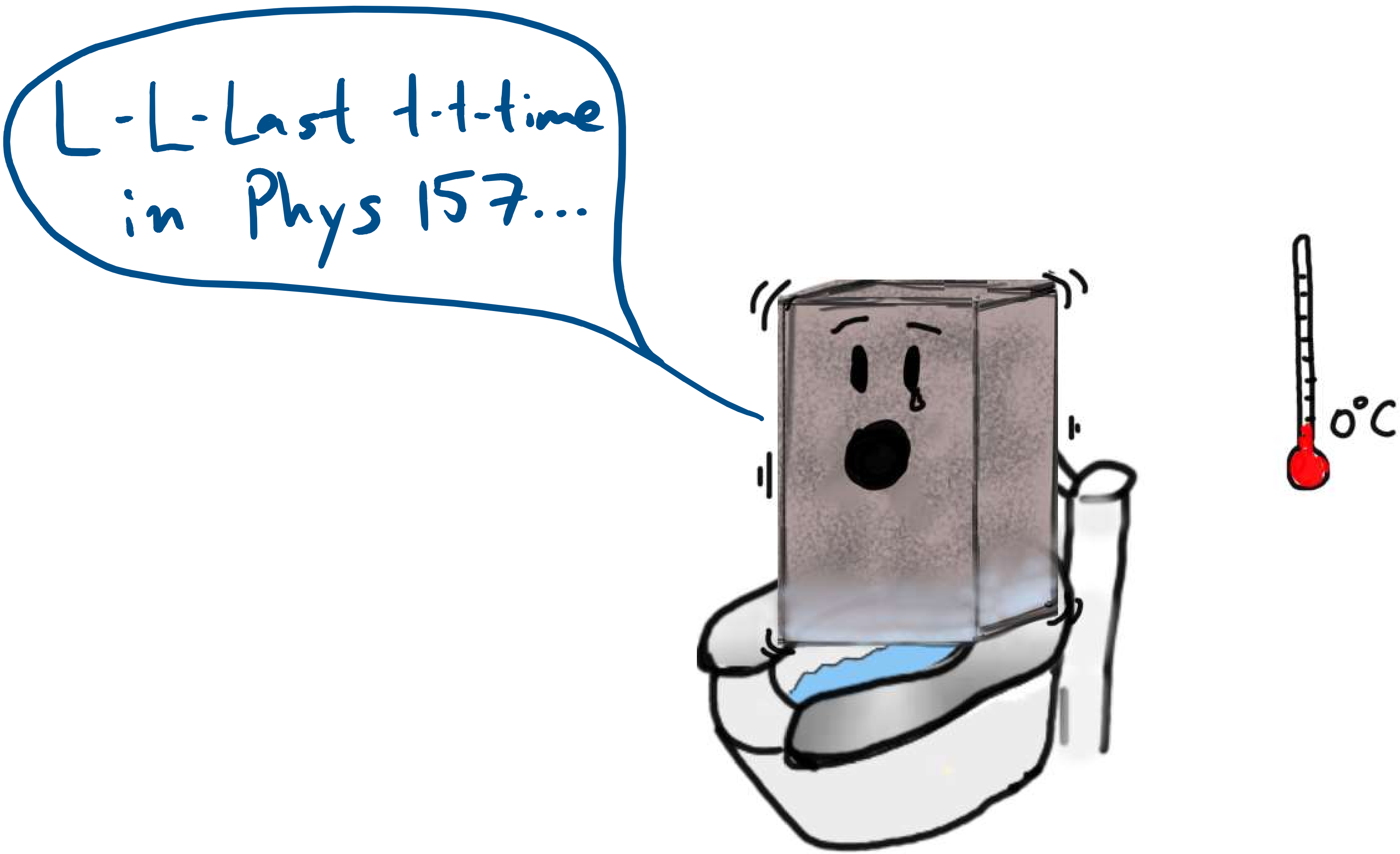
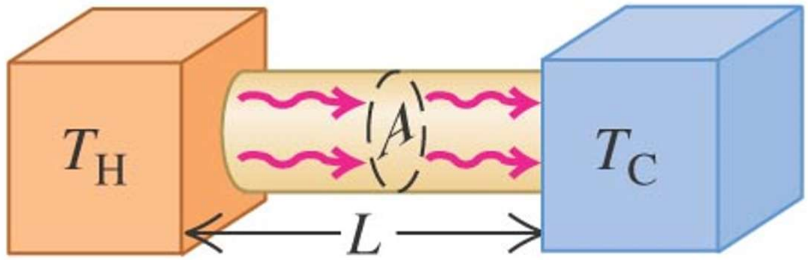
Learning goals:

- When heat is flowing steadily from an object with a higher thermal conductivity to an object with a lower thermal conductivity, explain how the heat currents can be the same
- Calculate heat flow or interface temperatures in systems with materials of various thermal conductivities
- Given the heat current into or out of an object, calculate the heat transferred in a given amount of time, or the temperature change of that object in a given amount of time


L-L-Last t-t-time in Phys 157...
0°C

THERMAL CONDUCTIVITY: Determines heat current from temperature gradient.

