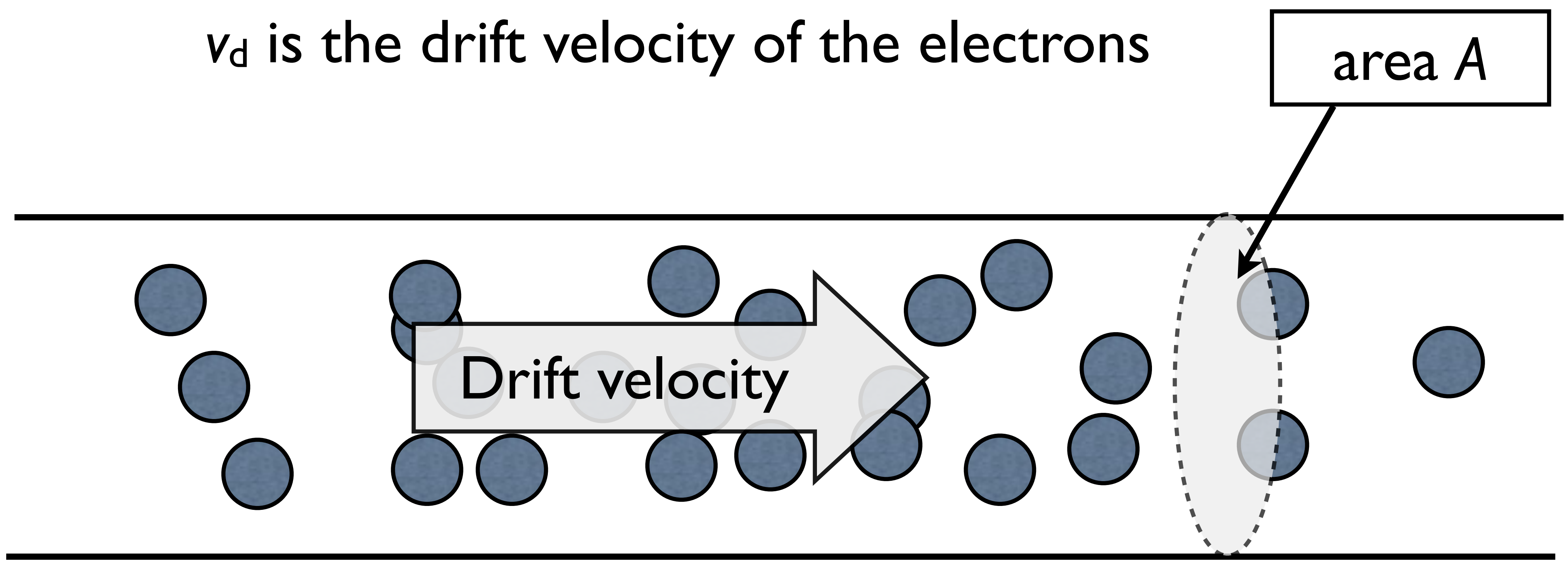
Current

The current, which is Coulomb's per second, is simply

$$I = e i_e = e n_e A v_\mathrm{d}$$

e is the charge is the electron
n_e is the density of electrons
A is the cross sectional area of the wire
v_d is the drift velocity of the electrons



Current and E-Field

Using the current from earlier, and the drift speed, the current is **proportional to the electric field!**

$$I = \frac{e^2 n_e \tau}{m_e} AE$$

which can be written as

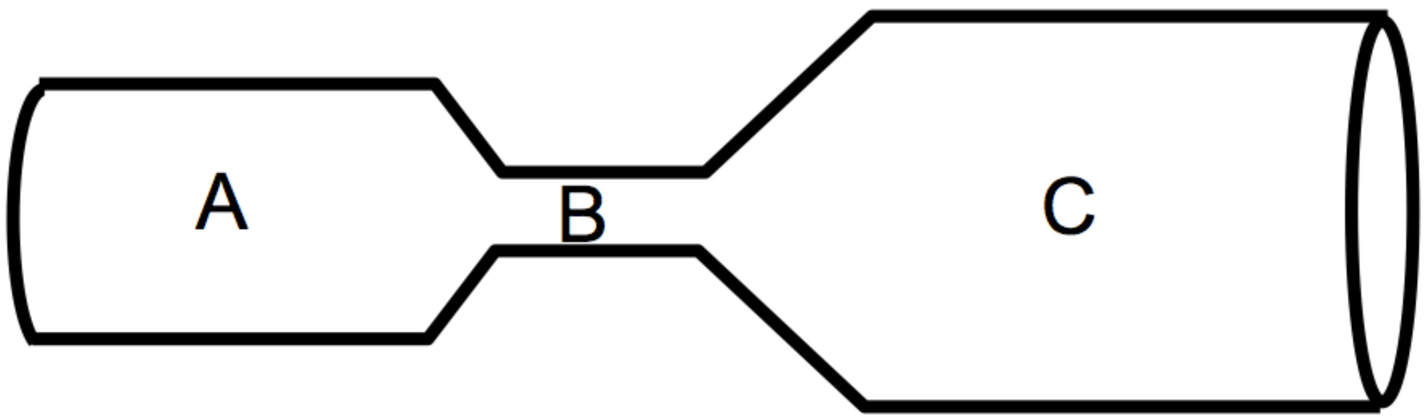
$$I = \sigma AE \quad \text{where} \quad \sigma = \frac{e^2 n_e \tau}{m_e}$$

The **material constant σ** is called the **conductivity** (SI unit = $\Omega^{-1}\,m^{-1}$).

The **inverse quantity** $\rho = \frac{1}{\sigma}$ is called the **resistivity** (SI unit: Ω m).

Clicker Question

A copper cylinder is machined to have the following shape. The ends are connected to a battery so that a current flows through the copper.

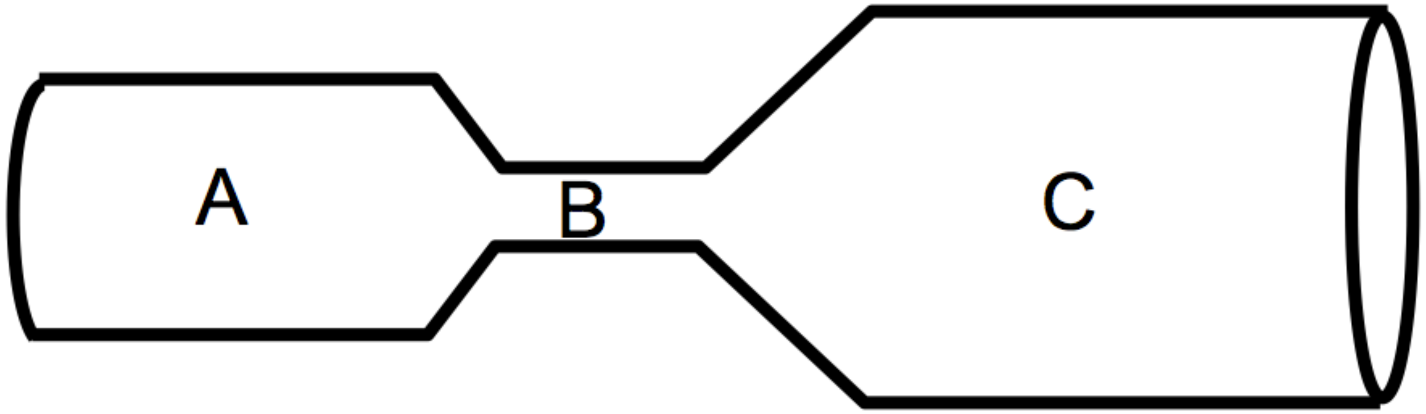


Which region has the greatest magnitude of current, I?

a) A
b) B
c) C
d) All three are the same
e) Not sure/ not enough info

Clicker Question

A copper cylinder is machined to have the following shape. The ends are connected to a battery so that a current flows through the copper.



Which region has the greatest magnitude of current, I?

a) A
b) B
c) C
d) All three are the same
e) Not sure/ not enough info

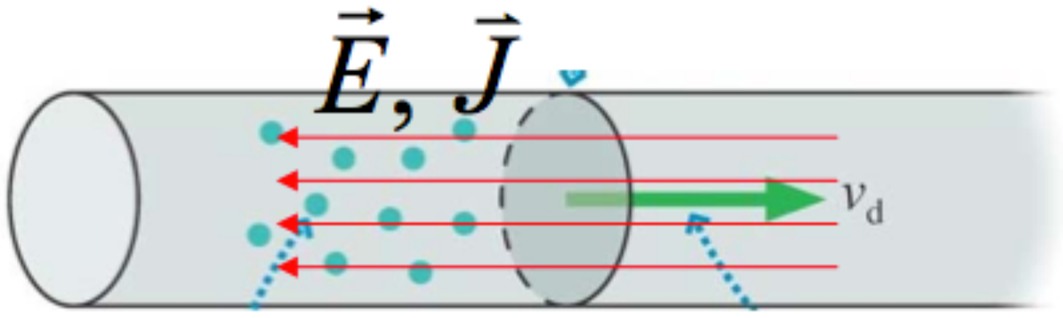
Current is conserved.
Think of water flow.

Current Density

The current divided by the cross sectional area gives us the current density,

$$J = \frac{I}{A} = \sigma E$$

A **microscopic quantity** that is proportional to the **electric field**.



Note: The current actually flows against the electric field.

