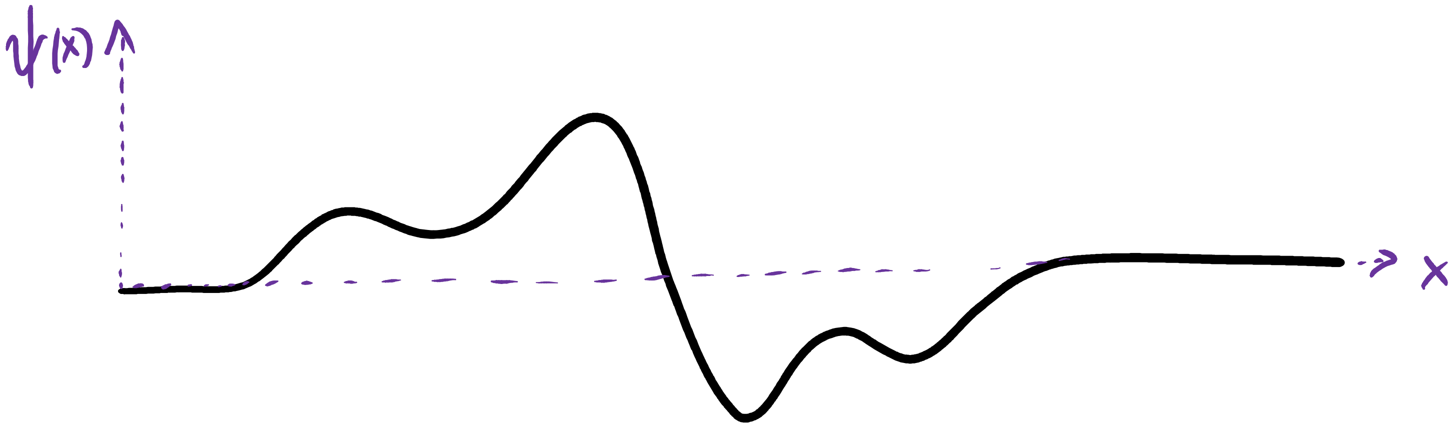
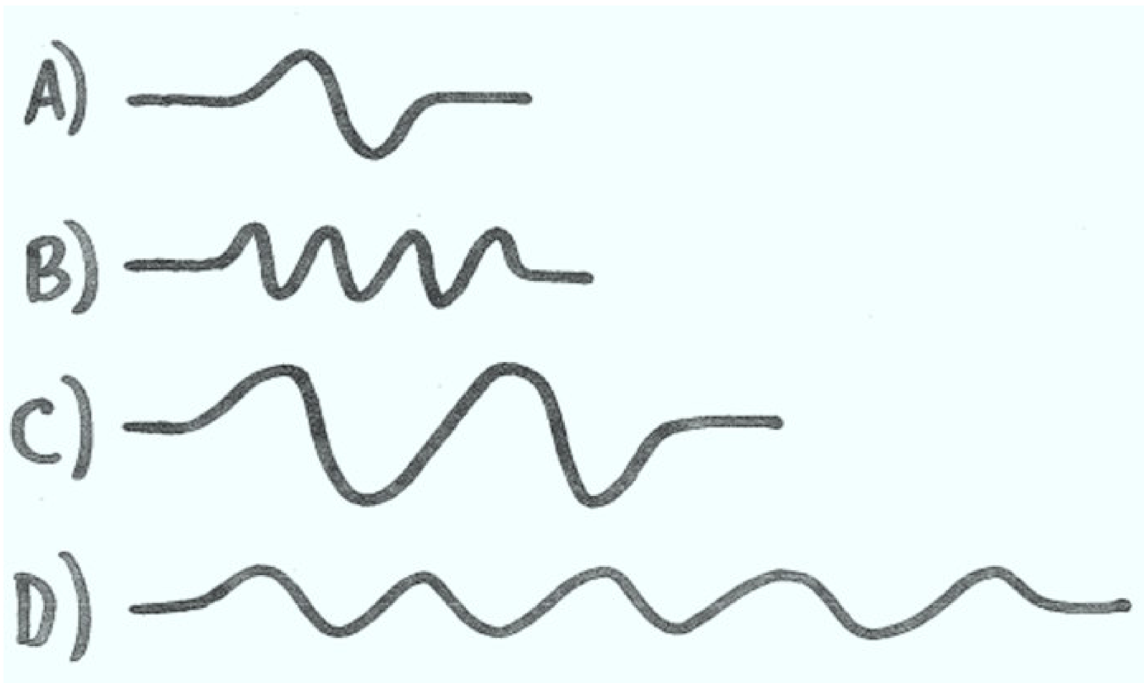
At time t=0, the wavefunction for a particle is shown below.

