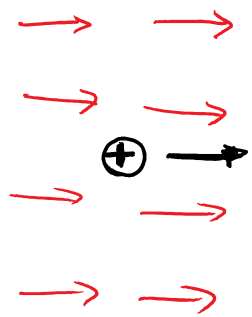
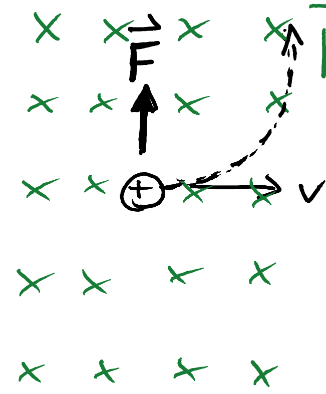
Summary of Electromagnetism

Electric & Magnetic fields produce forces on charged particles via:

$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$



$\vec{E}$ accelerates charge



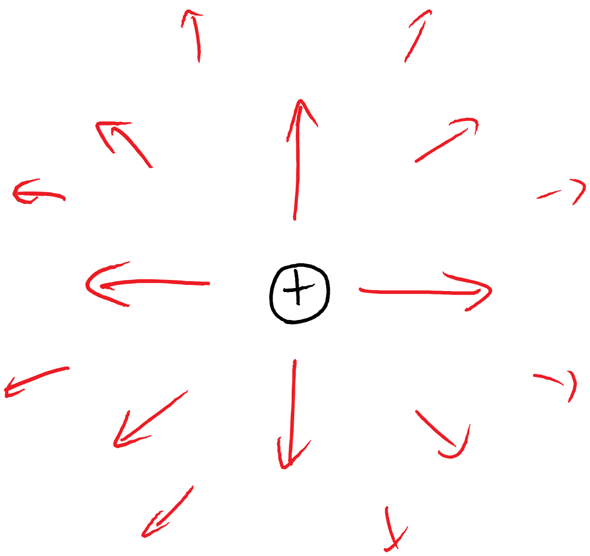
$\vec{B}$: changes direction

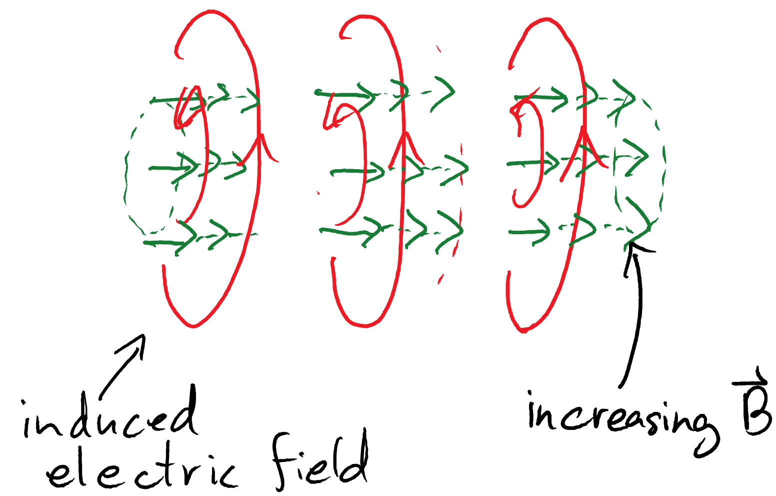
↓

circular/helical paths

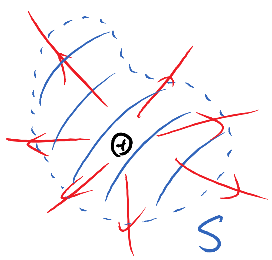
ELECTRIC FIELDS produced by

charges OR changing magnetic fields



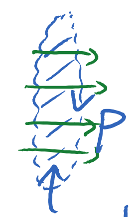


$$\int_S \vec{E}\cdot d\vec{A} = \frac{1}{\varepsilon_0} Q_{enc}$$



GAUSS' LAW

$$\int_P \vec{E}\cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

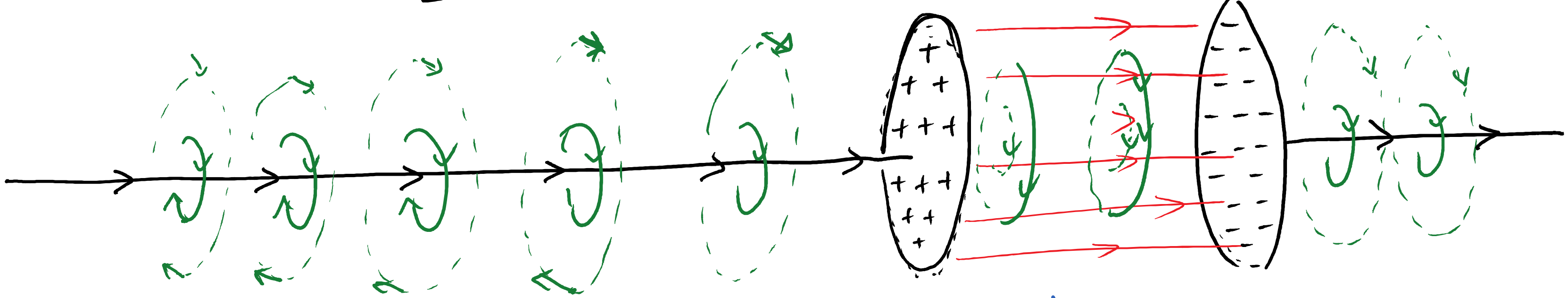


FARADAY'S LAW

can choose any surface ending on P to find Φ_B

MAGNETIC FIELDS produced by:

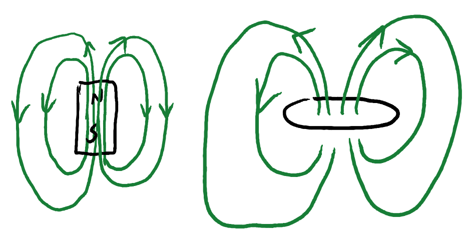
currents OR changing electric fields (NEW!)



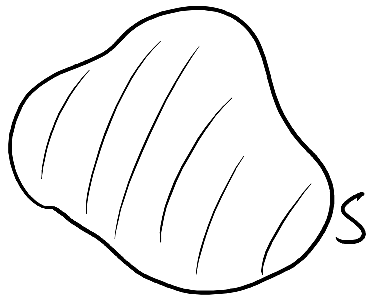
$$\underbrace{\int_P \vec{B}\cdot d\vec{l} = \mu_0 I_{enc}}_{\text{AMPERE'S LAW}} + \underbrace{\varepsilon_0 \mu_0 \frac{d\Phi_E}{dt}}_{\text{MAXWELL TERM}}$$

