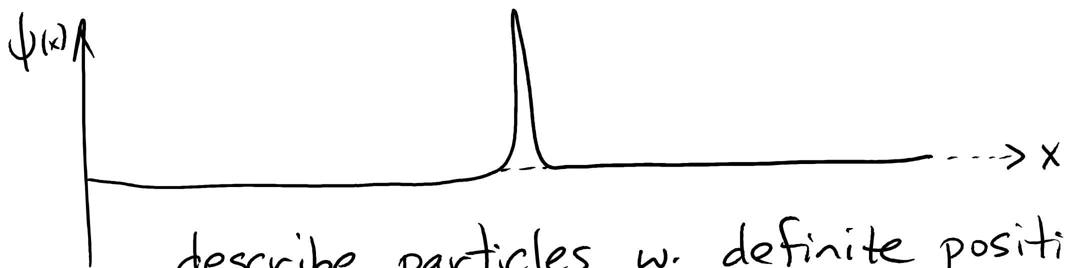
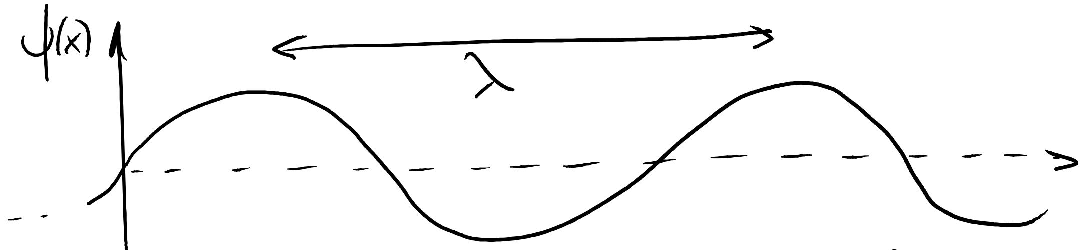
Last time: special wavefunctions

POSITION EIGENSTATES:



describe particles w. definite position

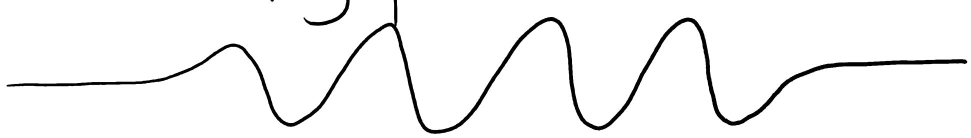
MOMENTUM EIGENSTATES



describe particle w. definite momentum

$$p = \frac{h}{\lambda}$$

Real traveling particle: described by WAVEPACKET



Can write as

(see slide w. clicker questions) for illustration

sum of position eigenstates	OR	sum of momentum eigenstates
$\Downarrow$		$\Downarrow$
range of possible positions Δx		range of possible momenta Δp

Heisenberg Uncertainty Principle:

$$\Delta x \cdot \Delta p \geq \frac{h}{4\pi}$$

if $\Delta x \to 0$ must have $\Delta p \to \infty$

* need to add up big range of wavelengths to make a narrow wavepacket. *

if $\Delta p \to 0$ must have $\Delta x \to \infty$

* wave with very accurately defined wavelength must be very spread out *

Question:

How quickly does this wavepacket move?

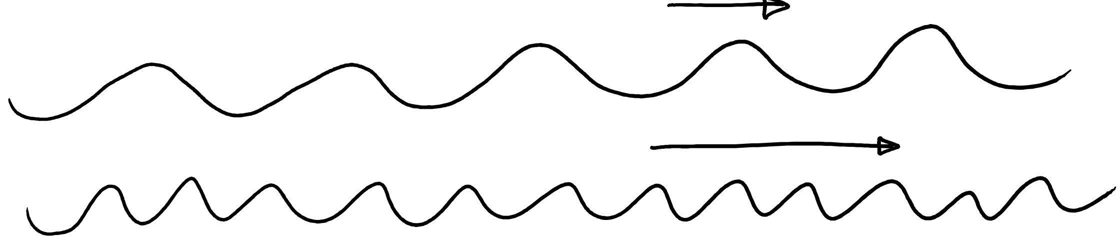
- expect packet to travel at velocity $\frac{p}{m}$ = particle speed

$$V = \frac{p}{m} = \frac{h}{\lambda m}$$

* packet also spreads out since finite packet length $\Rightarrow$ some uncertainty in momentum *

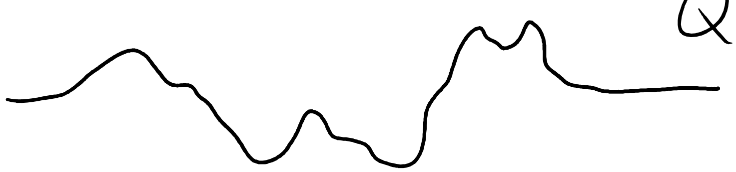
- in limit of long packet (pure wave), no spreading, so complete time dependence is just travelling sinusoidal wave:

