Physics Tutorial - Currents and Circuits

Question 1: Ion Channels

The biochemistry that takes place inside cells depends on various elements that are dissolved in water as ions. The ions enter cells through narrow pores in the cell membrane known as ion channels. Each ion channel, which is formed from a specialized protein molecule, is selective for one kind of ion. Measurements with microelectrodes have shown that a 0.30 nm diameter potassium ion (K^+) channel carries a current of 1.8 pA.

a) How many potassium ions pass through if the ion channel opens for 1.0 ms?

$$N_{ions} = \frac{I}{q} \Delta t = \frac{1.8 \times 10^{-12} C/s}{1.602 \times 10^{-19} C} (0.001 s) = 11235 \text{ ions}$$

b) What is the current density in the ion channel?

$$\text{Current density} \quad J = \frac{I}{A} = \frac{1.8 \times 10^{-12} A}{\pi (0.15 \times 10^{-9} m)^2} = 2.5 \times 10^7 A/m$$

huge!

c) The membrane wall is 8 nm thick and the potential across the membrane is 80 mV. What is the conductivity of the ion channel? *Hint: Remember that J = σ E. How do you find E?*

$$\sigma = \frac{J}{E} = \frac{Jd}{V} = \frac{(2.5 \times 10^7 A/m)(8 \times 10^{-9} m)}{80 \times 10^{-3} V} = 2.5 (\Omega \cdot m)^{-1}$$

From $E = \frac{V}{d}$ for uniform field

Question 2: The Unbearable Brightness of Being a Light Bulb

The figure to the right shows five identical bulbs connected to an ideal battery. All the bulbs are glowing. Rank in order, from brightest to dimmest, the brightness of bulbs A to E. Explain. *(It may be helpful to remember from class that the power dissipated across two points is P = V²/R).*
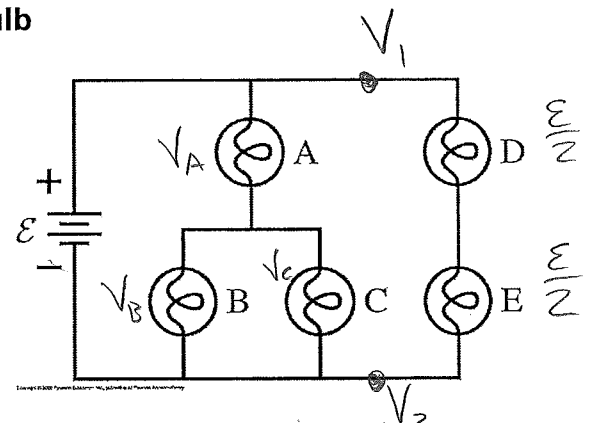
$V_1 - V_2 = \mathcal{E}$. D and E have the same resistance, so $\frac{\mathcal{E}}{2}$ is lost over each. $V_D = V_E = \frac{\mathcal{E}}{2}$. Because B and C are in parallel $V_B = V_C$.
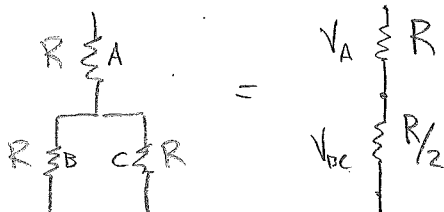
The equivalent resistance of B and C is less than that of A, so more voltage is lost over A than B or C, $V_A = \frac{2\mathcal{E}}{3}$ $V_{BC} = \frac{\mathcal{E}}{3}$.


So $\boxed{A > D = E > B = C}$

The equivalent resistance is 4Ω.