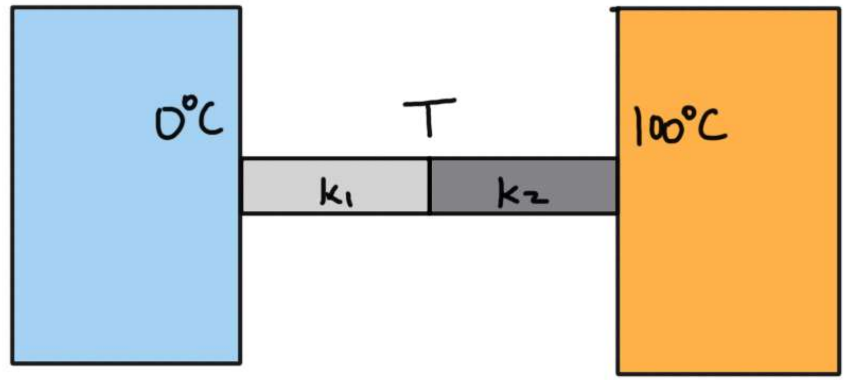


$$H = kA\underbrace{\frac{T_H - T_c}{L}}_{\text{temperature gradient}}$$

Heat current
"
Heat per time

Thermal conductivity

Two materials of equal dimensions but different thermal conductivities are placed side to side between objects kept at 0ºC and 100ºC, and a steady heat flow is established. If $k_1 > k_2$, we can say that the temperature T in the middle is:



A) Equal to 50ºC

B) Greater than 50ºC

C) Less than 50ºC

EXTRA: How would you calculate the temperature.

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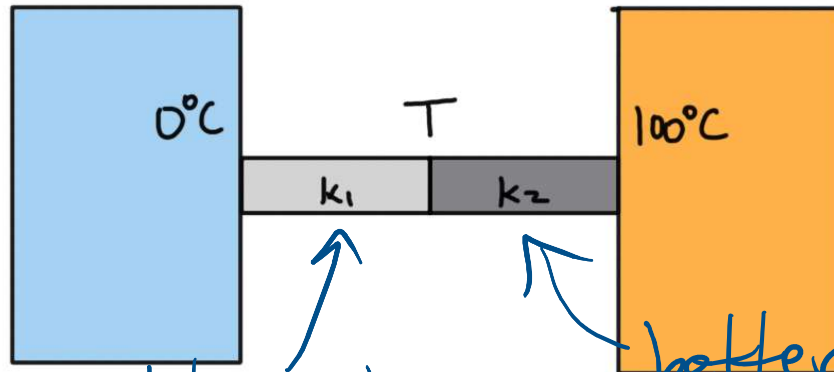