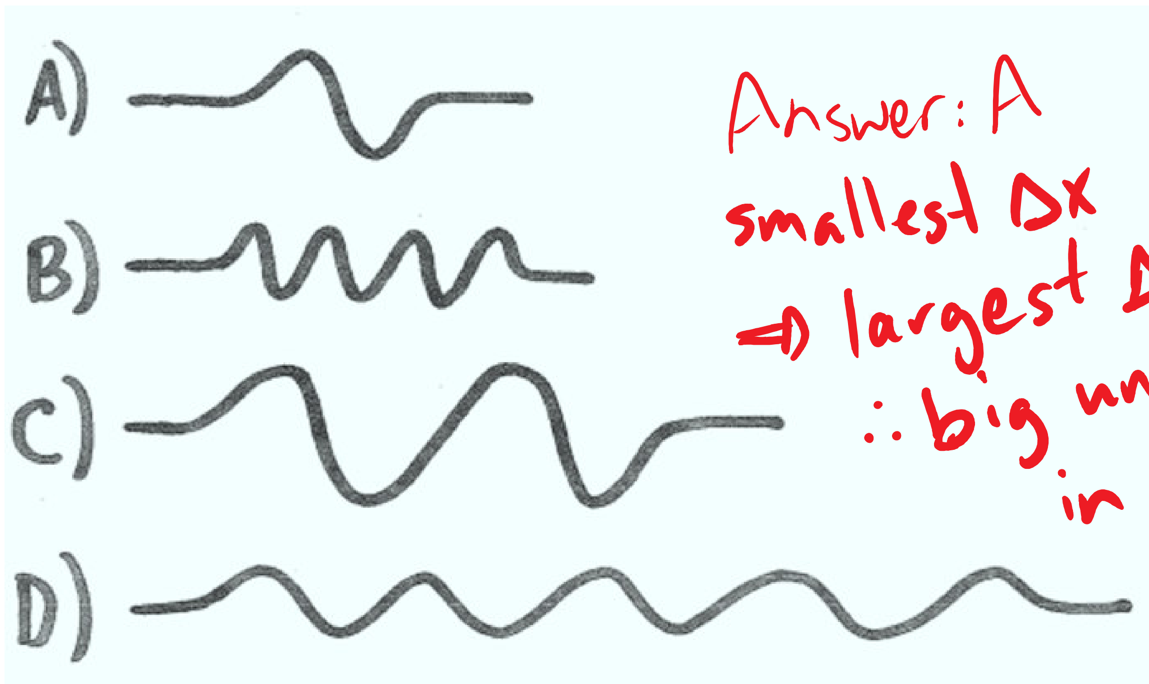


How can we determine (in principle) the wavefunction at a later time t=T?

Wavepackets for travelling particles tend to spread out as they move. Which of the wavepackets below will spread out the fastest?



Which of the wavepackets below will spread out the fastest?



Answer: A

smallest Δx

$\Rightarrow$ largest Δp

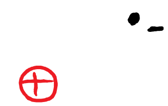
$\therefore$ big uncertainty in velocity

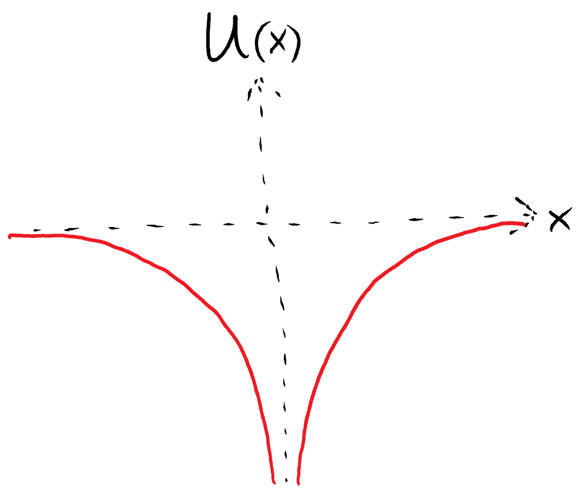
SO FAR: Pretty boring physics

wave packets travel & spread.