Question 3: Grounded

The three horizontal lines in the bottom left of the circuit is the symbol for ground. Ground sets the potential at that point equal to zero. Find the potential at point a. (*Hint: find the potential that is dropped across each component. Use these drops, or gains, to relate the potential at ground to that at point* a)
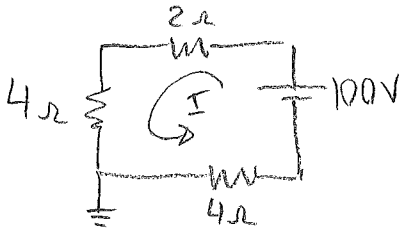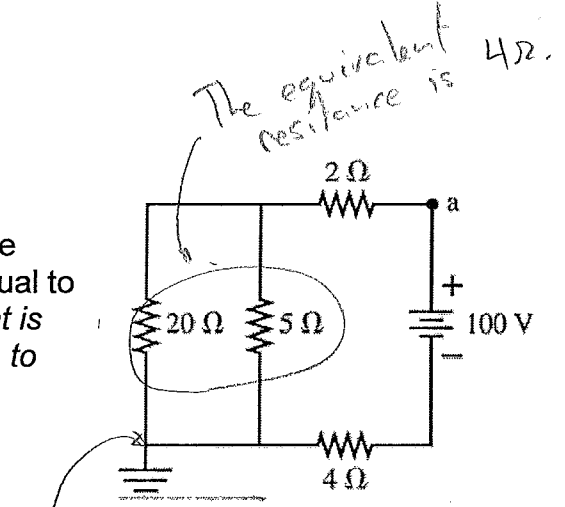
Kirchhoff tells us that

$$100v - (2Ω + 4Ω + 4Ω)I = 0 \implies I = 10A$$

$V = 0$

We can use this current to find the voltage dropped across all the resistors. If we start at ground and move across the 4Ω resistor we know that 40v is lost. When we move across the 100v battery we gain 100v, so $V_a = 60v$.

$V = 60v$

$V = 0$

$\Delta V = 40V$

$V = -40v$
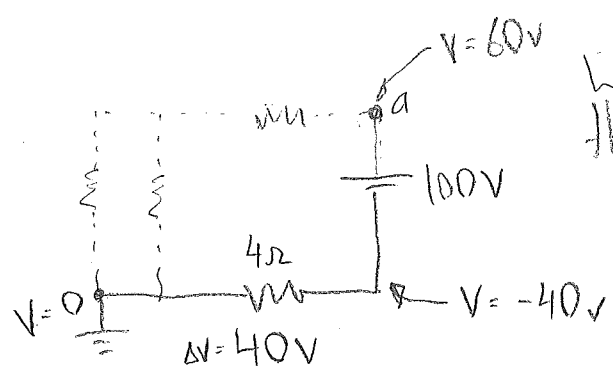
Question 4: A resistor?

In the circuit shown a new resistor is added between points A and B. Does the light bulb get brighter, dimmer, or stay the same? Explain.
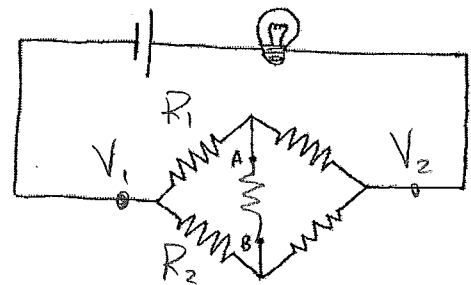
Because $R_1 = R_2$, the same amount of voltage must be lost across them. This means that $V_A = V_B$. We know that $P = \frac{V^2}{R}$, where V is the voltage drop across R. Because $V_A = V_B$, $V = 0$ and no power is dissipated.

Question 5: The time constant

The circuit shown includes a light bulb with resistance R, a capacitor with capacitance C, and a battery with voltage V. When the switch is flipped from A to B, the process is not instantaneous. Let's estimate the amount of time it takes to discharge the circuit.
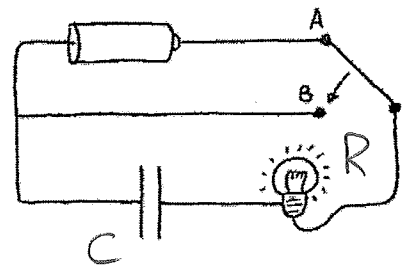
a) Looking at the circuit, what are the possible quantities the discharge time could depend on?