Clicker Question

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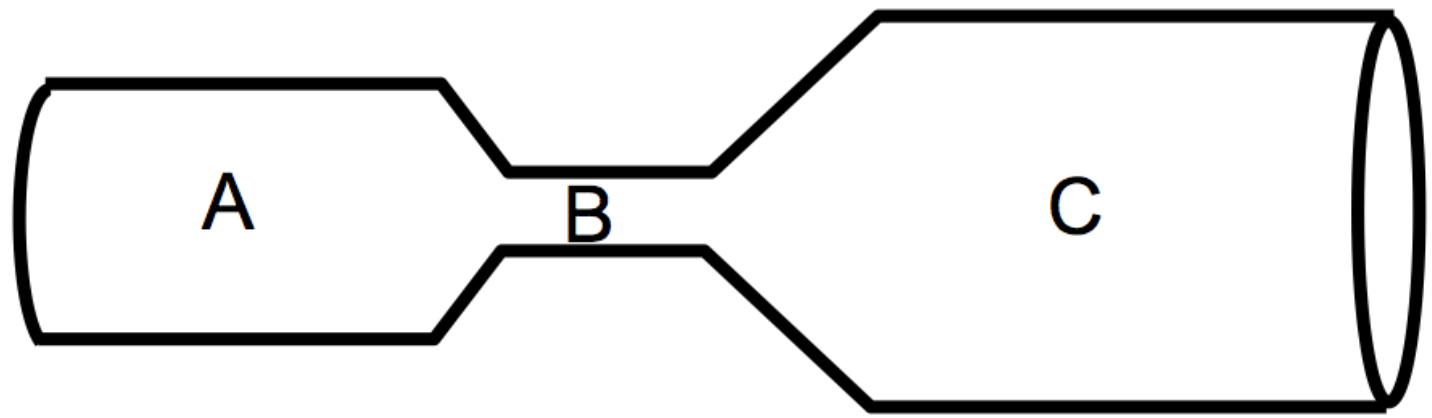

Which region has the greatest current density |J|?

a) A
b) B
c) C
d) All three are the same
e) Not sure/ not enough info

Clicker Question

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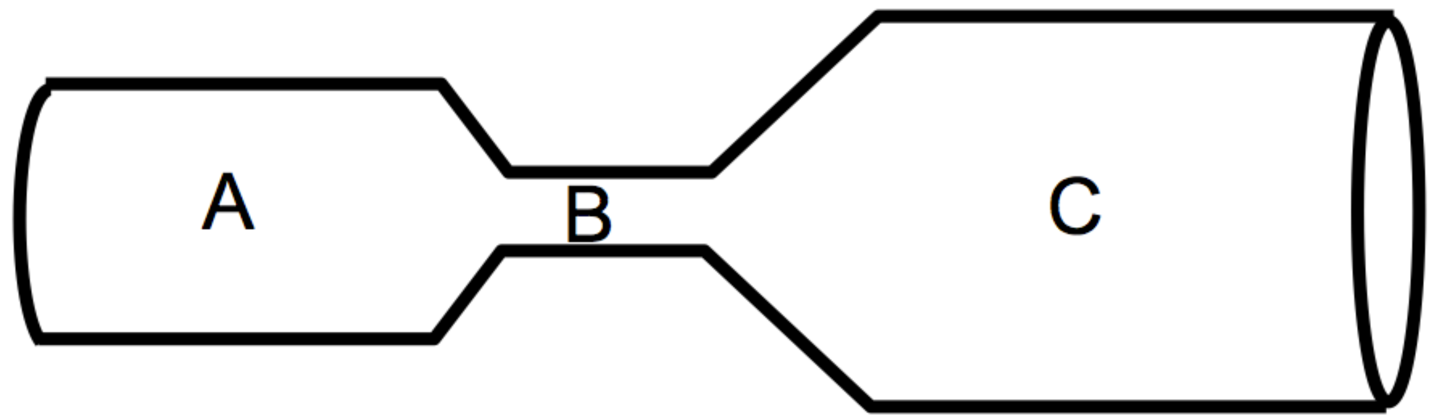


Which region has the greatest current density |J|?

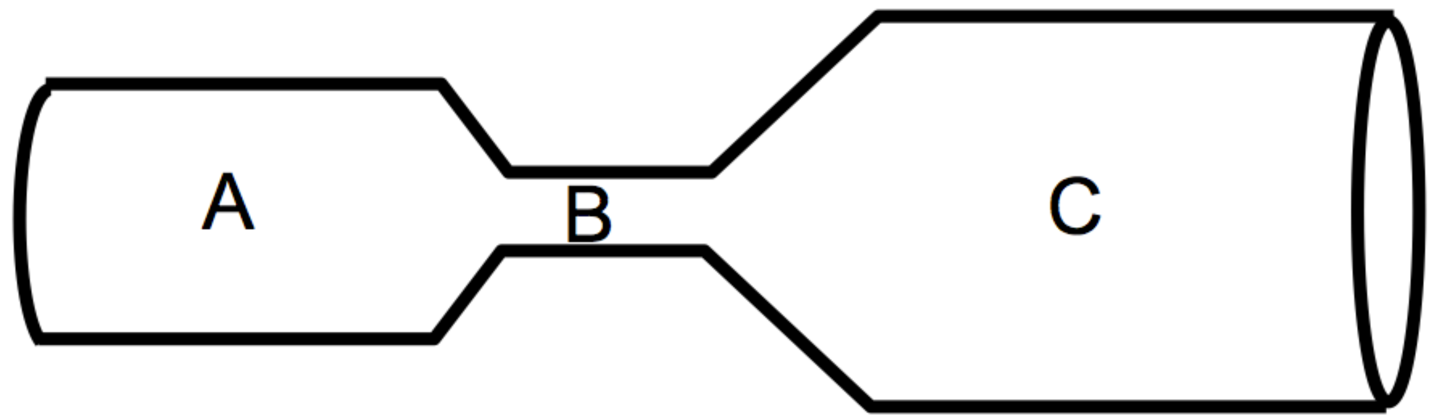
a) A
b) B
c) C
d) All three are the same
e) Not sure/ not enough info

Current is conserved so all the electrons must flow faster in the narrower part.

This also means that B has **greatest electric field (question from PIAZZA)**!

Clicker Question

A copper cylinder is machined to have the following shape. The ends are connected to a battery so that a current flows through the copper.

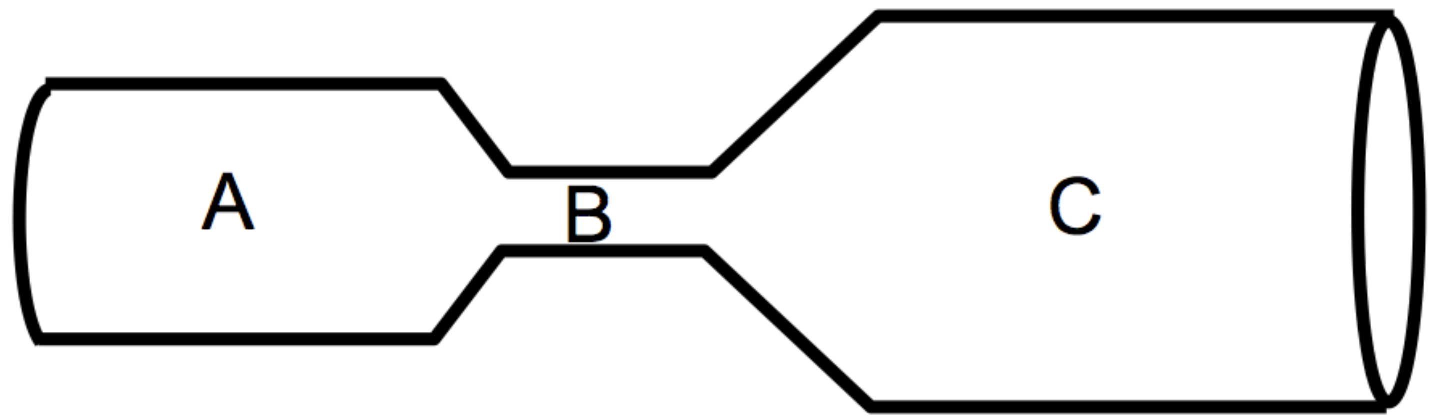


Which region has the greatest conductivity σ?

a) A
b) B
c) C
d) All three are the same
e) Not sure/ not enough info

Clicker Question

A copper cylinder is machined to have the following shape. The ends are connected to a battery so that a current flows through the copper.



Which region has the greatest conductivity σ?

a) A

b) B

c) C

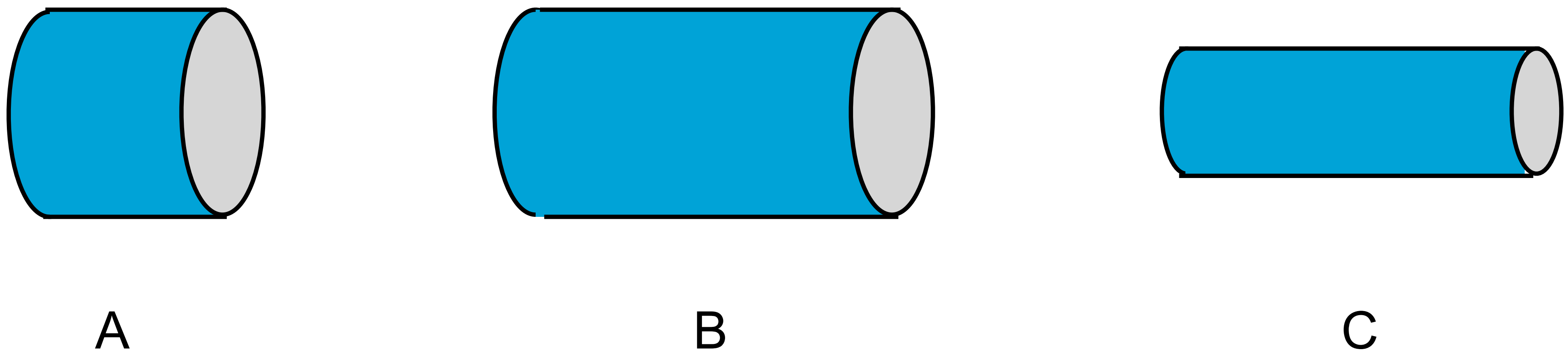
d) All three are the same

e) Not sure/ not enough info

Conductivity and resistivity are **microscopic properties** of the material.