What if we have forces?

e.g. Coulomb force of proton in H atom.



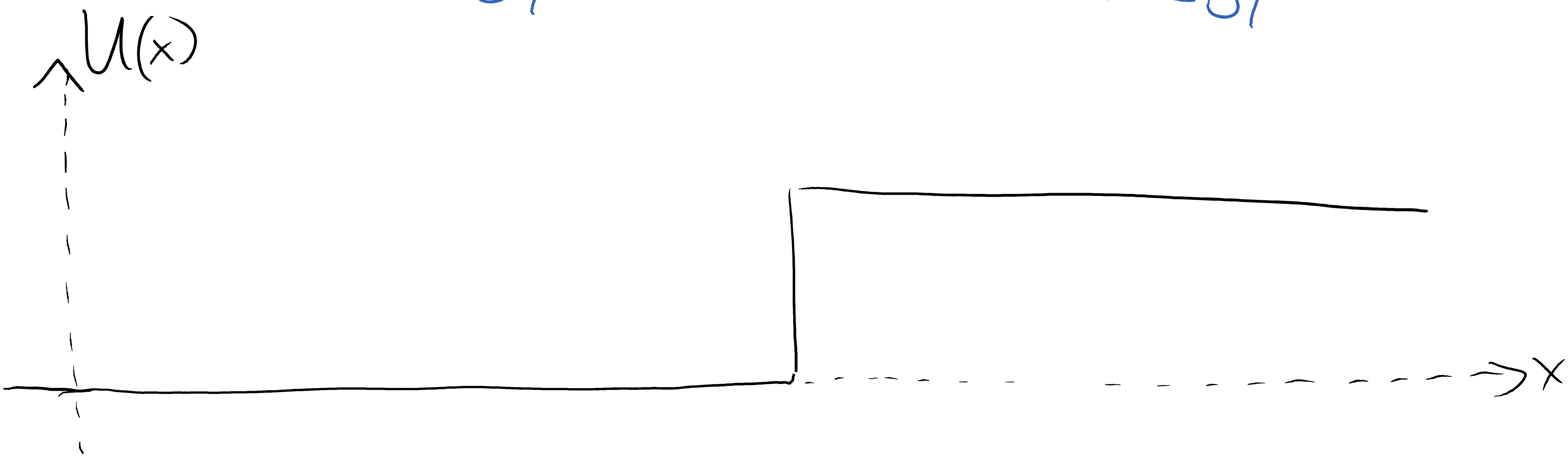


describe by potential $U(x)$

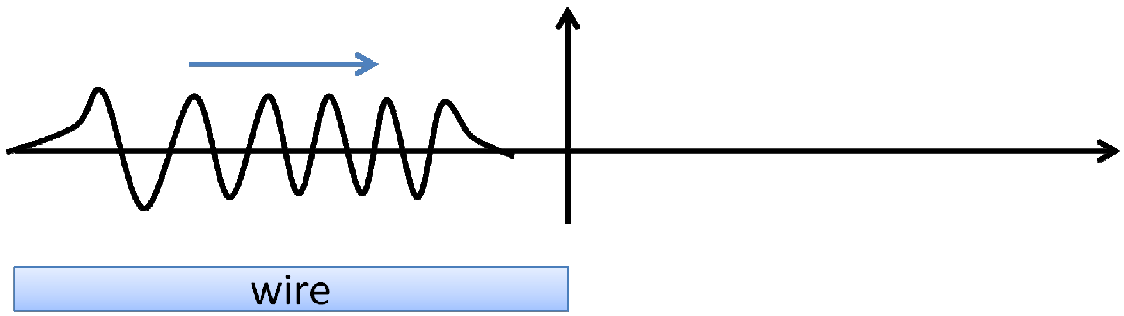
Simple example: locally constant potential