The two quantities are R and C.

b) Can you combine these quantities such that you get something that has units of time?

$V = IR$

$$[R] = \left[\frac{V}{I}\right] = \frac{(\text{potential})(\text{time})}{(\text{charge})}$$

$$[C] = \left[\frac{Q}{V}\right] = \frac{(\text{charge})}{\text{potential}}$$

$$[RC] = \frac{(\text{potential})(\text{time})}{(\text{charge})}\left(\frac{\text{charge}}{\text{potential}}\right) = (\text{time}). \quad RC \text{ has units time.}$$

c) Use Kirchhoff's loop law to get a differential equation describing the RC circuit. Remember, the voltage drop across a capacitor is given by $V = Q/C$ and current can be written in terms of the charge as $I = -dQ/dt$ (i.e., the current is positive as the charge on the capacitor decreases).
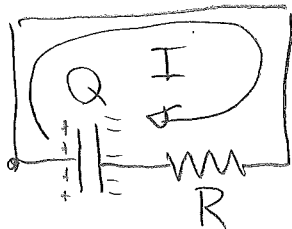
$$-IR + \frac{Q}{C} = 0 \qquad \text{from Kirchhoff.}$$

$$\Rightarrow \boxed{\frac{dQ}{dt} = \frac{Q}{RC}}$$

d) Find a solution for the equation you found in c).

A solution is $Q(t) = Q_0 e^{-t/RC}$. We can use guess a check to verify. Notice that we can define $\tau = RC$ to be the characteristic time of the decay. This gives us an estimate for the time it takes to discharge the capacitor. It's also what we guessed in b)!
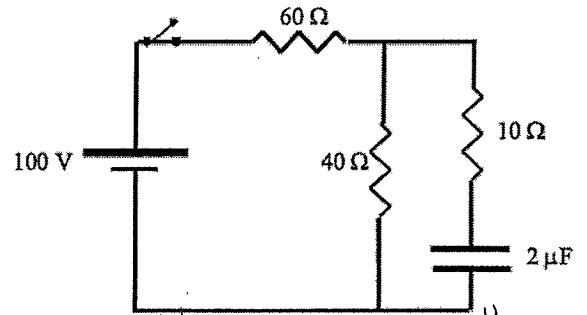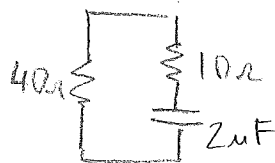
Question 6: RC Circuits

a) The switch has been closed for a long time, what is the potential across the capacitor?

When the switch is on for a long time the capacitor has no current flowing in it. The voltage across the capacitor is the same as the 40Ω resistor = 40V.

b) When the switch is opened what value of R is used for the time constant?

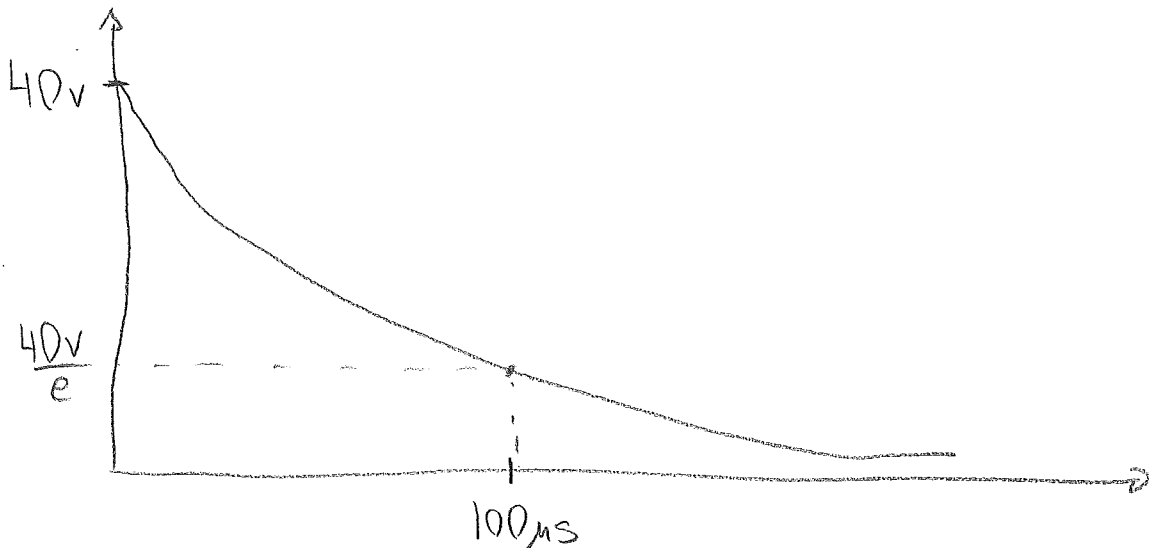The 60Ω resistor is not part of the circuit any more. The resistance is 50Ω.