better conductor

better insulator

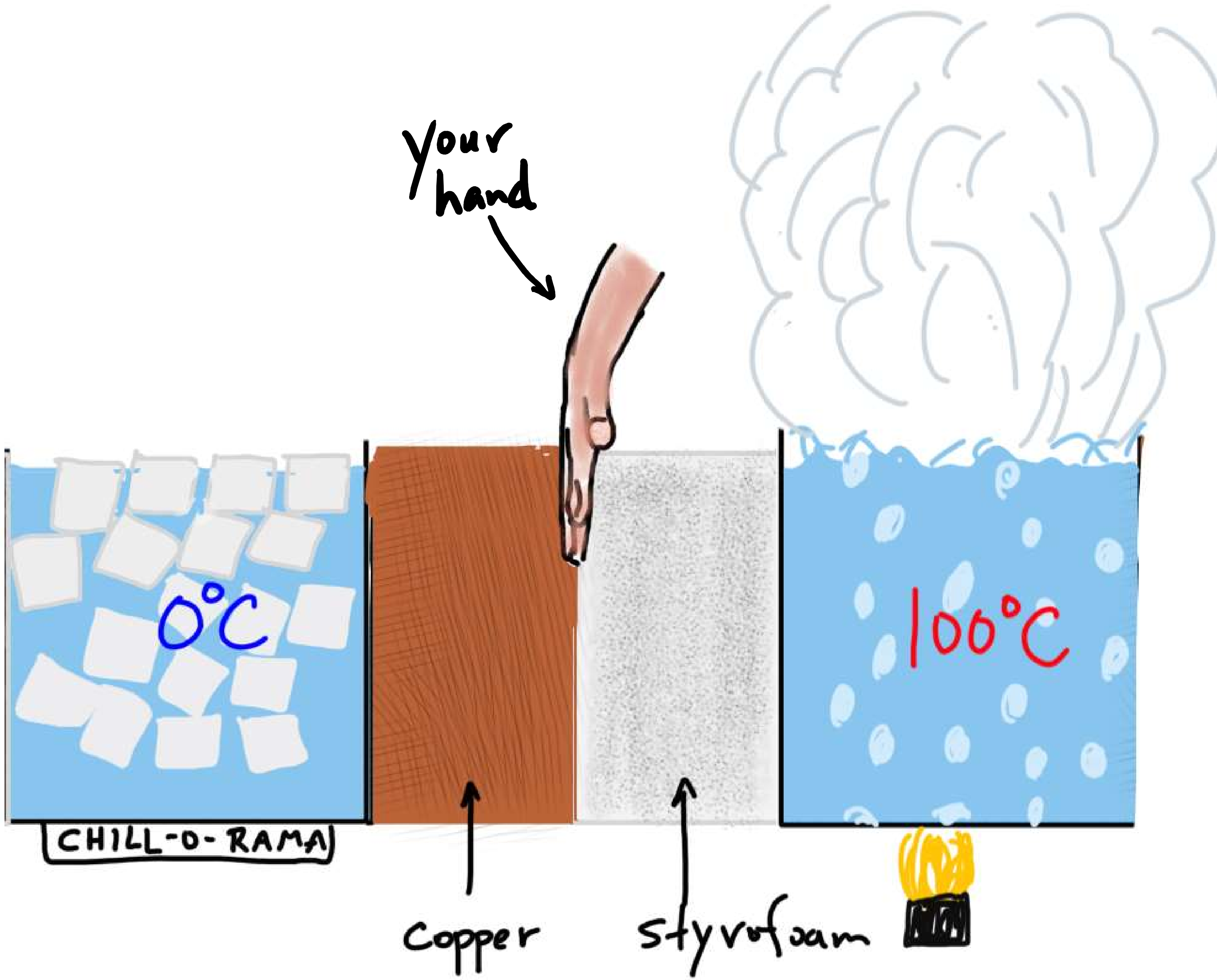
A) Equal to 50ºC

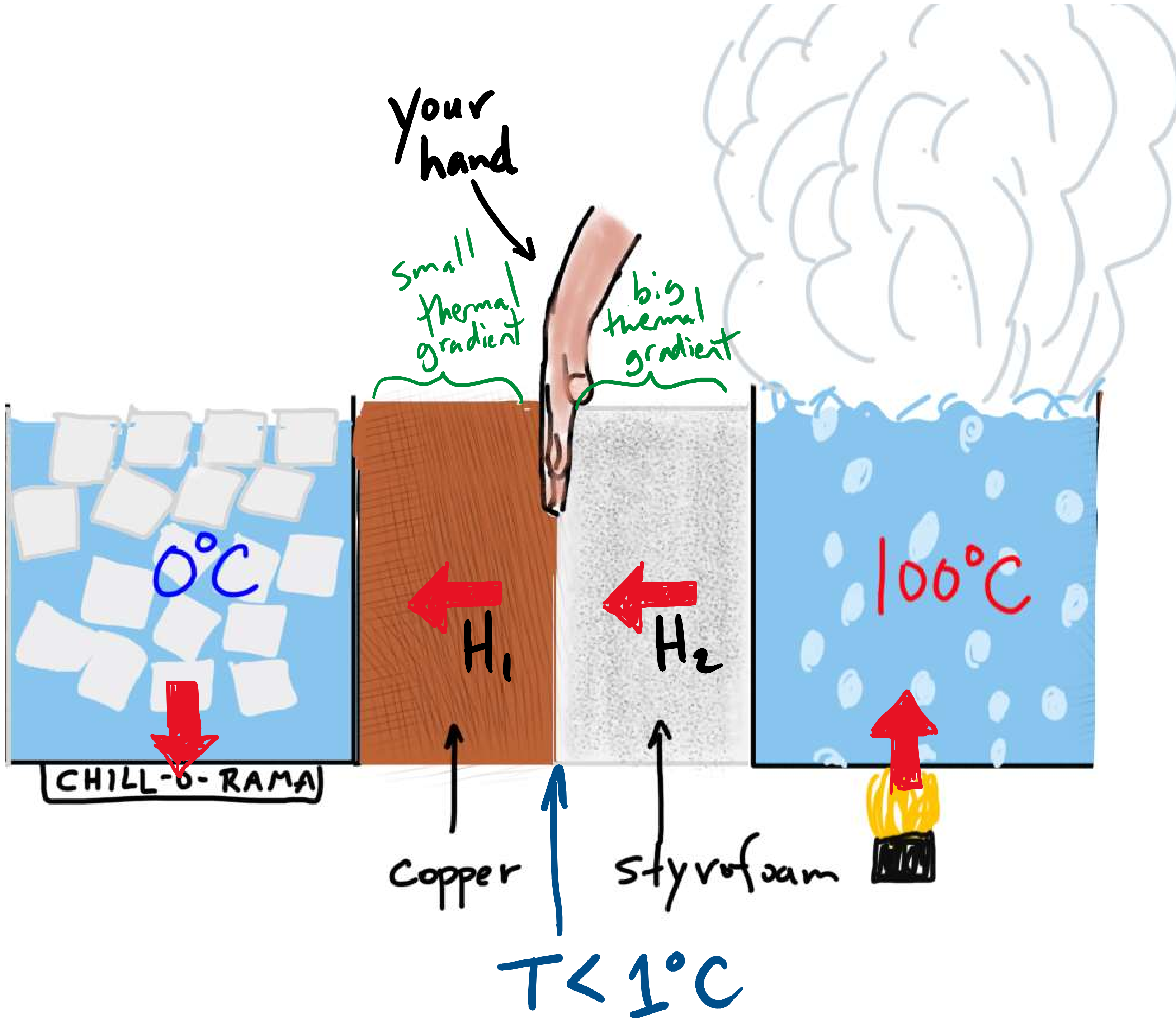
B) Greater than 50ºC

C) Less than 50ºC

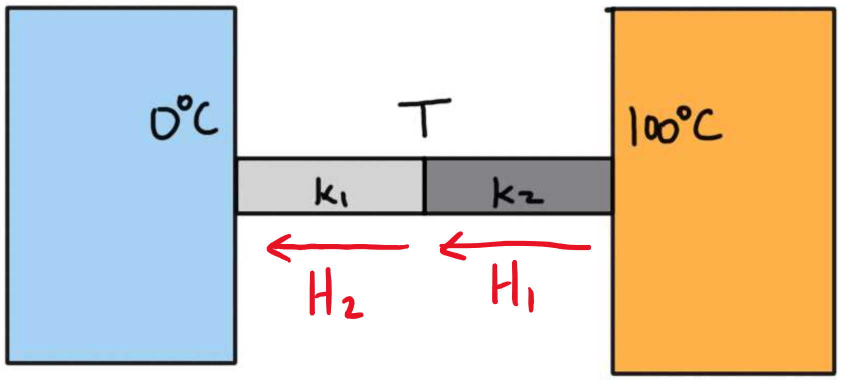
EXTRA: How would you calculate the temperature.

see intuition on next slide


your hand
0°C
100°C
CHILL-O-RAMA