$v \propto \frac{1}{\lambda}$



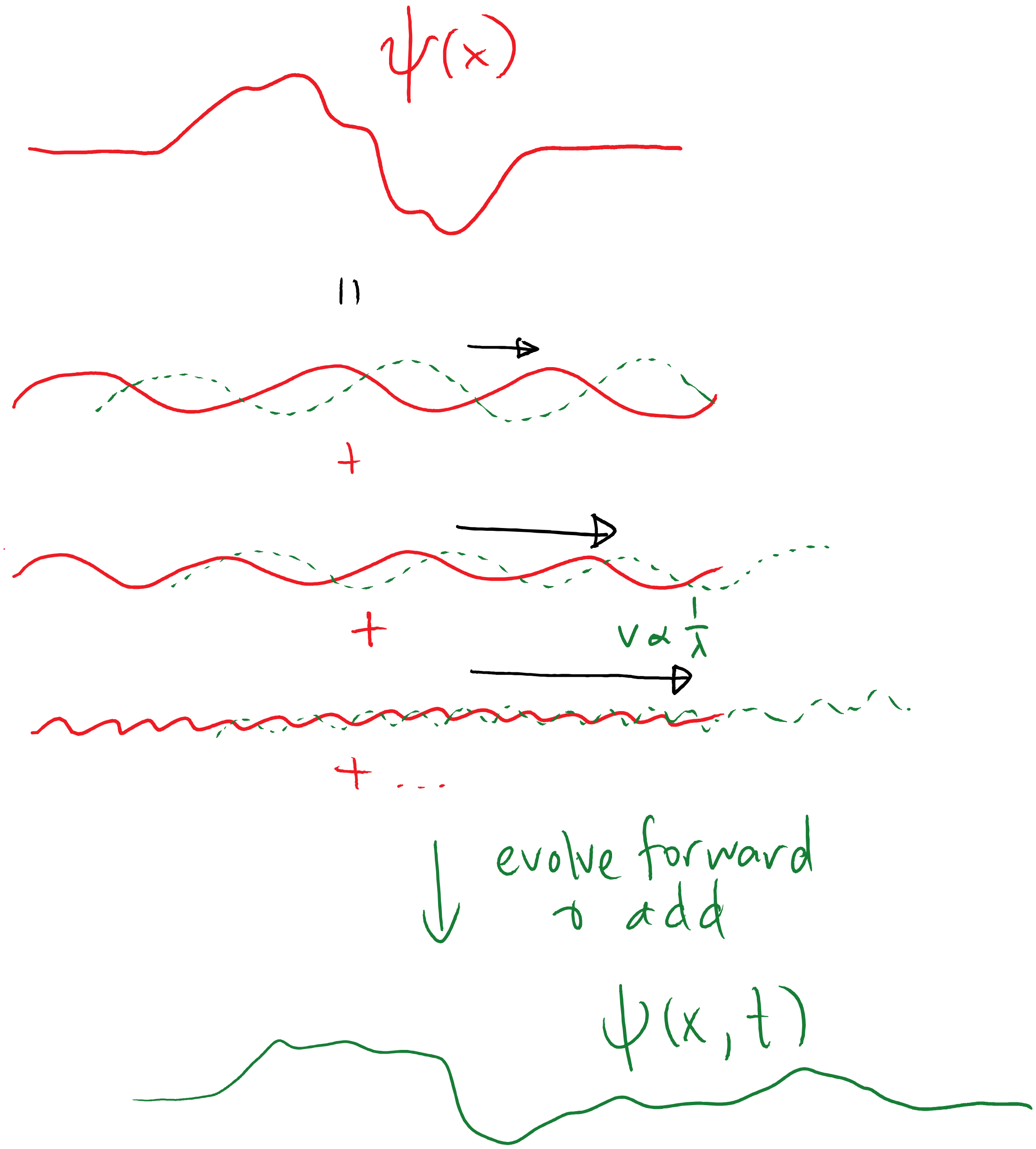
aside: using $f = \frac{v}{\lambda}$, $\lambda = \frac{h}{p}$, we get:

$$hf = \frac{p^2}{2m} = E$$ (same as for photons)



Q: How can we find time dependence of general wavefunction?

A:
1. Write $\psi(x, t=0)$ as a sum of pure waves
2. Evolve pure waves as above to time t
3. Add them up again to get $\psi(x,t)$!



This procedure is equivalent to solving a differential eqn. called SCHRÖDINGER'S EQUATION.

$$\frac{d\psi}{dt} = \text{constant} \times \frac{d^2\psi}{dx^2}$$

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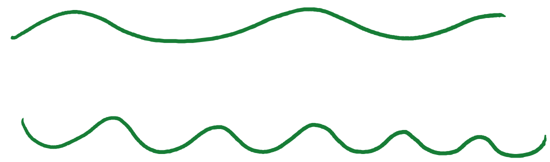
prop. to frequency

prop. to $\frac{1}{(\text{wavelength})^2}$

- says waves w. half the wavelength (i.e. particle w. double the momentum) have 4x the frequency (i.e. particle has 4x the energy)



- also: if ψ_1 and ψ_2 are solutions, so is $\psi_1 + \psi_2$