inside wire:
lower potential energy

outside wire:
higher potential energy

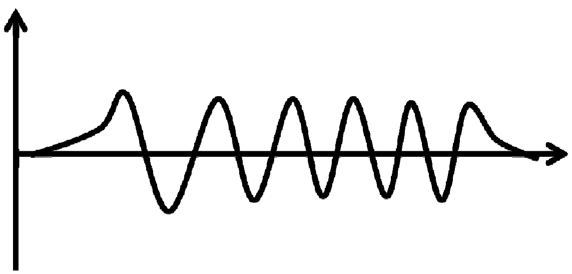


CLICKER QUESTION:

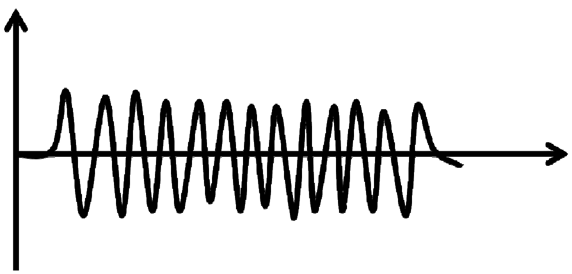


An electron travelling along a wire with a nearly definite momentum has just enough energy to escape from the wire. If the wavefunction just before the electron exits the wire is shown above, which of the following might represent the wavefunction in the region x>0 outside the wire sometime later?

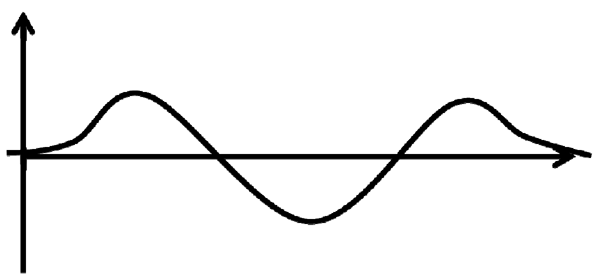
A



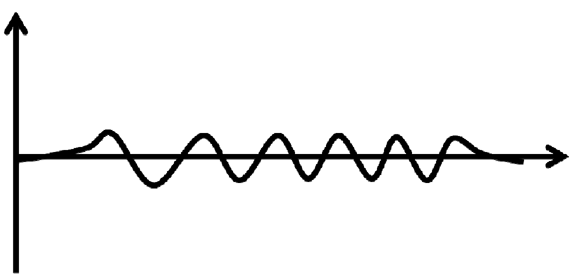
B



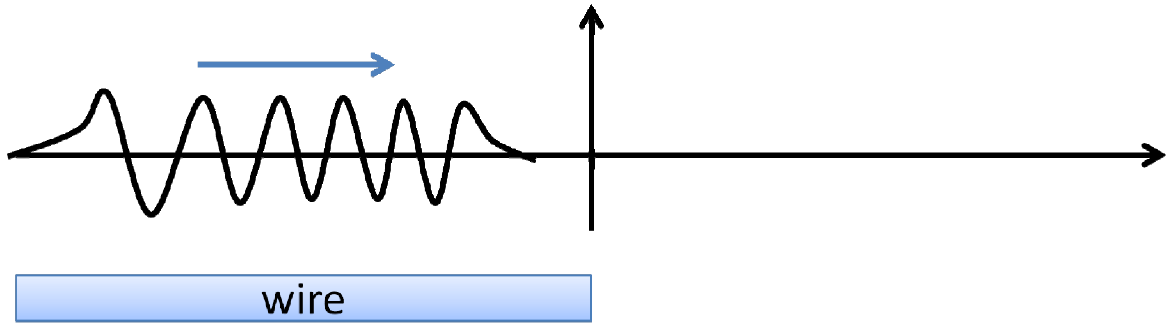
C



D

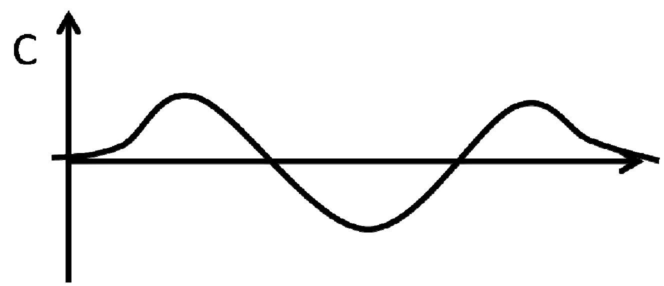


Extra: what can we say about the frequency of the wave in the x > 0 region?