Clicker Question

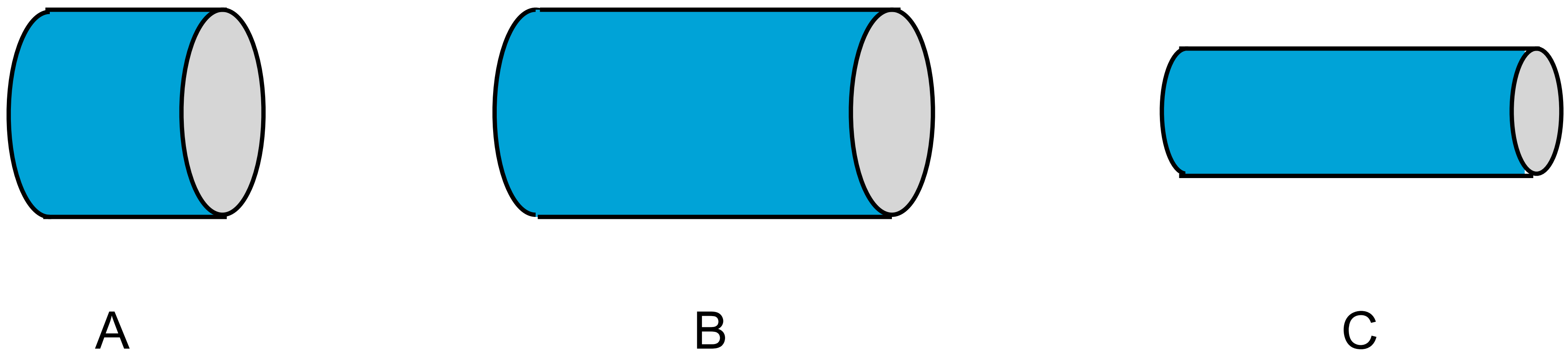
Rank these tubes by how hard it would be to push stuff (say towels) through them at the same rate.



a) A harder than B harder than C
b) C harder than B harder than A
c) B harder than C harder than A
d) A harder than C harder than B
e) C harder than A harder than B

Clicker Question

Rank these tubes by how hard it would be to push stuff (say towels) through them at the same rate.



a) A harder than B harder than C
b) C harder than B harder than A
c) B harder than C harder than A
d) A harder than C harder than B
e) C harder than A harder than B

Resistance and Resistivity

We can combine $I = \sigma A E$ and $V = Ed$ (but we're going to rename *d* to *L*) to get

$$I = \sigma \frac{A}{L} V$$

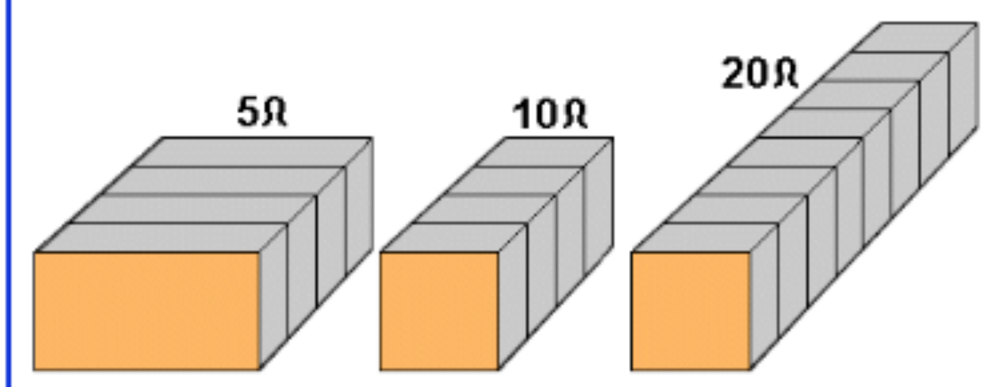
Ohm's Law !

$$\Delta V = IR$$

The **resistance R** of a real wire depends on the cross sectional area A and its length L:

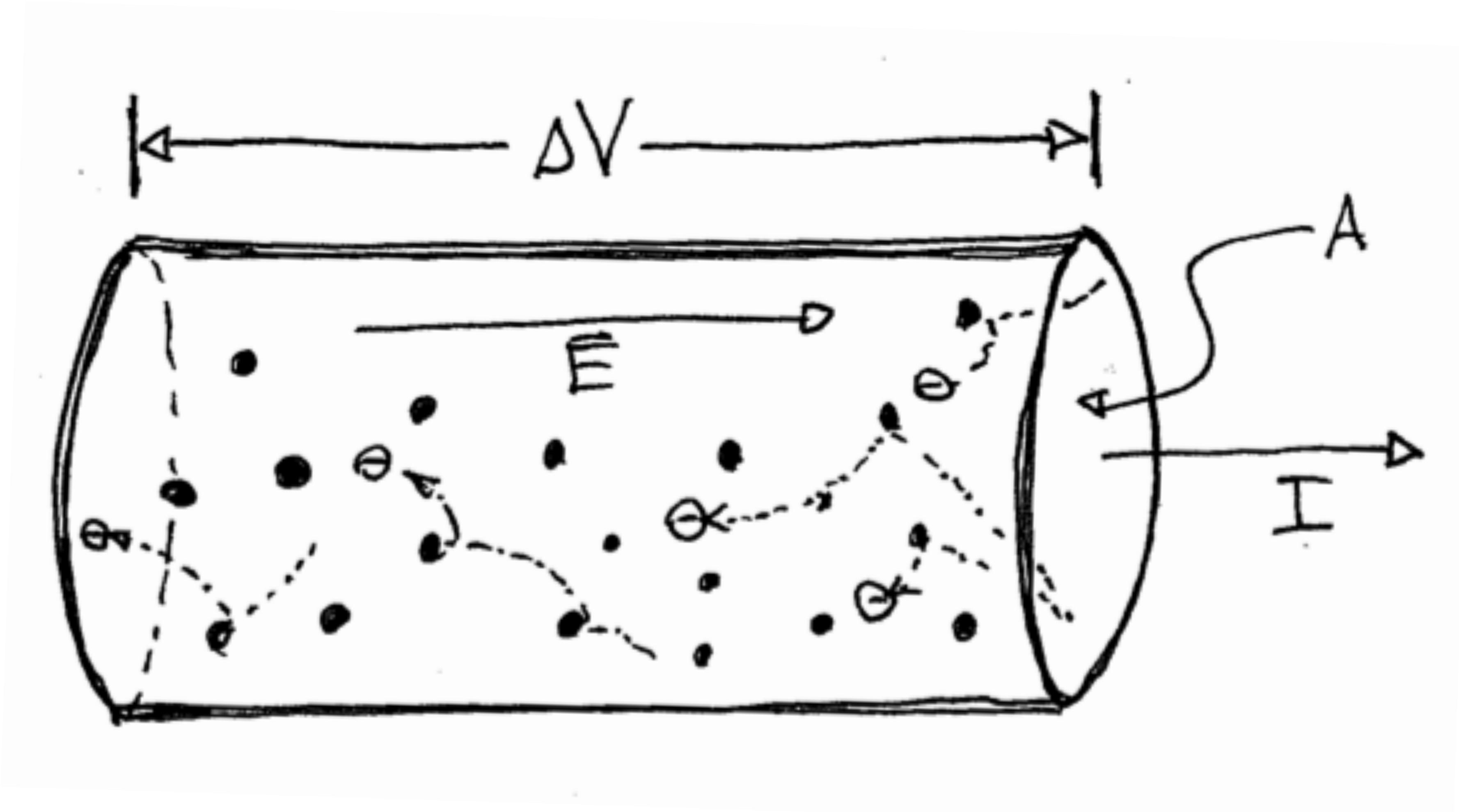
$$R = \rho \frac{L}{A}$$

A macroscopic property of the whole wire depending on the length and cross sectional area.



resistance of wire drops if wire has larger cross-sectional area but increases with length

Resistance



Higher voltage leads to a **larger electric field** and an increase in drift speed and and **increase in current**.

This leads to more collisions, more vibrations, and more energy loss (heat, light).