NO MAGNETIC CHARGES: $\int \vec{B}\cdot d\vec{A} = 0$

* Magnetic field never originate from somewhere, always complete loops *



QUESTION: Consider a closed surface S. Define:

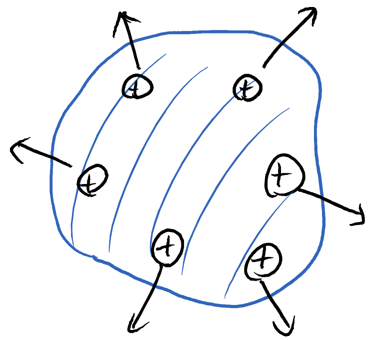


Q_{encl} = charge inside S

I_{out} = the net current out through surface S

Find a mathematical relation between Q and I based on the fact that charge can't be created or destroyed.

Why do we need the Maxwell term?



charge conservation:

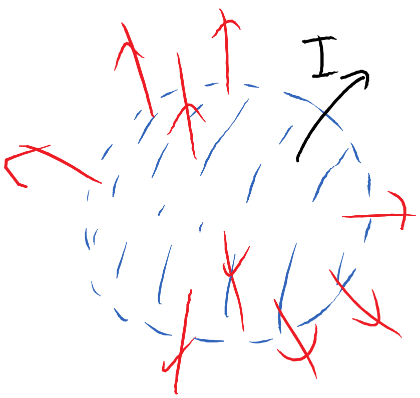
$$I_{\text{out through surface}} = -\frac{dQ_{\text{encl}}}{dt}$$

Ampere's Law w. Maxwell term:

$$\int_P \vec{B}\cdot d\vec{s} = \mu_0\left(I_{\text{enc}} + \varepsilon_0\frac{d\Phi_E}{dt}\right)$$

vanishes for closed surface

$\therefore$ Right side needs to vanish for closed surface.



closed surface: $I_{\text{out}} + \varepsilon_0\frac{d\Phi_E}{dt} = 0$

$$\Rightarrow I_{\text{out}} + \frac{d}{dt}\left[\varepsilon_0\int \vec{E}\cdot d\vec{A}\right] = 0$$

$$\Rightarrow I_{\text{out}} = -\frac{dQ_{\text{encl}}}{dt}$$

Maxwell term gives charge conservation.