copper
styrofoam


your hand
small thermal gradient
big thermal gradient
0°C
100°C
H1
H2
CHILL-O-RAMA
Copper
Styrofoam
T< 1°C

$$\boxed{H = kA\frac{T_H - T_C}{L}}$$



$k_1 > k_2$

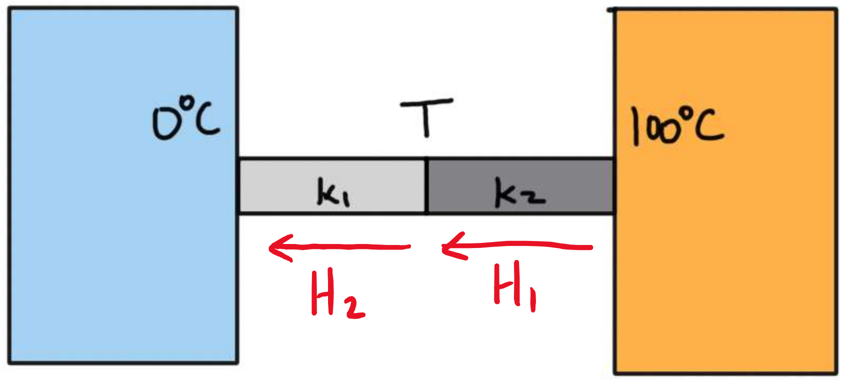
Calculate T in terms of k_1 and k_2

Hint: what are H_1 and H_2 and how are they related to each other?

