① For this problem, we need to look up some information. First (according to the Wikipedia article on pupils) a typical pupil diameter is 1.5mm in bright light, and 8mm in dim light. So the area is

$$A = \pi \left(\frac{d}{2}\right)^2 = \begin{cases} 7.1 \times 10^{-6}\ m^2 & \text{bright light} \\ 2 \times 10^{-4}\ m^2 & \text{dim light.} \end{cases}$$

Next, we need to know how large the flux of energy in sunlight is. This is known as the solar constant, and is about $1.4\ kW/m^2$. This is spread over a range of wavelengths, but concentrated in the visible range, so for our estimate, we can just pick an average wavelength, which is around 550nm. Now, the flux of energy through our pupil will be some total power given by

$$P = 7.1 \times 10^{-6}\ m^2 \times 1.4 \times 10^3\ W/m^2$$

$$\approx 9.9 \times 10^{-3}\ J/s$$

This translates to a number of photons per second of

$$N = \frac{P}{E_{photon}} = \frac{P}{(hc/\lambda)} = \frac{9.9 \times 10^{-3}\ J/s \times 550 \times 10^{-9}\ m}{3 \times 10^8\ m/s \times 6.6 \times 10^{-34}\ J \cdot s}$$

$$\approx \boxed{2.8 \times 10^{16} \text{ per second}}$$

b) The intensity of a magnitude 30 star is

$$2.512^{30+26.7} = 4.8\times10^{22}$$

times less than the intensity of light from the sun. The Hubble telescope diameter is

$$\frac{2.4\text{m}}{1.5\text{ mm}} = 1600$$

times bigger than our pupil, so the area is 2.56×10^{6} times larger. We have flux = intensity times area. So, the flux of photons from the mag. 30 star into the Hubble telescope is

$$\frac{2.56\times10^{6}}{4.8\times10^{22}}$$

times the flux of solar photons into our eyes. This gives

$$\frac{2.56\times10^{6}}{4.8\times10^{22}}\times 2.8\times10^{16} = 1.5 \text{ photons/second}$$

③ In order to produce the intensity pattern shown after lots of photons hit the screen, the probability of a single photon hitting the screen in the region $2\text{mm} < x < 4\text{mm}$ must be $\frac{1}{4}$. If we send in just two photons, the chances of both hitting in $2\text{mm} < x < 4\text{mm}$ would be $\frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$.

The chances of neither hitting is $\frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$.

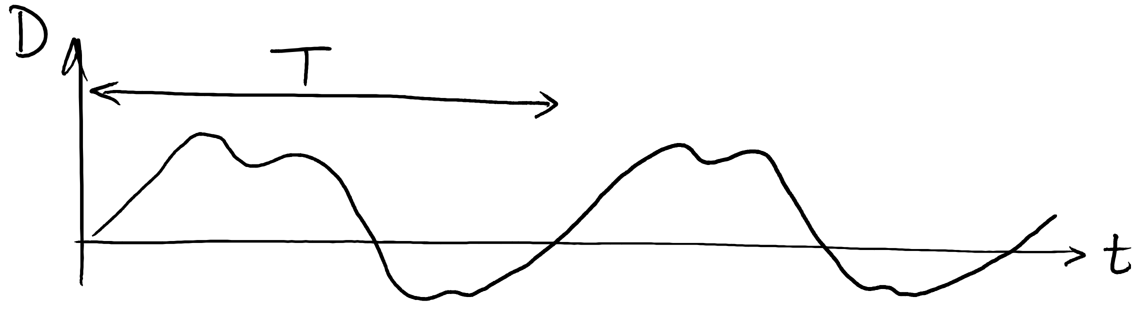
The chances of just one hitting are

$$1 - \frac{1}{16} - \frac{9}{16} = \frac{3}{8}.$$

(another way to get this is to say that the chances of the first hitting in $2 < x < 4$ and the second not are $\frac{1}{4} \times \frac{3}{4} = \frac{3}{16}$. The chances of the first not hitting and the second hitting are $\frac{3}{4} \times \frac{1}{4} = \frac{3}{16}$. So combining, we get $\frac{3}{8}$).

(3)

Let's plot the history graph for a musical note:



The note sounds like a note because it is periodic, with some period $T = \frac{1}{\text{freq.}}$.

To recognize that the sound is periodic, it must last at least a few periods. So the minimum length of a note is $\frac{1}{\text{freq}} \times$ (a number greater than 1).

A particle in QM with well-defined momentum has a particular wavelength in its wavefunction. This only makes sense if the wave fn. has a width of one or more of these wavelengths. So we have uncertainty in position.