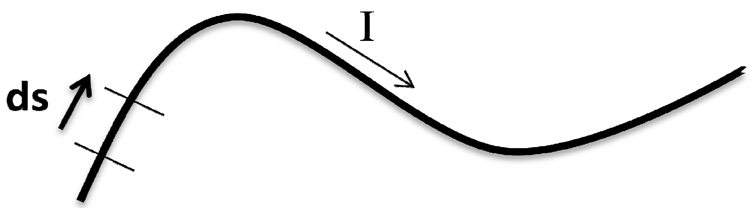
Question 1



In class, we saw the result for the magnetic field of a moving point charge and the force from a magnetic field on a point charge. In this question, we'll understand how to use these to find the field from a current in a wire or the force on a current-carrying wire. Consider the little segment of wire shown above.

a) Suppose the charges in the wire have an average velocity v in the direction of the current (imagine here that they are positive charges flowing in the direction of the current). How much time dt does it take for the charges to travel the distance ds? (*answer in terms of ds and v)*

$dt = ds / v$

b) In this amount of time, how much charge flows into the segment? (*answer in terms of ds, v, and I, using your previous result for dt)*

$Q = I\, dt = I\, ds / v$

c) Using your result from b), rewrite Q **v** (the moving charge in the segment times its velocity) in terms of the current I and the vector **ds**.

$Q\, \mathbf{v} = I\, \mathbf{ds}$

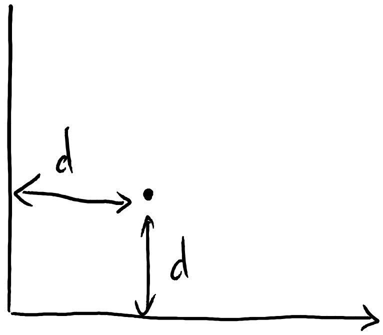
Using this result, we can start with equation (32.4) for the field from a point charge to get the field from the small segment of wire shown above. To get the field from the whole wire, we need to integrate this over the wire.

d) Read Example 32.3 (The magnetic field of a long, straight wire) to see an example of this. What is the result for the magnitude of the field at a distance d from a wire carrying current I?

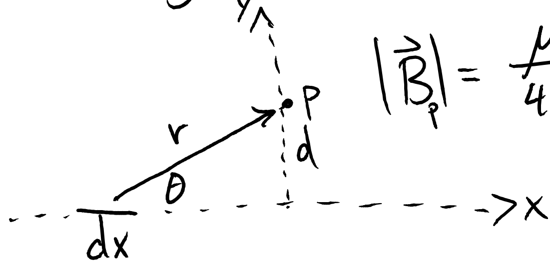
$B = \mu_0\, I / (2\, \pi\, d)$

Use the result to find the magnetic field 1cm away from a long straight wire carrying a current of 1A. Answer in Tesla (T):

$B = \mu_0\, I / (2\, \pi\, d) = (4\, \pi \times 10^{-7}\ N/A^2)\ (1\ A)/(2\, \pi\ 0.01m) = 2 \times 10^{-5}\ T$



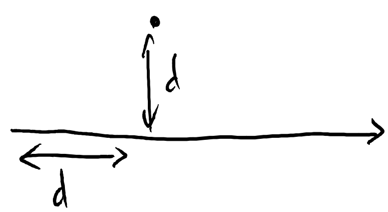
The magnetic field from all segments of this wire is out of the page, so we only need to add up the magnitudes. The field calculation is the same as example 32.3 for an infinite wire, but we now change the limits on the integration. We have for a segment dx as shown:



$$|\vec{B}_p| = \frac{\mu_0}{4\pi} I \frac{|d\vec{s} \times \hat{r}|}{r^2} = \frac{\mu_0 I}{4\pi} \frac{dx}{r^2} \cdot \sin\theta$$

$$= \frac{\mu_0 I}{4\pi} \frac{dx}{d^2 + x^2} \cdot \frac{d}{\sqrt{d^2 + x^2}}$$

So for the lower segment:



$$|\vec{B}| = \frac{\mu_0 I}{4\pi} d \int_{-d}^{\infty} \frac{dx}{(d^2+x^2)^{3/2}}$$

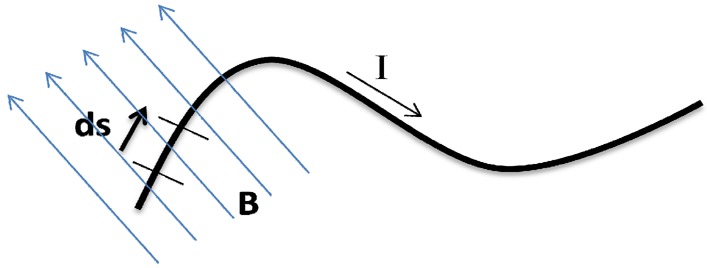
$$= \frac{\mu_0 I}{4\pi} d \cdot \left. \frac{x}{d^2(d^2+x^2)^{\frac{1}{2}}} \right|_{-d}^{\infty}$$

$$= \frac{\mu_0 I}{4\pi d} + \frac{\mu_0 I}{4\pi d} \cdot \frac{\sqrt{2}}{2}$$

The result from the other segment is

$$|\vec{B}| = \frac{\mu_0 I}{4\pi} d \int_{-\infty}^{d} \frac{dx}{(d^2+x^2)^{3/2}}.$$