Click A if you have an answer

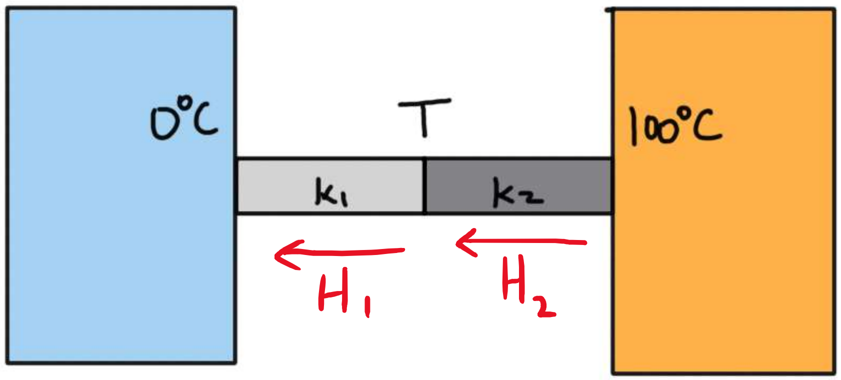
Click B if you are stuck

$$\boxed{H = kA\frac{T_H - T_C}{L}}$$



$k_1 > k_2$

Calculate T in terms of k_1 and k_2



$k_1 > k_2$

+ steady flow

- Energy conservation $\Rightarrow H_1 = H_2$

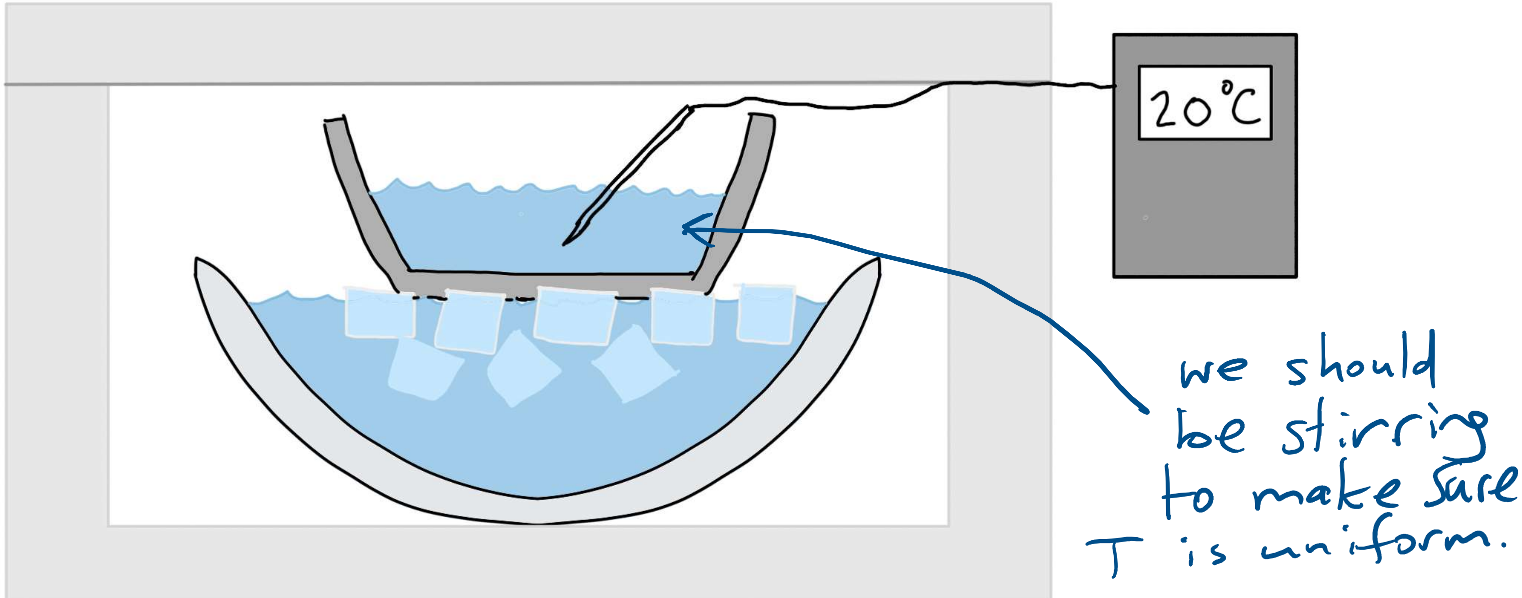
- $$k_1 \cdot A \cdot \frac{T - 0^\circ C}{L} = k_2 \cdot A \cdot \frac{100^\circ - T}{L}$$

$$k_1 (T - 0^\circ C) = k_2 (100^\circ - T)$$

bigger

smaller

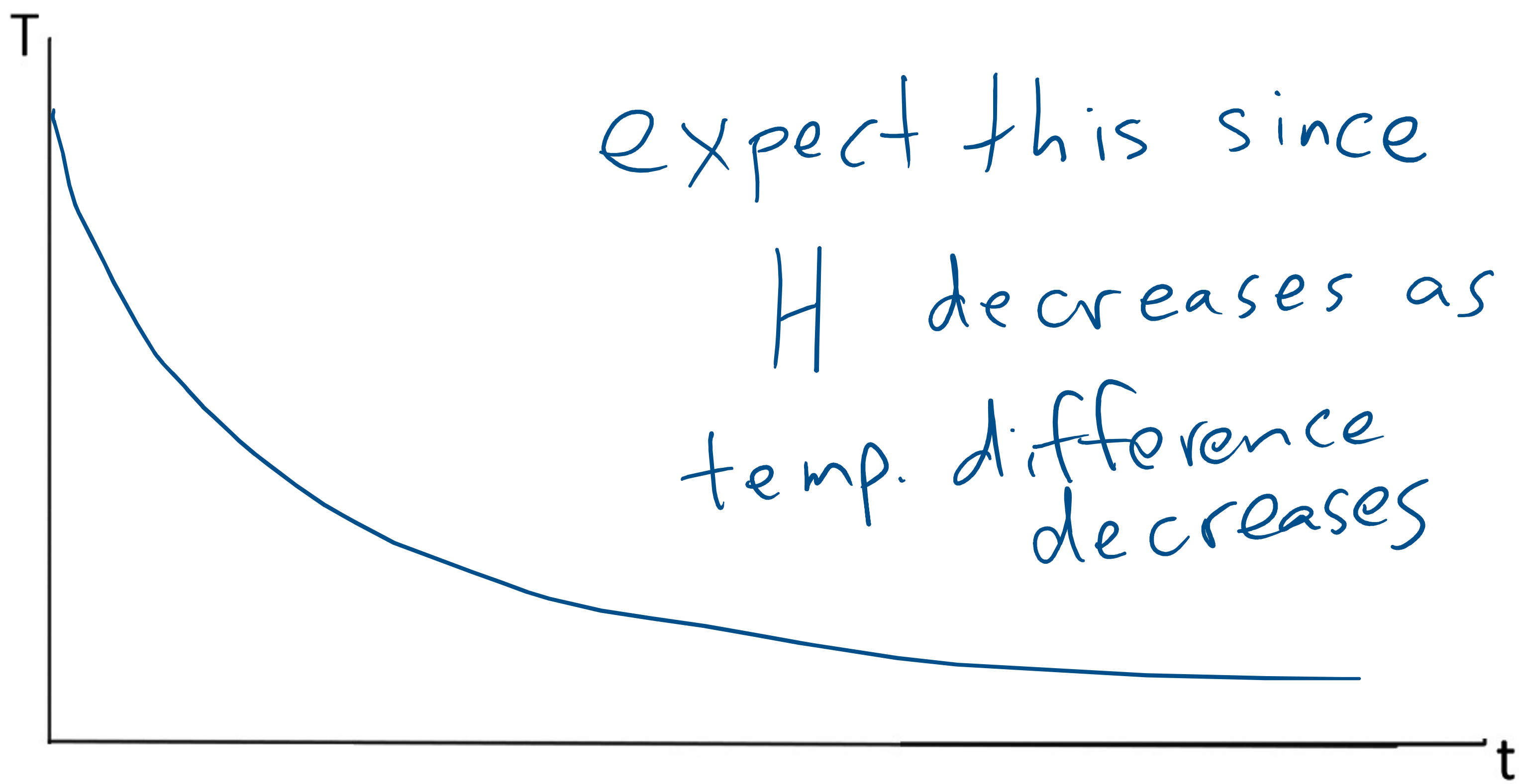
$$T = \frac{k_2}{k_1 + k_2} \times 100^\circ C$$