An electron travelling along a wire with a nearly definite momentum has just enough energy to escape from the wire. If the wavefunction just before the electron exits the wire is shown above, which of the following might represent the wavefunction in the region $x>0$ outside the wire sometime later?

Correct answer:



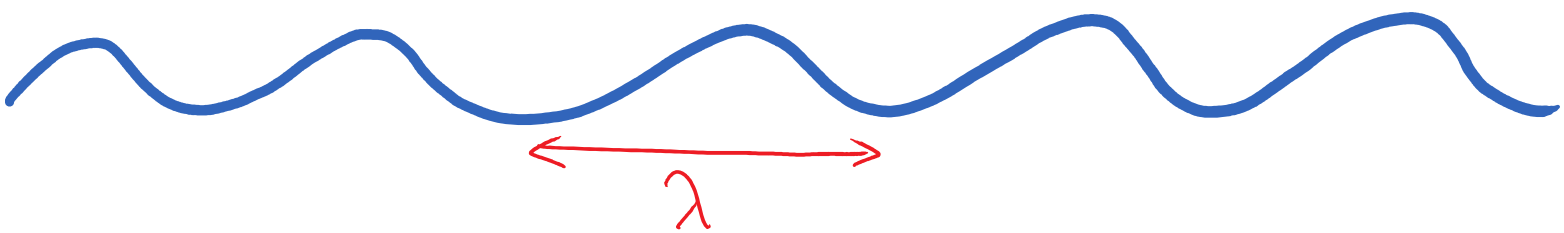
By energy conservation, the electron must have less kinetic energy in the region where the potential is larger. Thus, its momentum will be smaller, so its wavelength will be larger here.

Extra part: the frequency should be the same everywhere, since frequency corresponds to total energy (or since the oscillation in the $x>0$ region is caused by the oscillation in the $x<0$ region, so the frequencies should match).

Lesson:

In regions with larger U:

- same frequency (since same total energy)
- larger wavelength (since smaller momentum)



Still have: $\boxed{p = \frac{h}{\lambda}}$ and $\boxed{hf = E} = \frac{1}{2}mv^2 + U = \frac{p^2}{2m} + U$

$$\Rightarrow f = \frac{1}{h}\left(\frac{h^2}{2m\lambda^2} + U\right)$$

- Sinusoidal waves move with $v = \lambda \cdot f$
- Evolution for general wavefunctions by superposition

This is equivalent to a differential eqn:

$$\frac{d\psi}{dt} = (\text{number}) \cdot \frac{1}{m} \frac{d^2\psi}{dx^2} + U \cdot \psi$$

potential energy

☆ Correct eqn. even when U depends on x ☆

This is the full SCHRÖDINGER EQUATION

= Newton's 2nd Law for wavefunctions

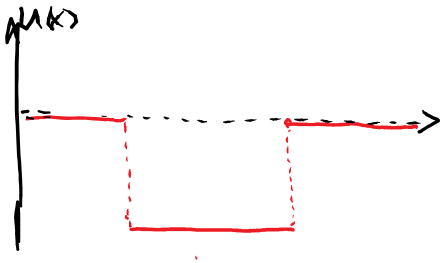
replaces:

$$\frac{dv}{dt} = \frac{F}{m}$$

$$\frac{dx}{dt} = v$$

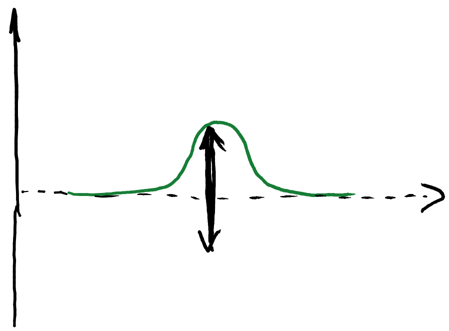
Simulate via computer:

- Find that valley in potential can "trap" wavefunction



- trapped wavefunctions oscillate w. specific frequencies: standing waves!