Battery Resistor Circuit

Power Dissipated in a Resistor

The **drop in voltage across a resistor** means that energy is being lost:

$$\Delta U = q\Delta V$$

The **rate of energy lost** is the power:

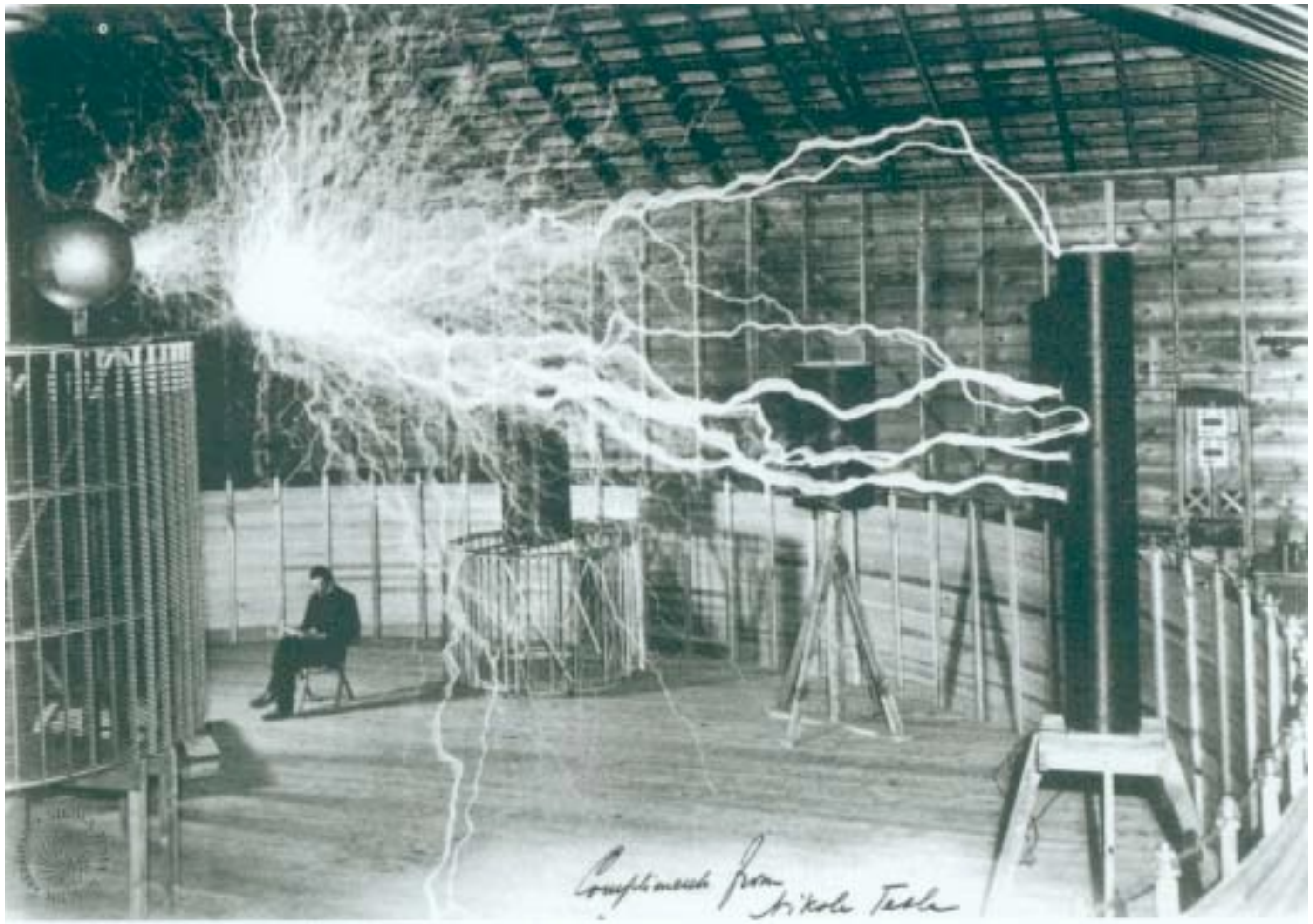
$$P = \frac{dU}{dt} = \frac{dq}{dt}\Delta V = I\Delta V$$

Using Ohm's Law:

$$P = I^2 R = \frac{V^2}{R}$$

This is the power dissipated in a component with resistance *R* and current *I* flowing through it.

Circuit Analysis



Circuits Galore
Problems 1 and 2

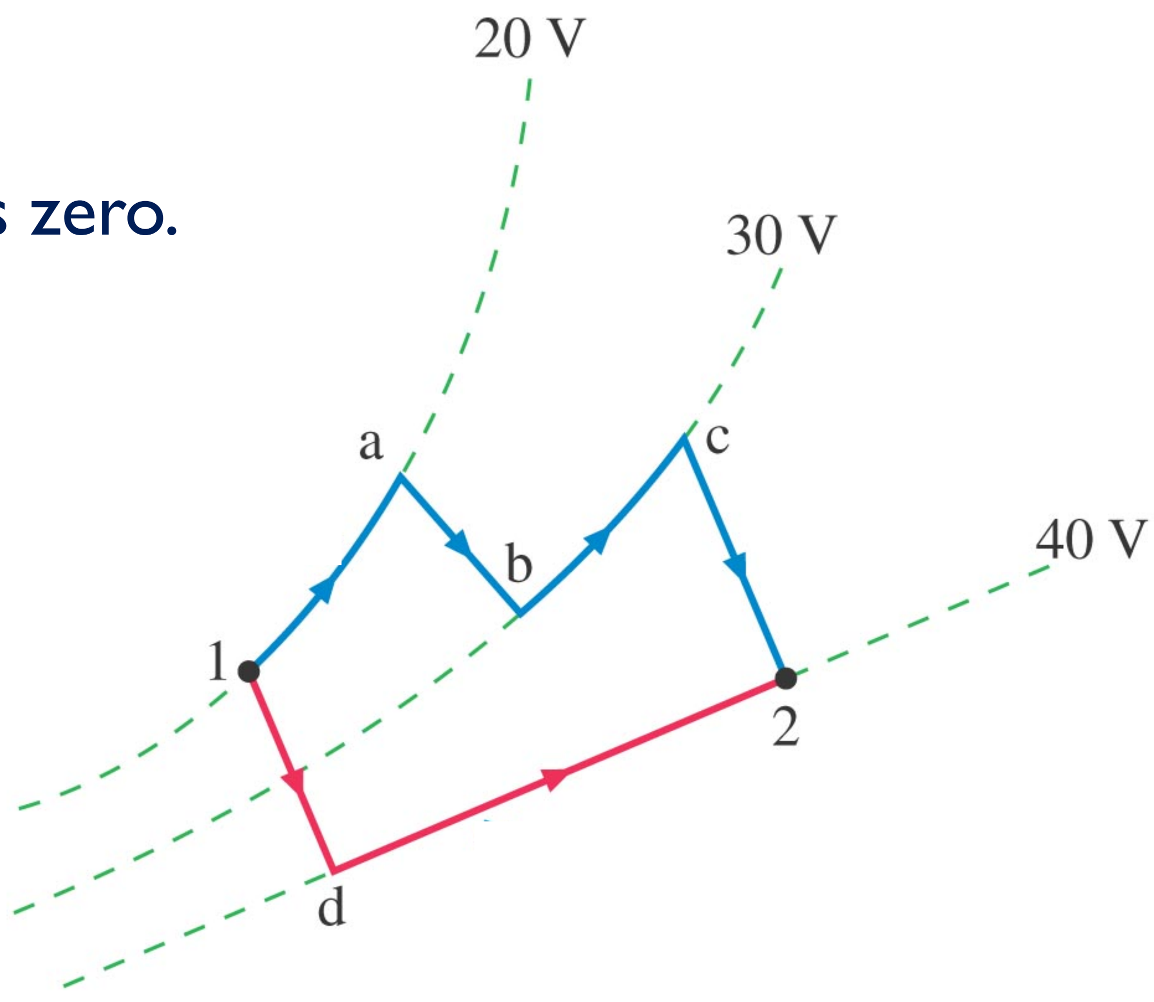
Kirchhoff's Loop law

A foundation of circuit analysis.

The loop law comes from path independence:

$$\Delta U = -\oint \vec{F} \cdot d\vec{s} = -q \oint \vec{E} \cdot d\vec{s} = 0$$

The change in energy around a closed path is zero.



We then know that *U* = *qV* gives

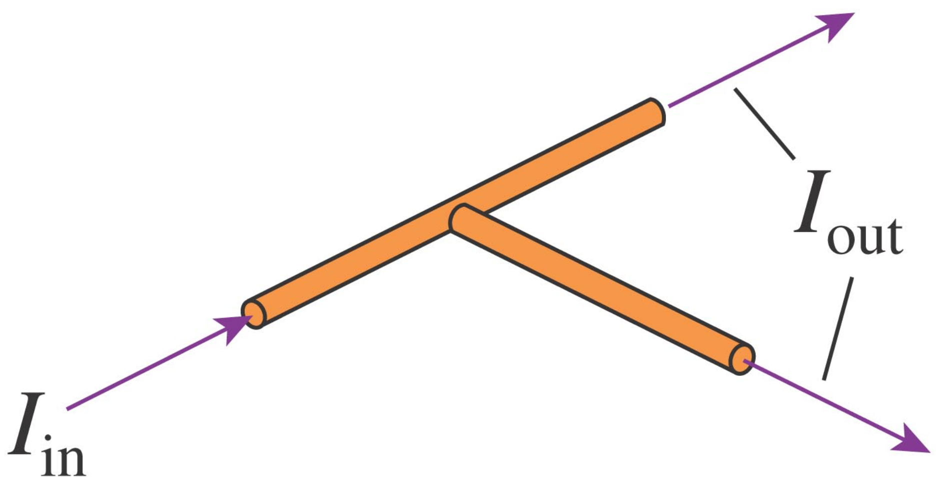
$$\Delta V_{\text{loop}} = 0$$

Which is the loop law.