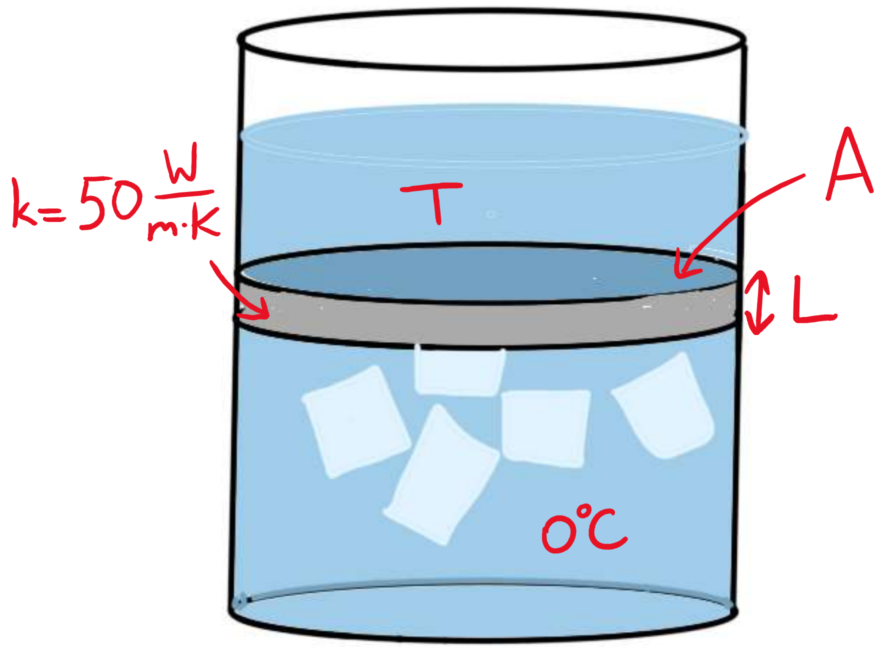
A small steel bowl of water at room temperature (T_0 = 20℃) is placed in contact with ice water in (T = 0℃). Sketch a graph of how you expect the temperature of the water in the smaller bowl to change with time, assuming that there is always some ice remaining and the system is isolated from its environment.

Sketch a graph showing how you expect the temperature of the water in the small bowl to change with time.

Sketch a graph showing how you expect the temperature of the water in the small bowl to change with time.



Let's understand this quantitatively

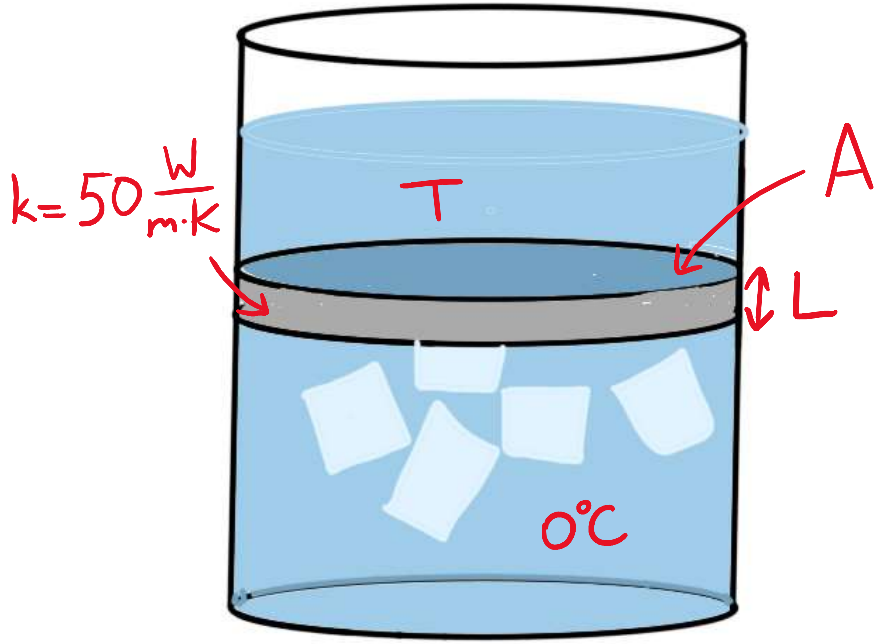


$M = 0.25\,kg$

$A = 0.01\,m^2$

$L = 2\,mm$

Question: What is the change in temperature dT of the water on top that occurs in a small time dt = 1 second?