$\Downarrow$



interpretation: bound particles have specific "QUANTIZED" energies

WHY: Confined waves have specific frequencies/wavelengths

Same as guitar string / kelp horn.

Which wavefunctions simply oscillate with frequency f
i.e. correspond to particles with energy E?

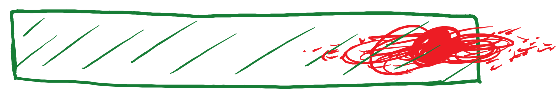
Look for solution

$$\psi(x,t) = \Psi(x) \times \text{oscillating function of time}$$

Works if

$$-\frac{\hbar^2}{2m}\frac{d^2\Psi}{dx^2} + U(x)\,\Psi = E\,\Psi$$

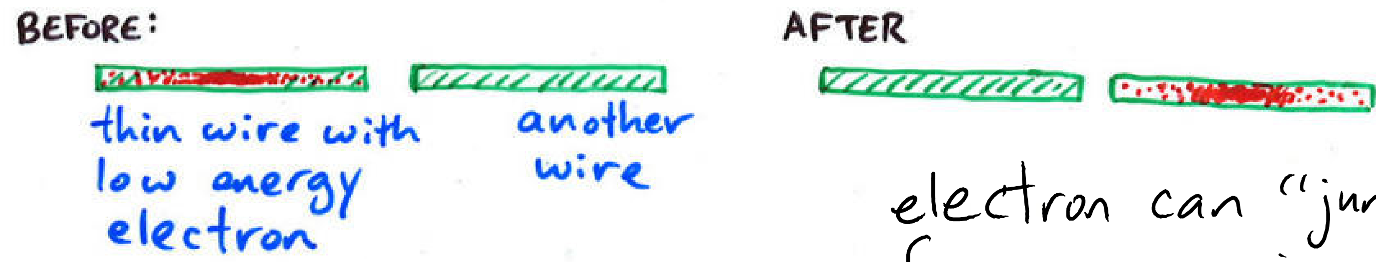
This is the "TIME-INDEPENDENT SCHRÖDINGER EQUATION"
(that you saw in chem.)



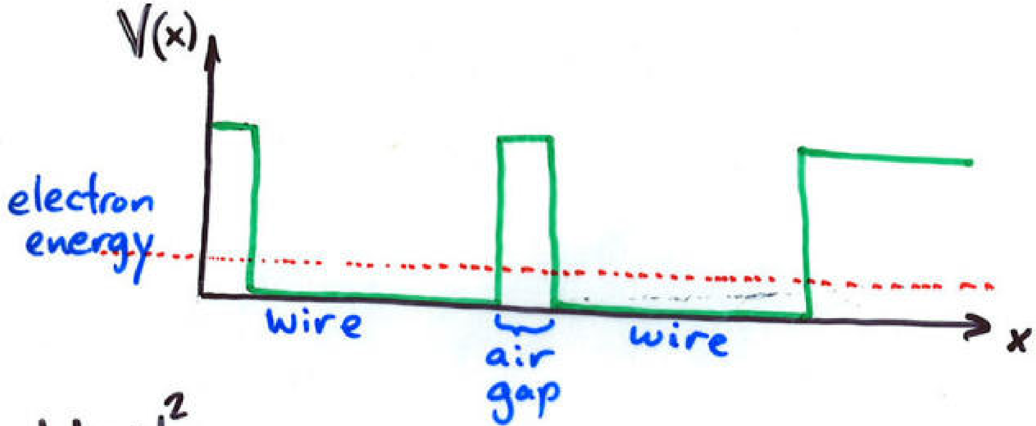
Cool fact: wavefunctions for trapped particles are never completely confined inside region where $E < U$.