$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

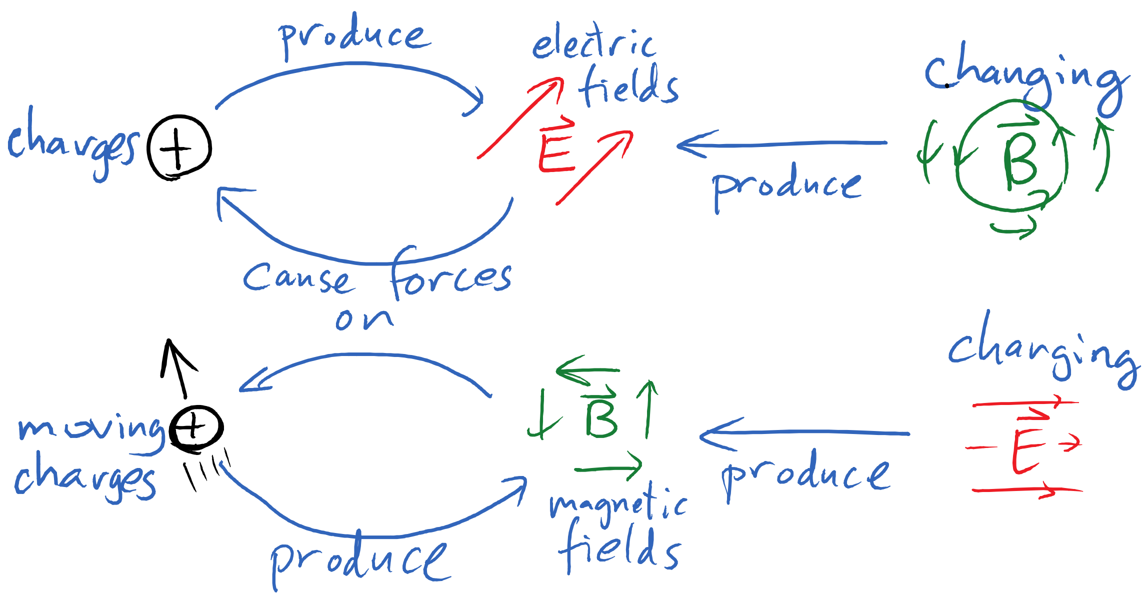
$$\int \vec{E} \cdot d\vec{A} = \frac{1}{\varepsilon_0} Q_{enc}$$

$$\int \vec{B} \cdot d\vec{A} = 0$$

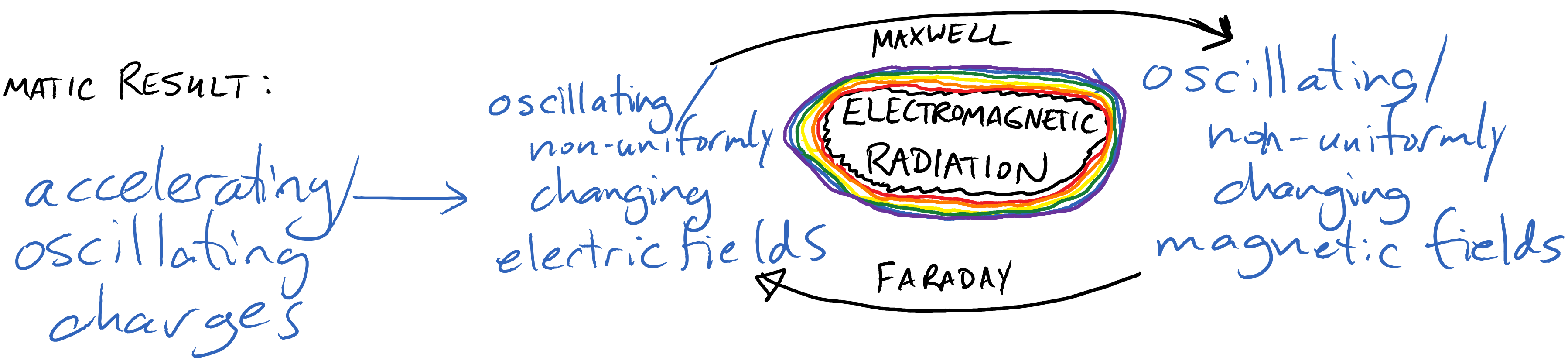
$$\int \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

$$\int \vec{B} \cdot d\vec{s} = \mu_0\left(I_{encl} + \varepsilon_0 \frac{d\Phi_E}{dt}\right)$$