Kirchhoff's Laws

Kirchhoff's Laws are summarized as:

$$\sum I_{in} = \sum I_{out} \quad \text{at a junction}$$



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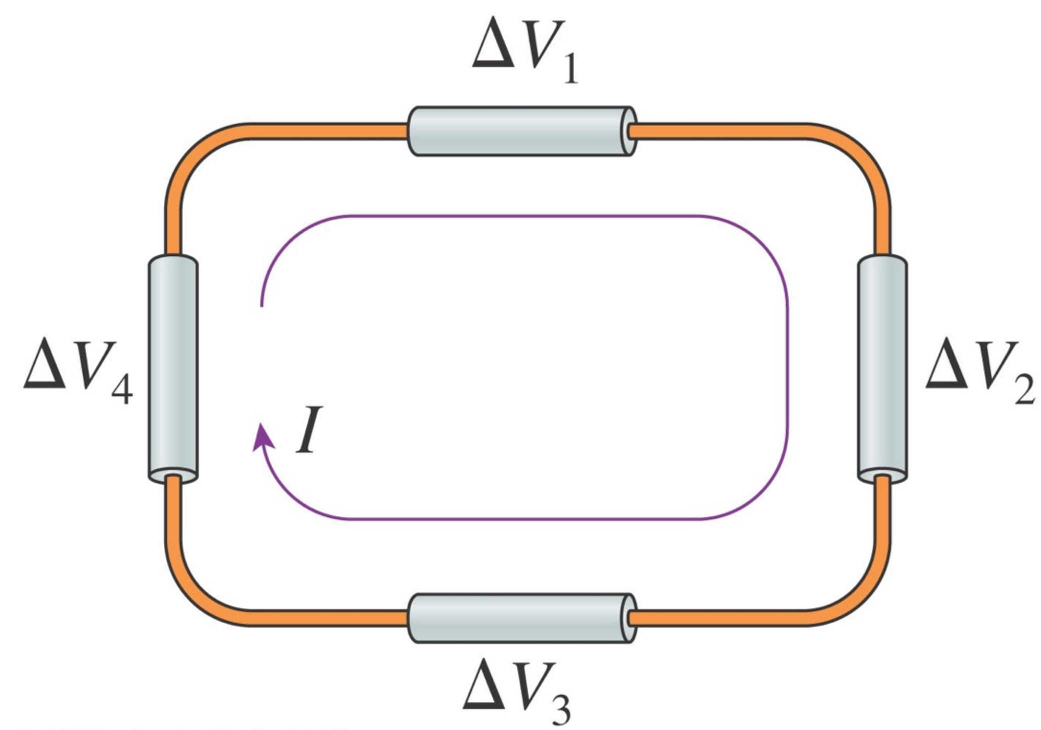
$$\sum_i \Delta V_i = 0 \quad \text{around closed loop}$$



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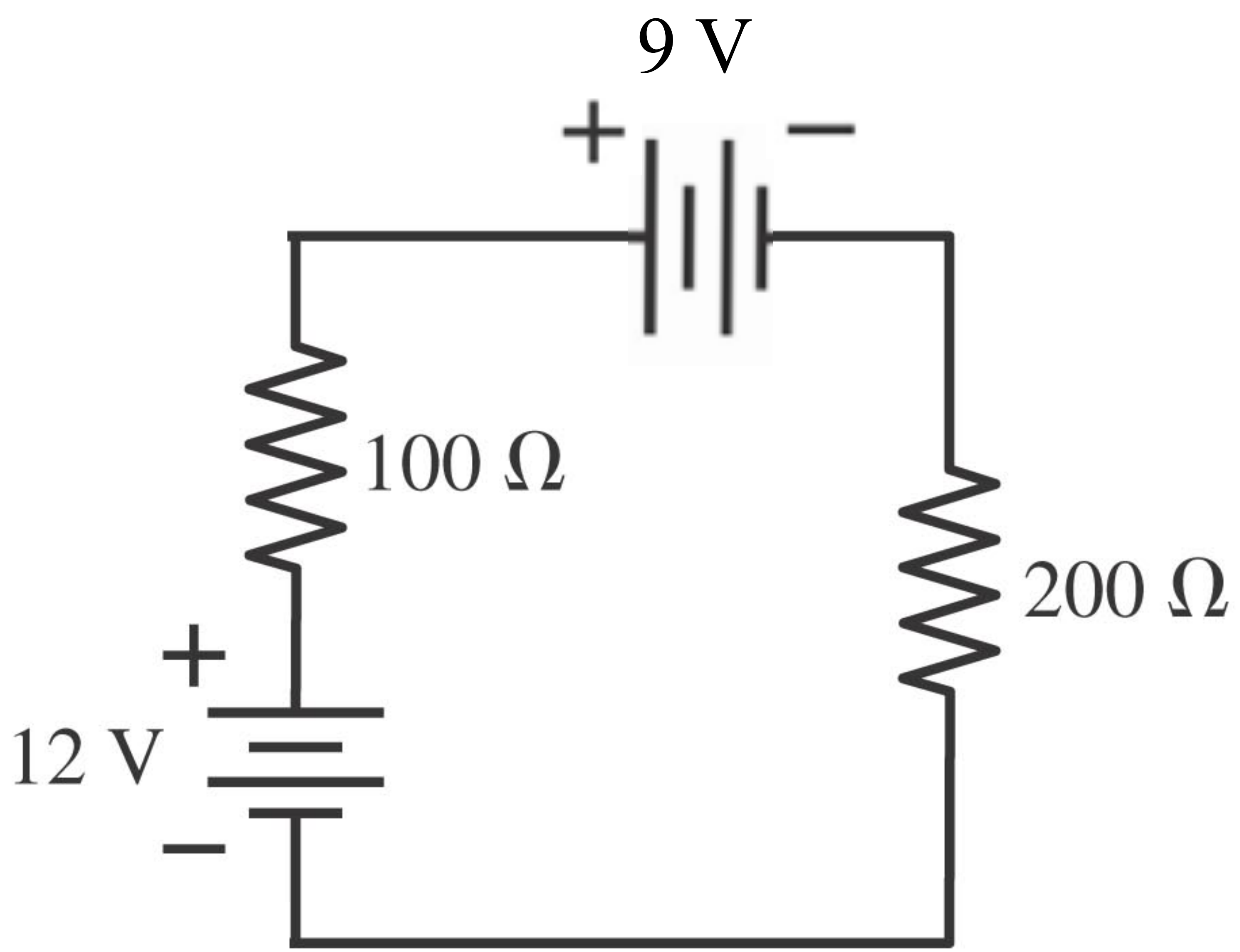
$$\Delta V_1 + \Delta V_2 + \Delta V_3 + \Delta V_4 = 0$$

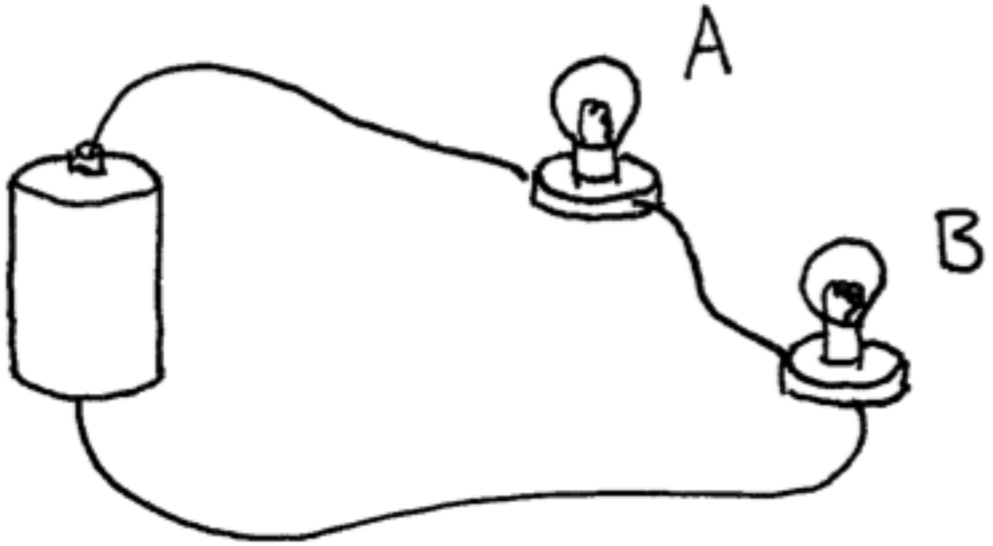
The Loop Law in Action

Find the current running through the circuit.



Clicker Question

Which light bulb is brighter?