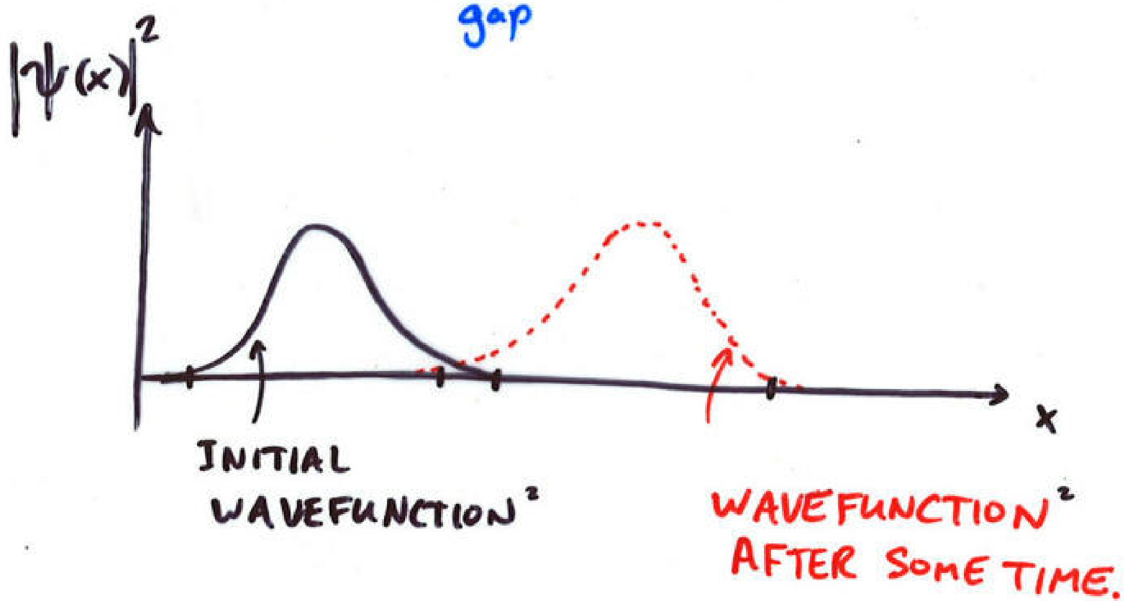
Small probability of finding particle where it doesn't have enough energy to be.

DRAMATIC CONSEQUENCE: TUNNELING

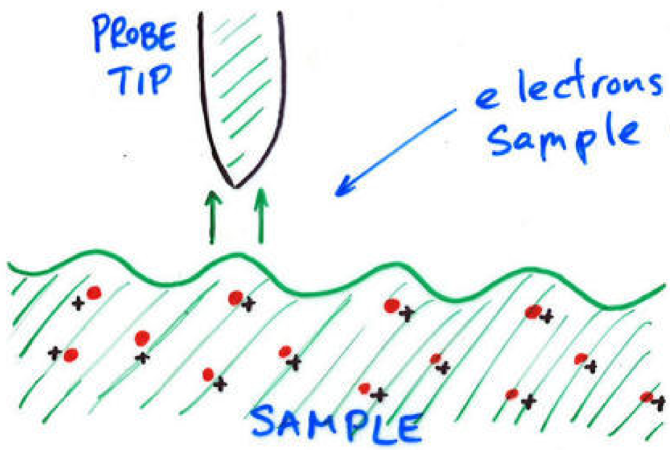


electron can "jump" from one wire to another through region with $U > E_{TOT}$!