c) Calculate the charge on the capacitor after two time constants of time have past.

$$\tau = 50\Omega (2\mu F) = 100\mu s \qquad Q_o = CV = 2\mu F \, 40v$$
$$= 8 \times 10^{-5} C$$

$$Q(2\tau) = (8 \times 10^{-5} C) e^{-2\tau/\tau}$$

$$= \frac{8 \times 10^{-5} C}{e^2} = 1.08 \times 10^{-5} C$$

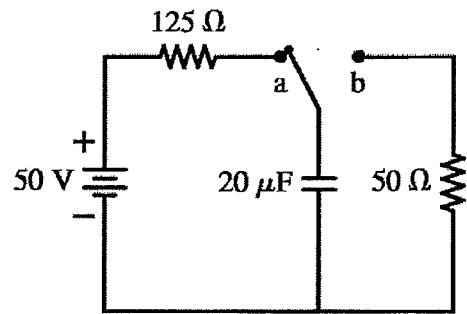d) Plot the voltage across the capacitor as a function of time. $\quad V = V_o \, e^{-t/\tau}$

$40v$

$\dfrac{40v}{e}$

$100\mu s$

Extra Practice 1: There and Back Again

The switch in the figure to the right has been in position **a** for a very long time. It is suddenly switched to position **b** for 1.25 ms and then flipped back to **a**.


a) Plot the potential across the capacitor as a function of time. What does the plot of the voltage across the resistor look like?

b) Calculate how much energy was dissipated by the resistor (i.e., how much energy is lost by the capacitor?)

Extra Practice 2: Switched.

When the switch is thrown to the left in the figure, the plates of the capacitor C_1 acquire a potential difference 100V and C_2 is initially uncharged. The switch is now thrown to the right. What are the final charges Q_1, Q_2 on the corresponding capacitors?
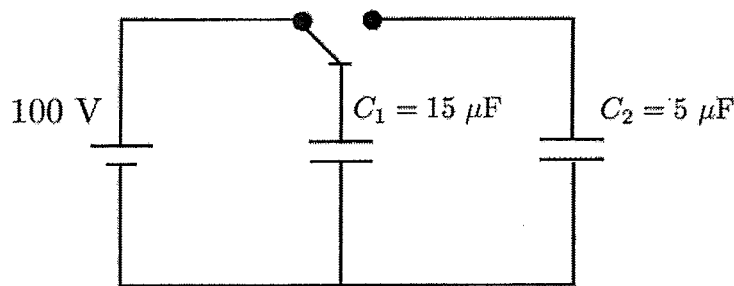
The trick is to realize that the charge is conserved and spread over both capacitors. What is the voltage across each capacitor after the switch is flipped?

Extra Practice 3: Two Loop Circuit

Find the set of equations that describes the current going through each resistor. Define a current in each independent leg of the circuit. The idea is to use Kirchhoff's loop law to get three equations that contain the three unknown current.

(If you want more work, solve for the currents)