$M = 0.25\,kg$

$A = 0.01\,m^2$

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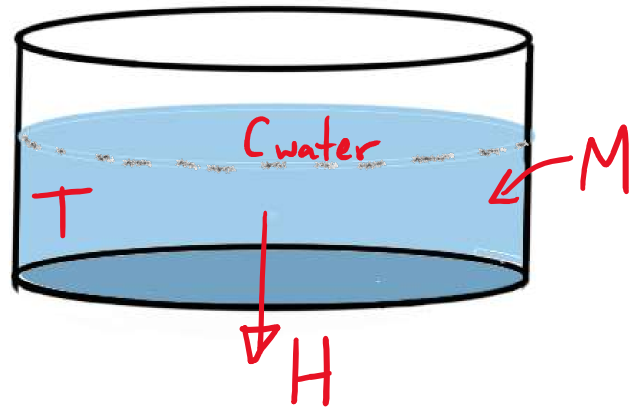
Question: What is the change in temperature dT of the water on top that occurs in a small time dt = 1 second?

Strategy: first consider each part separately

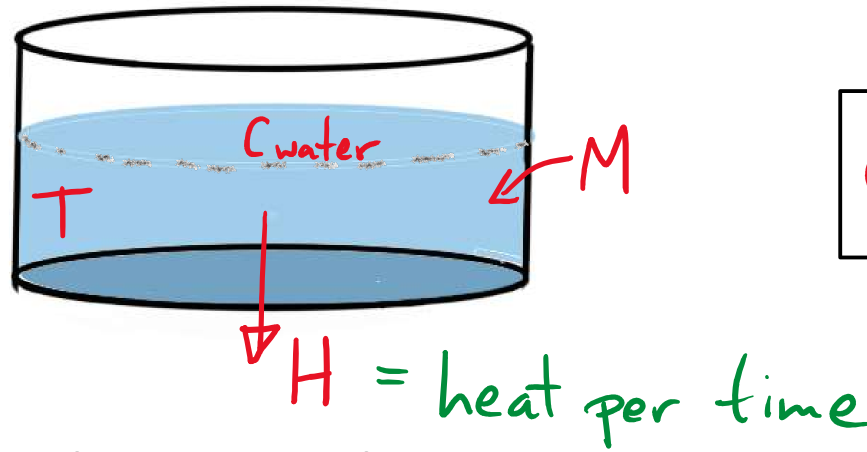


$$Q = Mc\,\Delta T$$

A heat current **H** flows out of the water on top. In a time **dt**, what is the change **dT** in the temperature of this water?

A) $-\dfrac{H}{Mc}$ B) $-\dfrac{Hdt}{Mc}$ C) $-\dfrac{H}{Mc\,dt}$ D) -H dt E) -H / dt

Hint: how much heat leaves the water during this time?



$$Q = Mc\,\Delta T$$

A heat current **H** flows out of the water on top. In a time **dt**, what is the change **dT** in the temperature of this water?

Hint: how much heat leaves the water this time?

In time dt, heat $H \times dt$ flows out of water, so

$$Q = -H\,dt$$

We have: $dT = \dfrac{Q}{Mc}$ so $dT = -\dfrac{H}{mc}\,dt$

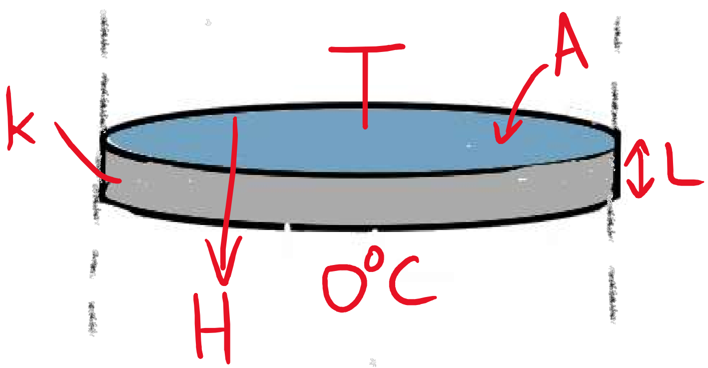
(Answer B)

What is the change in temperature dT of the water on top that occurs in a small time dt?

$$\boxed{Q = Mc\Delta T}$$

So far: $dT = -\frac{H}{Mc}dt$

What is H?