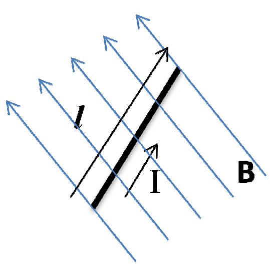
This has the same answer, so the net field is: $\frac{\mu_0 I}{2\pi d}\left(1 + \frac{\sqrt{2}}{2}\right)$.



Now let's find the force on the same small segment of wire from some other magnetic field (not the one created by this segment of wire)

e) Use equation (32.17) for the force on a point charge together with your result from c) to find the force on this segment of wire in a magnetic field.

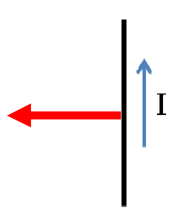
Starting with $\mathbf{F} = Q\,\mathbf{v} \times \mathbf{B}$ and replacing $Q\,\mathbf{v} \rightarrow I\,\mathbf{ds}$ we get $\mathbf{F} = I\,\mathbf{ds} \times \mathbf{B}$



f) When there is a long straight segment of wire and the magnetic field is constant, the result above applies to the whole segement (replacing **ds** with the vector **l** along the wire with magnitude equal to the length). Make this replacement and check your result with equation 32.25 from the text.

$\mathbf{F} = I\,\mathbf{l} \times \mathbf{B}$

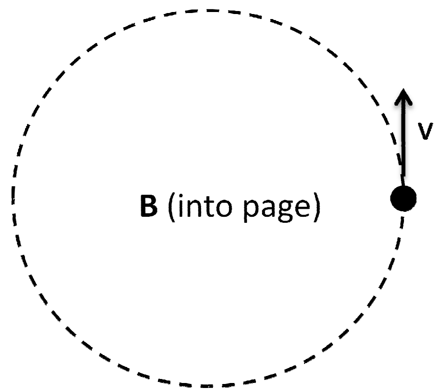
e) How much force is there on 10cm of wire carrying 1A in a magnetic field of 1T aligned perpendicular to the wire? Indicate the direction of the force on the wire if the magnetic field points into the page.



$|\mathbf{F}| = I\,\mathbf{l} \times \mathbf{B} = 1A\,(0.1m)\,(1T) = 0.1N$

Question 2

Charged particles traveling perpendicular to a constant magnetic field move in circular trajectories.



a) Find an expression for the radius of the circle, clearly explaining the logic. Answer in terms of q, B, v, and m, the mass of the particle.

For circular trajectories at constant speed, the acceleration is related to the speed and the radius of the circle by

$a = v^2/R$.

In this case, the acceleration is due to the magnetic force. Its magnitude is given by

$a = F/m = q\ v\ B\ /\ m$.

The velocity is always perpendicular to the magnetic field, so the $\sin(\theta)$ in the cross product is always equal to 1.

Equating our two expressions for the acceleration, we get:

$v^2/R = q\ v\ B\ /\ m$

so we find that

$R = mv\ /\ (q\ B)$

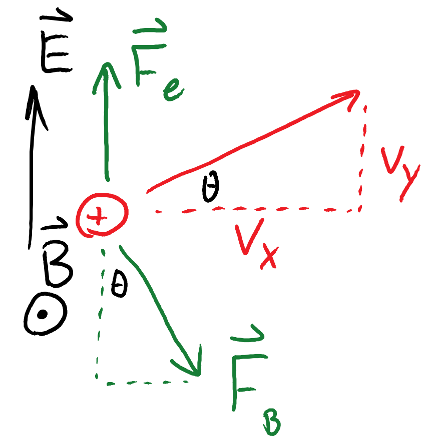
b) If the velocity increases, what happens to the period of the orbit?

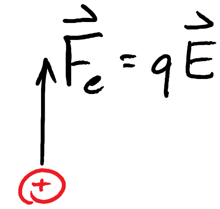
The period is T = circumference/ speed = $(2\ \pi\ R)\ /\ v$. Using the result from a), this gives

$T = 2\ \pi\ m\ /\ (q\ B)$

This doesn't depend on v, so the period doesn't change if the velocity increases.

PROBLEM 5:





We have both electric and magnetic forces on the plankton. Suppose that at some time, the velocity of the plankton is $\vec{v} = (v_x, v_y)$.

Then the electric force is

$$\vec{F}_E = \vec{E} q = (0, qE)$$

and the magnetic force is

$$\vec{F}_B = q(\vec{v} \times \vec{B})$$

From the diagram above, we have:

$$\begin{aligned} (F_B)_x &= |\vec{F}_B| \sin\theta \\ &= q |\vec{v} \times B| \sin\theta \\ &= q v B \times \frac{v_y}{v} \\ &= q B v_y \end{aligned}$$

Similarly,

$$(F_B)_y = -q\,|\vec{v}\times\vec{B}|\cos\theta$$
$$= -q\,v\,B\,\frac{v_x}{v}$$
$$= -\,q\,B\,v_x$$

So $\vec{F}_B = qB\,(v_y, -v_x)$.