SCANNING-TUNNELING MICROSCOPES

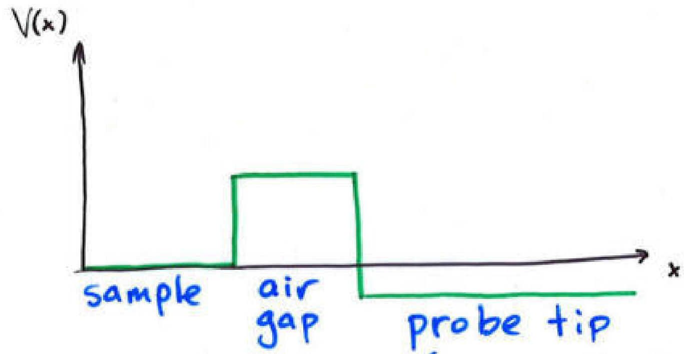


electrons can tunnel from sample to probe tip

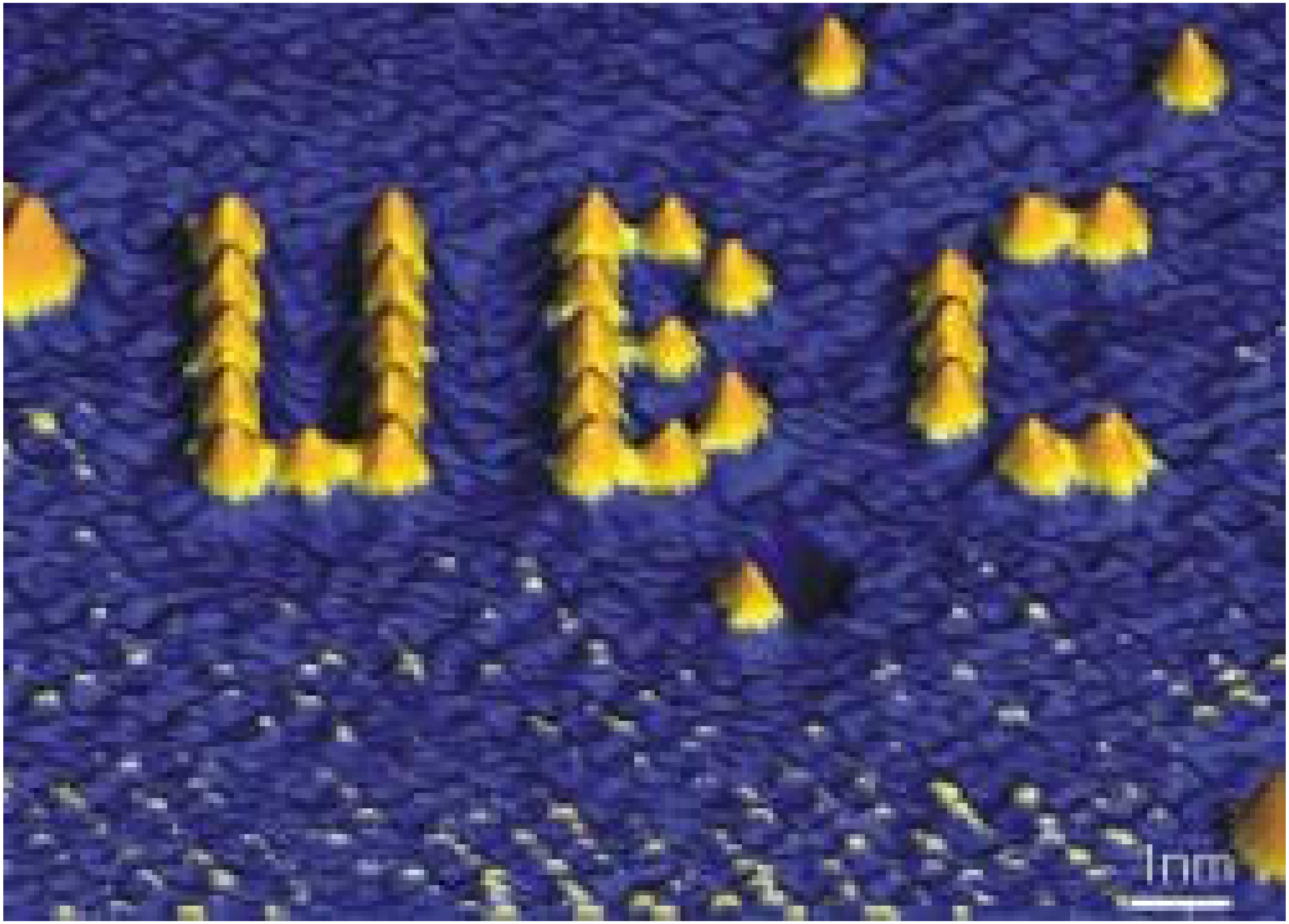
current $\propto$ rate of tunneling

$\propto e^{-a\cdot(\text{SEPARATION})}$

Move tip horizontally across surface + measure current to deduce height profile



(make V slightly <0 so net current of electrons into probe).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