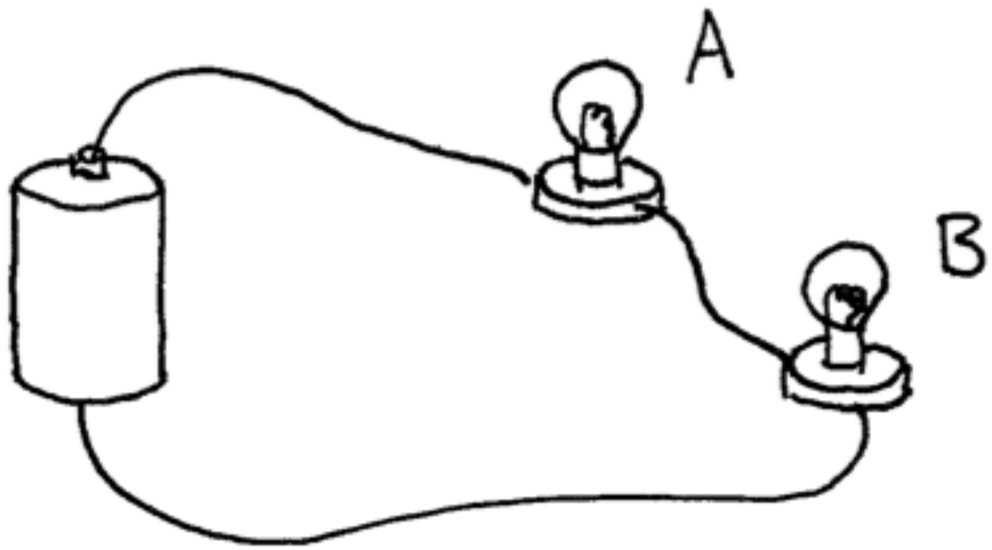
a) A

b) B

c) both the same

Clicker Question

Which light bulb is brighter?



a) A

b) B

c) both the same

Clicker Question

Which lightbulbs are brighter?

a) the bulbs in the circuit on the left

b) the bulb in the circuit on the right

c) the left circuit has one bulb brighter and one bulb dimmer than the bulb in the right circuit.

d) they're all the same brightness

Clicker Question

Which lightbulbs are brighter?

a) the bulbs in the circuit on the left

b) the bulb in the circuit on the right

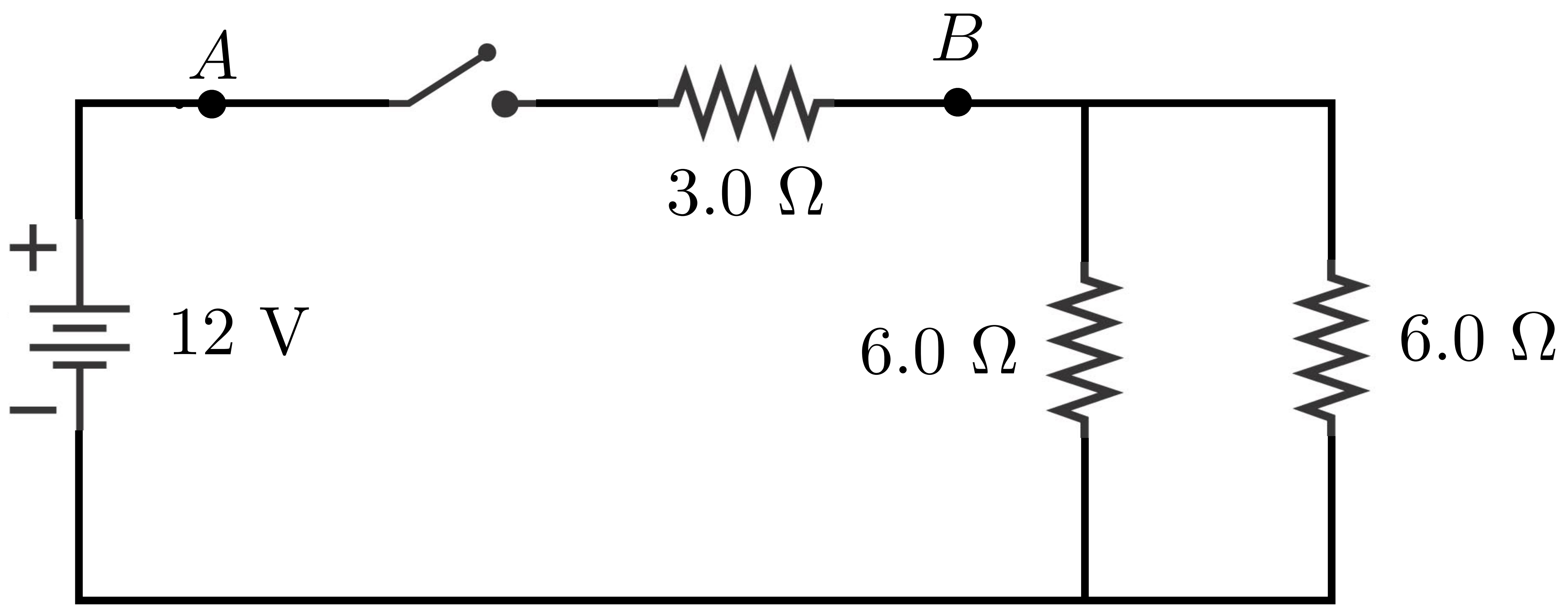
c) the left circuit has one bulb brighter and one bulb dimmer than the bulb in the right circuit.

d) they're all the same brightness

Circuits Galore
Problems 3 and 4

Clicker Question

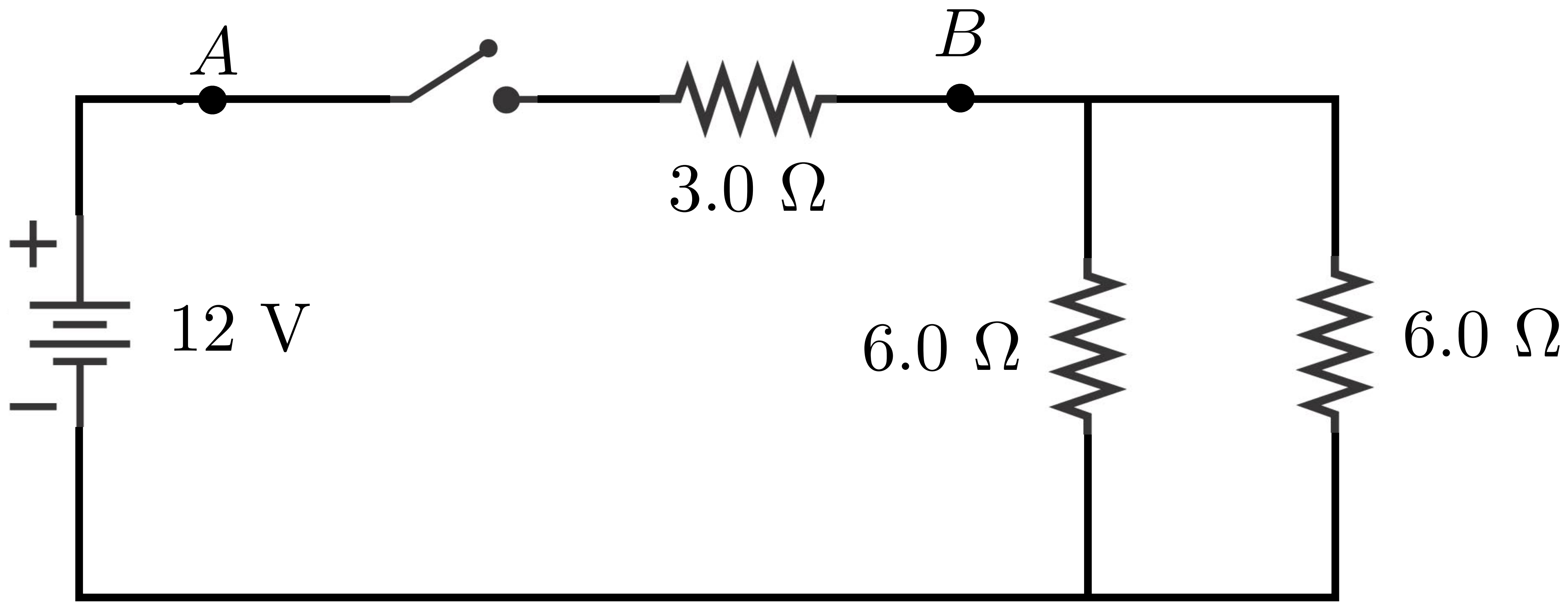
The switch is open. What is the potential difference between point *A* and *B?*



a) 0V
b) 3V
c) 6V
d) 9V
e) 12V

Clicker Question

The switch is open. What is the potential difference between point *A* and *B*?