Another Summary

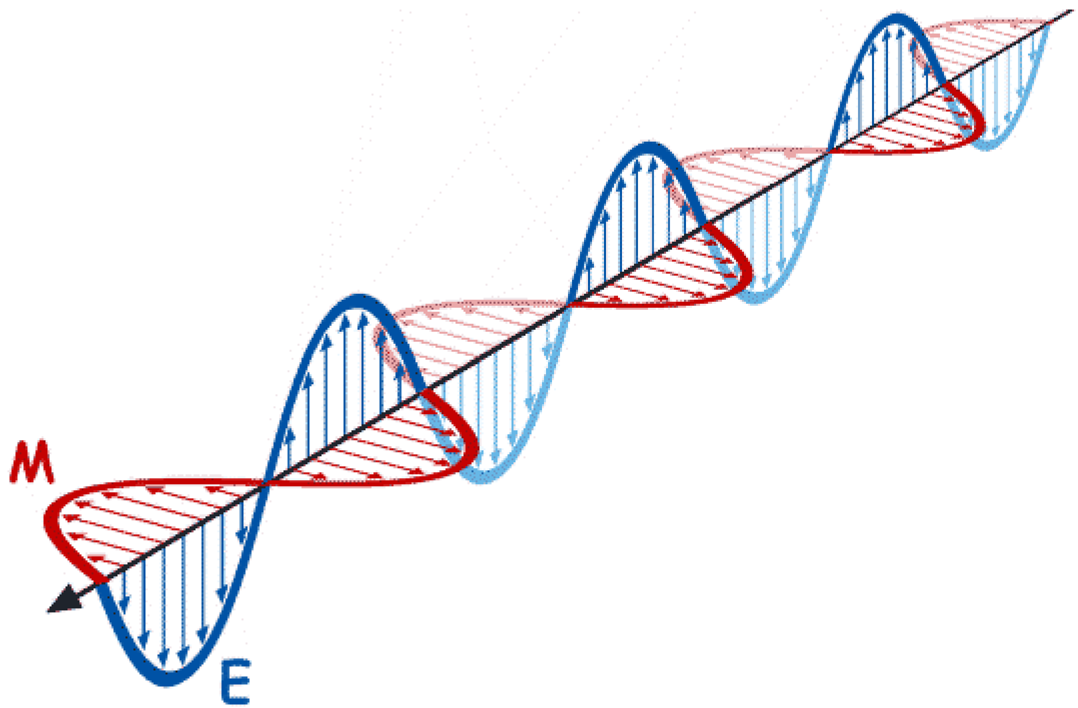


Dramatic Result:



Maxwell: E&M equations have wave solutions even if $Q = I = 0$ everywhere. $v = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = 3 \times 10^8 \, m/s$

Maxwell concluded that light is an electromagnetic wave & predicted existence of electromagnetic radiation with all wavelengths. (1864)

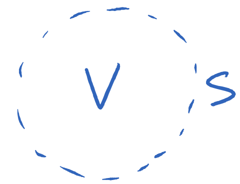


- $\vec{E}$ & $\vec{B}$ can exist on their own

- Carry energy density $\frac{\epsilon_0}{2}|\vec{E}|^2 + \frac{1}{2\mu_0}|\vec{B}|^2$

ASIDE: we can rewrite all of Maxwell's equations as differential equns.

e.g. $$\int_S \vec{E}\cdot d\vec{A} = \frac{1}{\varepsilon_0} Q_{encl}$$



$\downarrow$ STOKES THEOREM

charge density

$$\int \left(\frac{dE_x}{dx} + \frac{dE_y}{dy} + \frac{dE_z}{dz} \right) dV = \frac{1}{\varepsilon_0} \int \rho \, dV$$

True for all regions V so:

$$\frac{dE_x}{dx} + \frac{dE_y}{dy} + \frac{dE_z}{dz} = \frac{1}{\varepsilon_0} \rho \qquad + 7 \text{ others}$$

$$\vec{\nabla}\cdot\vec{E} = \frac{1}{\varepsilon_0} \rho$$

These are the equations that Maxwell found solutions to.