Look at steel bottom

$$\boxed{H = kA\frac{T_H - T_c}{L}}$$

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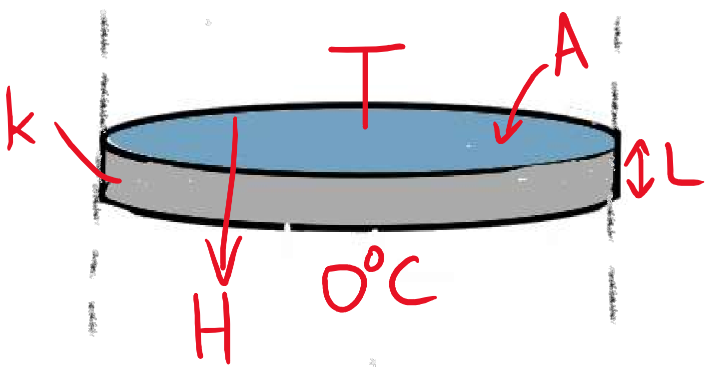
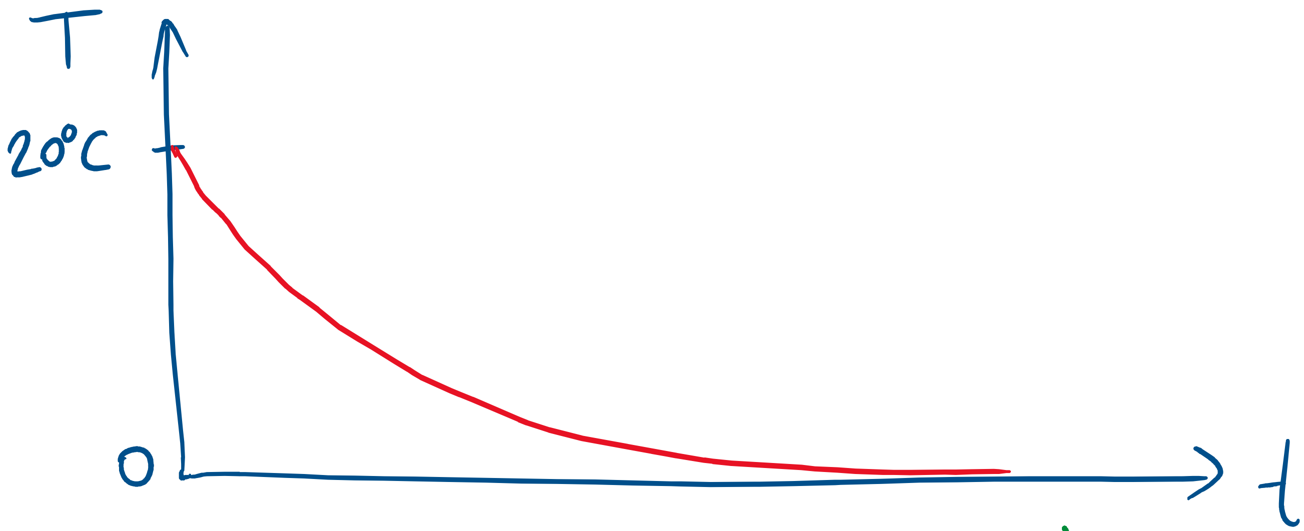


Look at steel bottom

$$\boxed{H = kA\frac{T_H - T_c}{L}}$$

$$H = kA\frac{T}{L}$$

Combine: $dT = -\frac{kA}{McL}T\,dt$



Previous slide: $\frac{dT}{dt} = -\left(\frac{kA}{McL}\right) \cdot T$

Prediction: slope of graph appoaches 0 as $T \to 0$

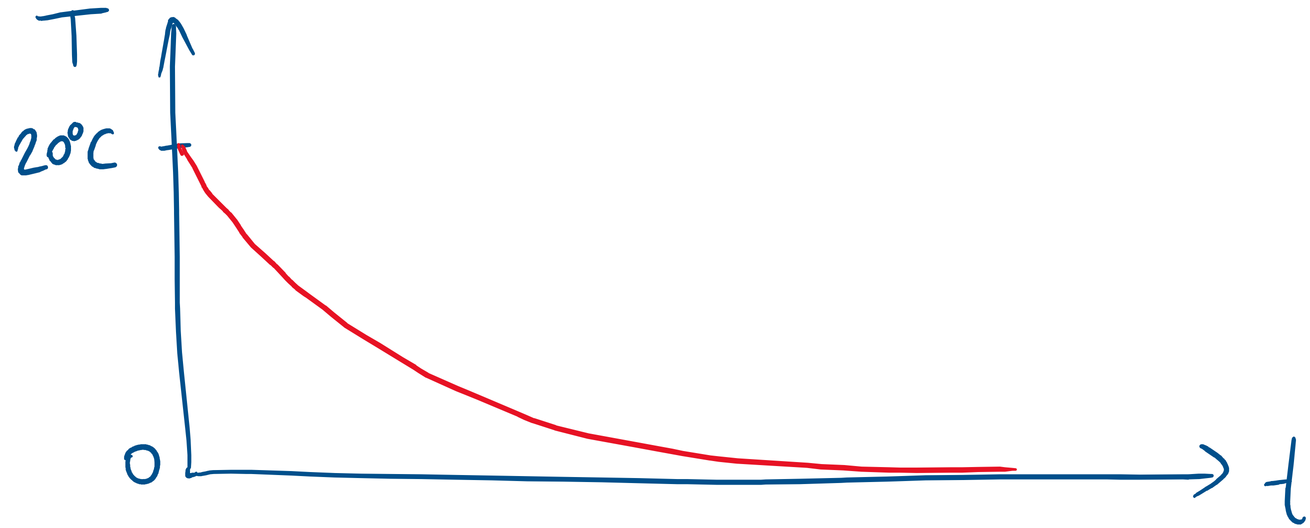
$$\frac{dT}{dt} = -\left(\frac{kA}{McL}\right) \cdot T$$

some constant

Rate of decrease of T is proportional to T.

Math: this means $T(t)$ is an EXPONENTIAL

$$T(t) = T(0)\, e^{-\frac{kA}{McL} \cdot t}$$



Prediction:

$$T(t) = 20^\circ C \times e^{-\frac{kA}{McL} \cdot t}$$