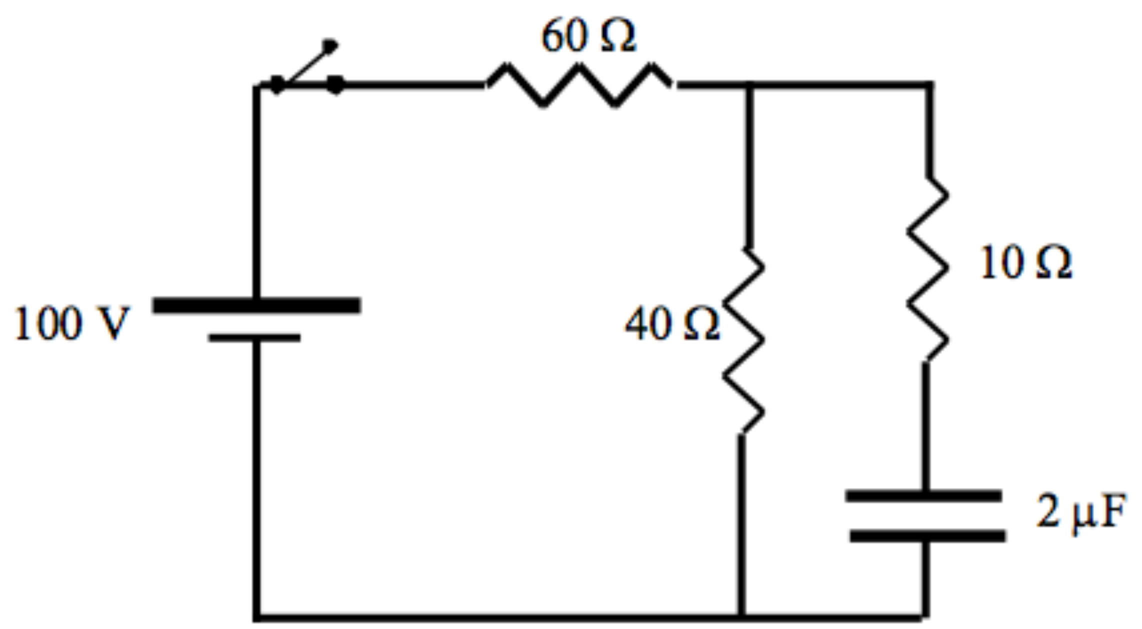


a) 0V
b) 3V
c) 6V
d) 9V
e) 12V

Circuits Galore
Problems 5, 6 and 7

Clicker Question

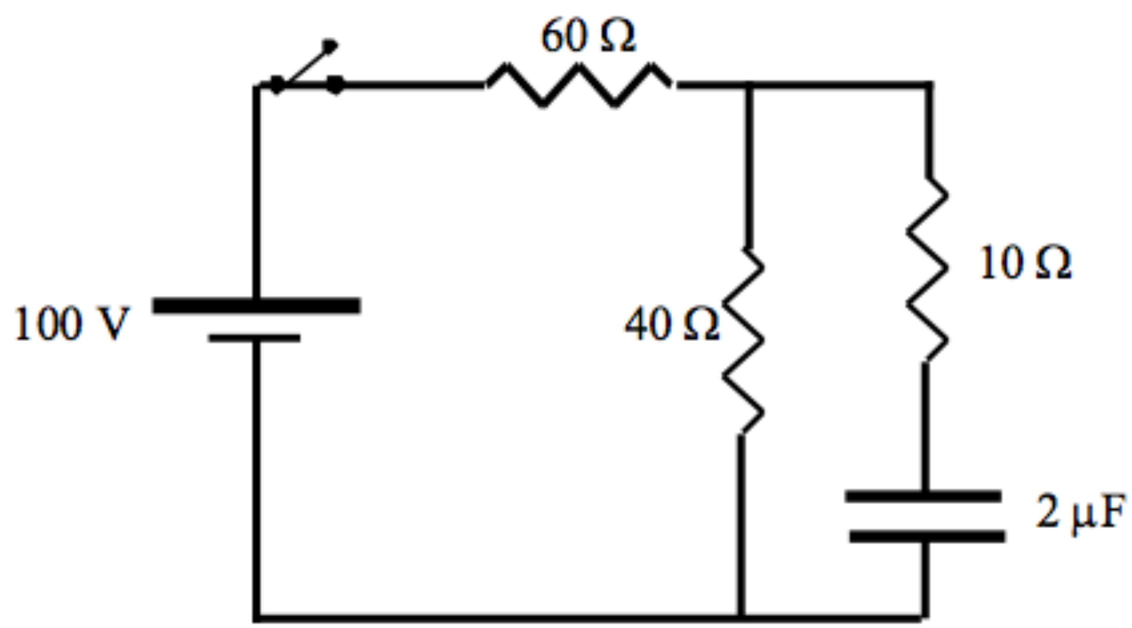
We close the switch. What is the voltage across the capacitor once the circuit has run for a while?



a) 0 V
b) 11.8 V
c) 40 V
d) 60 V
e) Unable to determine.

Clicker Question

We close the switch. What is the voltage across the capacitor once the circuit has run for a while?



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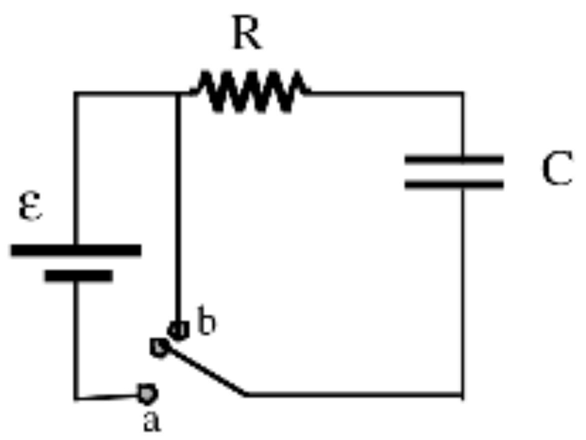
RC Circuits

Charging and Discharging Capacitors



ε=100 V
R=1000 Ω
C=1000 μF

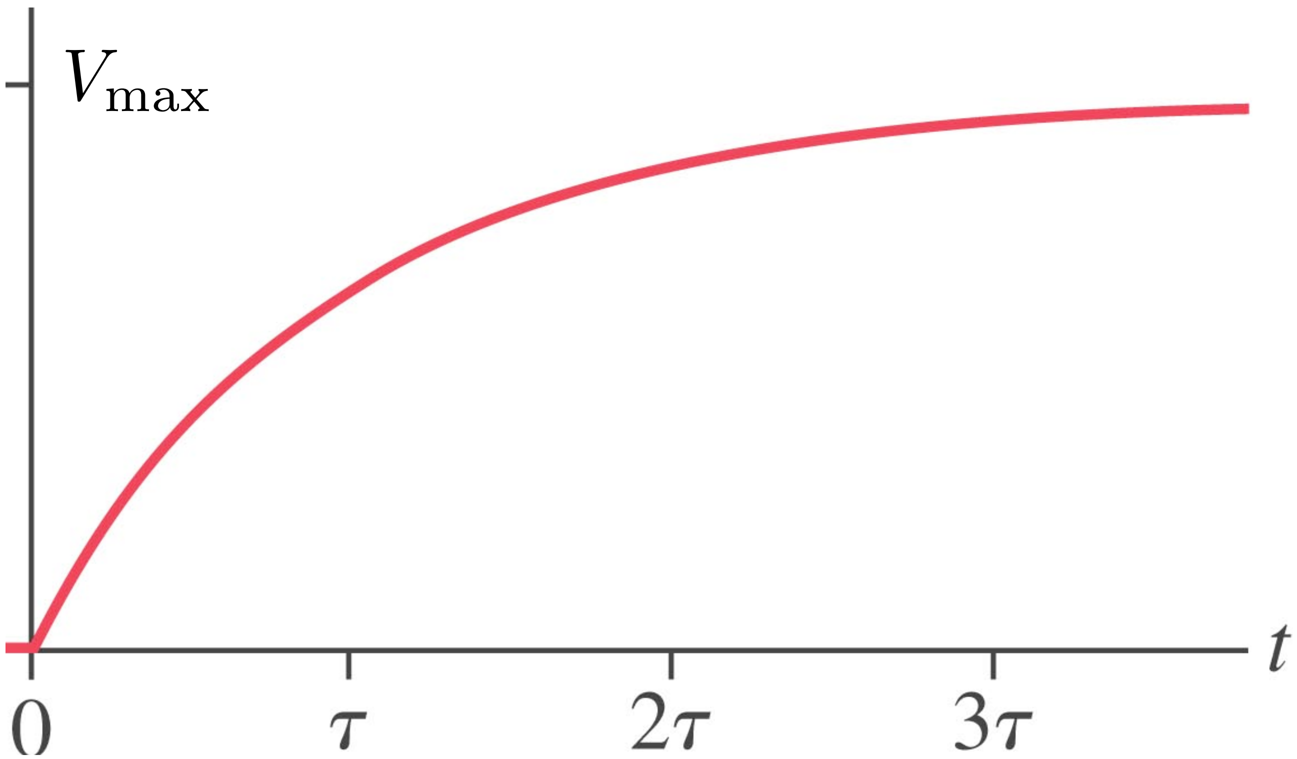
Switch to a ⇒ charge ⇒ long time ⇒ switch to b ⇒ discharge

		Charge		Discharge	
Quantity	**max value**	**t = 0**	**t = ∞**	**t = 0**	**t = ∞**
q					
ΔV_C					
I					
ΔV_R					