Using Newton's 2nd Law, we now have:

$$\frac{dv_x}{dt} = \frac{1}{m}F_x = \frac{qB}{m}\cdot v_y$$

$$\frac{dv_y}{dt} = \frac{1}{m}F_y = \frac{qE}{m} - \frac{qB}{m}v_x$$

From these, we can use Euler's method to find the trajectory (using also that $\frac{dx}{dt} = v_x$ and $\frac{dy}{dt} = v_y$).

Extra for experts

To solve the equations directly, we first notice that one solution is

$$v_y = 0 \qquad v_x = \frac{E}{B}$$

If the initial velocity is $(0, \frac{E}{B}) = (0, 1\,m/s)$, then the velocity will remain constant.

To find the general solution, we can make a substitution:

$$V_x = \frac{E}{B} + w_x \qquad V_y = w_y.$$

Then:

$$\frac{dw_x}{dt} = \frac{qB}{m} w_y$$

$$\frac{dw_y}{dt} = -\frac{qB}{m} w_x$$

We recognize these equations as the ones satisfied by sin and cos. Specifically for our case $w_x(0) = \frac{E}{B}$ and $w_y(0) = 0$, so we have:

$$w_x = \frac{E}{B} \cos\left(\frac{qB}{m} t\right) \qquad w_y = -\frac{E}{B} \sin\left(\frac{qB}{m} t\right)$$

So the solution is

$$V_x = \frac{E}{B} + \frac{E}{B} \cos\left(\frac{qB}{m} t\right) \qquad V_y = -\frac{E}{B} \sin\left(\frac{qB}{m} t\right)$$

This gives the velocities as functions of time.

To find the positions, we just anti differentiate:

$$\frac{dx}{dt} = \frac{E}{B} + \frac{E}{B}\cos\left(\frac{qB}{m}t\right) \quad \text{and} \quad x(0) = 0$$

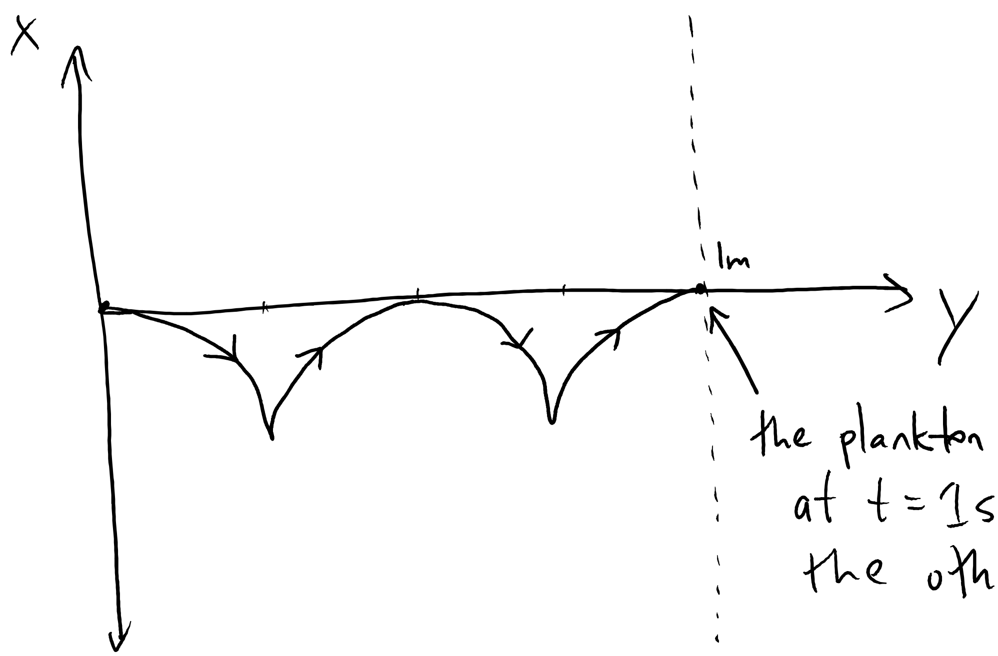
$$\Rightarrow \quad x(t) = \frac{E}{B}t + \frac{Em}{qB^2}\sin\left(\frac{qB}{m}t\right)$$

$$\frac{dy}{dt} = -\frac{E}{B}\sin\left(\frac{qB}{m}t\right) \quad \text{and} \quad y(0) = 0$$

$$\Rightarrow \quad y(t) = \frac{E}{B}\cos\left(\frac{qB}{m}t\right) - \frac{E}{B}$$

chose constant to make $y(0)=0$

The trajectory looks like this:



the plankton reaches 1 m at $t = 1s$, so ties with the other plankton.