RC Circuits

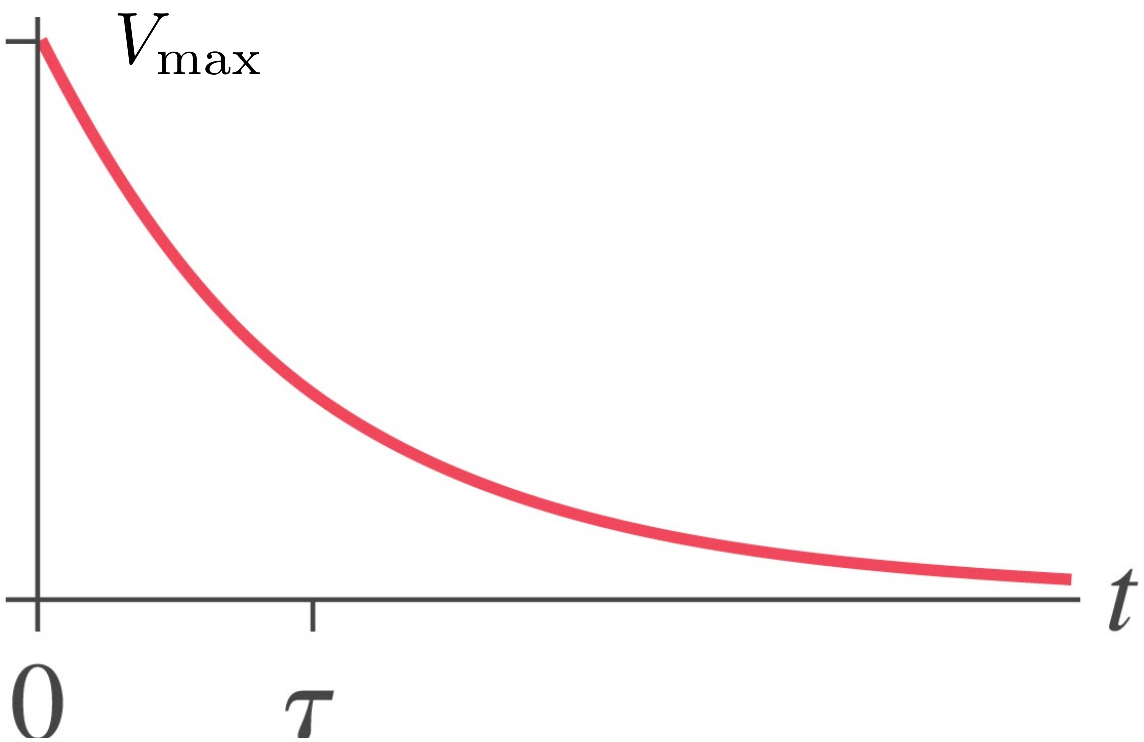
Charging and discharging a capacitor is governed by exponential laws.

charging:



$$V_C = V_{\max}\left(1 - e^{-t/RC}\right)$$

discharging:



$$V_C = V_{\max}\, e^{-t/RC}$$

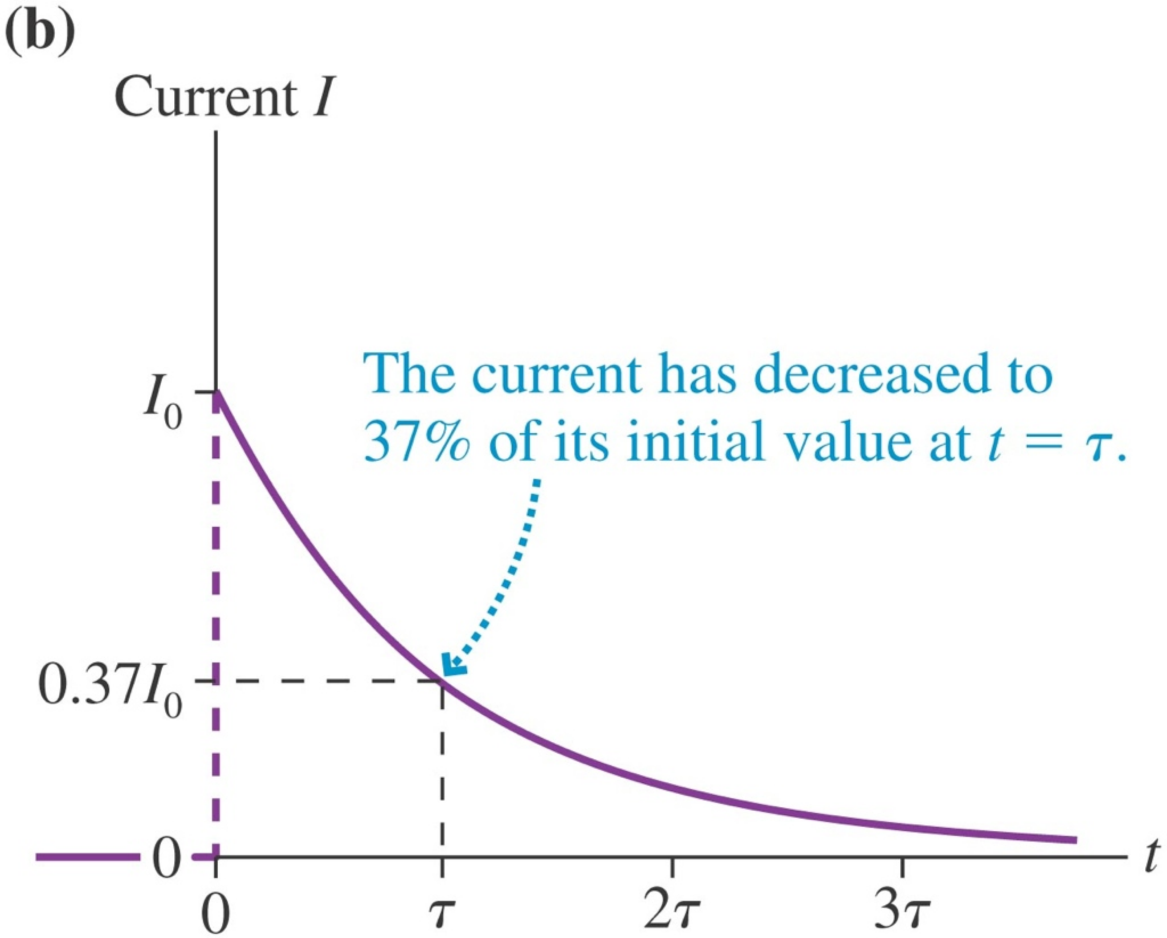
Circuit Construction Kit PhET

RC Circuits

We can define a time constant to characterize the exponential decay

$$\tau = RC$$

It's mathematically identical to the lifetime in radioactive decay.



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Clicker Question

This graph shows V_C of a capacitor that is separately discharged through three difference resistors.

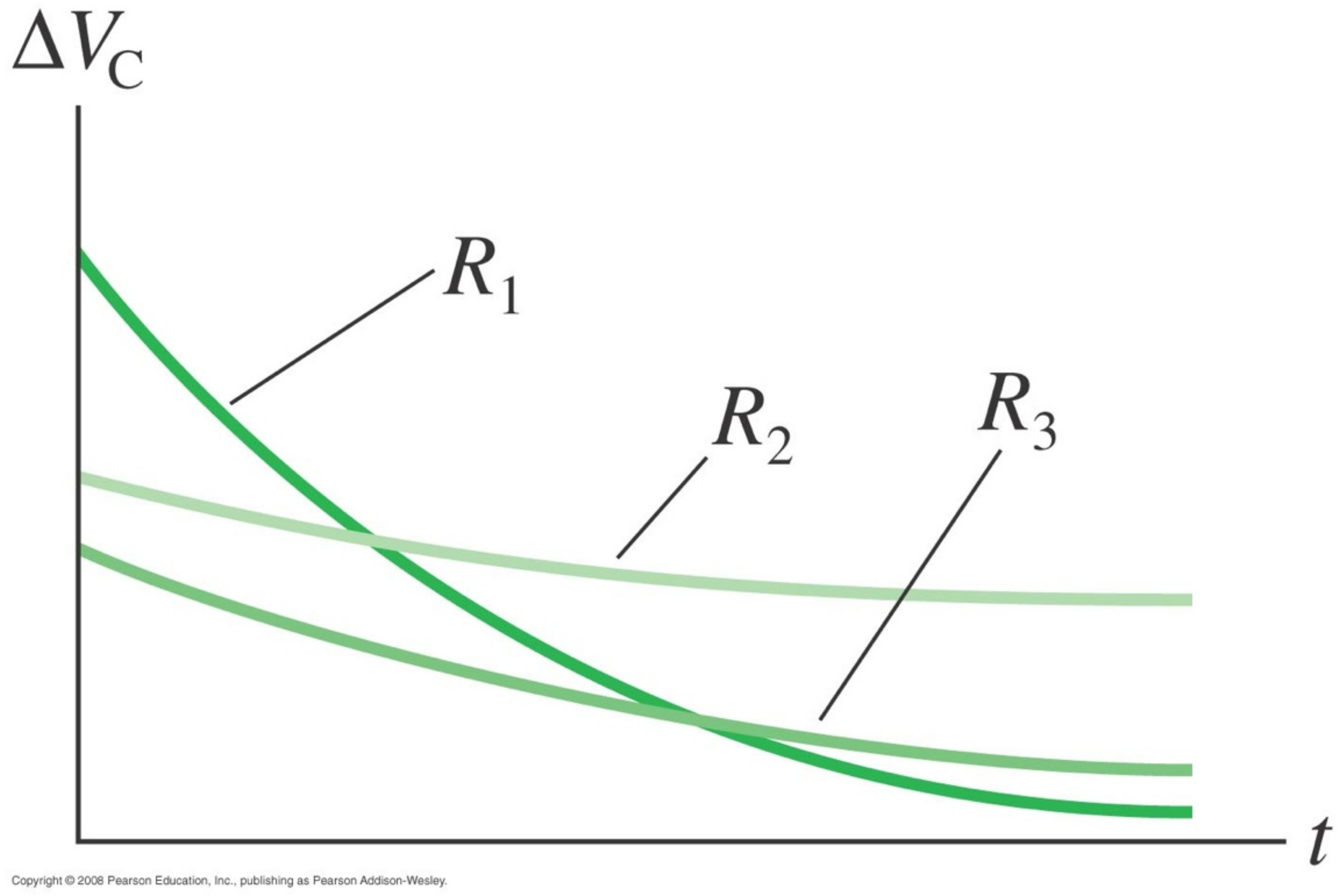
Rank the value of the resistance from smallest to largest.

a) $R_1 > R_2 > R_3$

b) $R_3 > R_2 > R_1$

c) $R_1 > R_3 > R_2$

d) $R_2 > R_3 > R_1$



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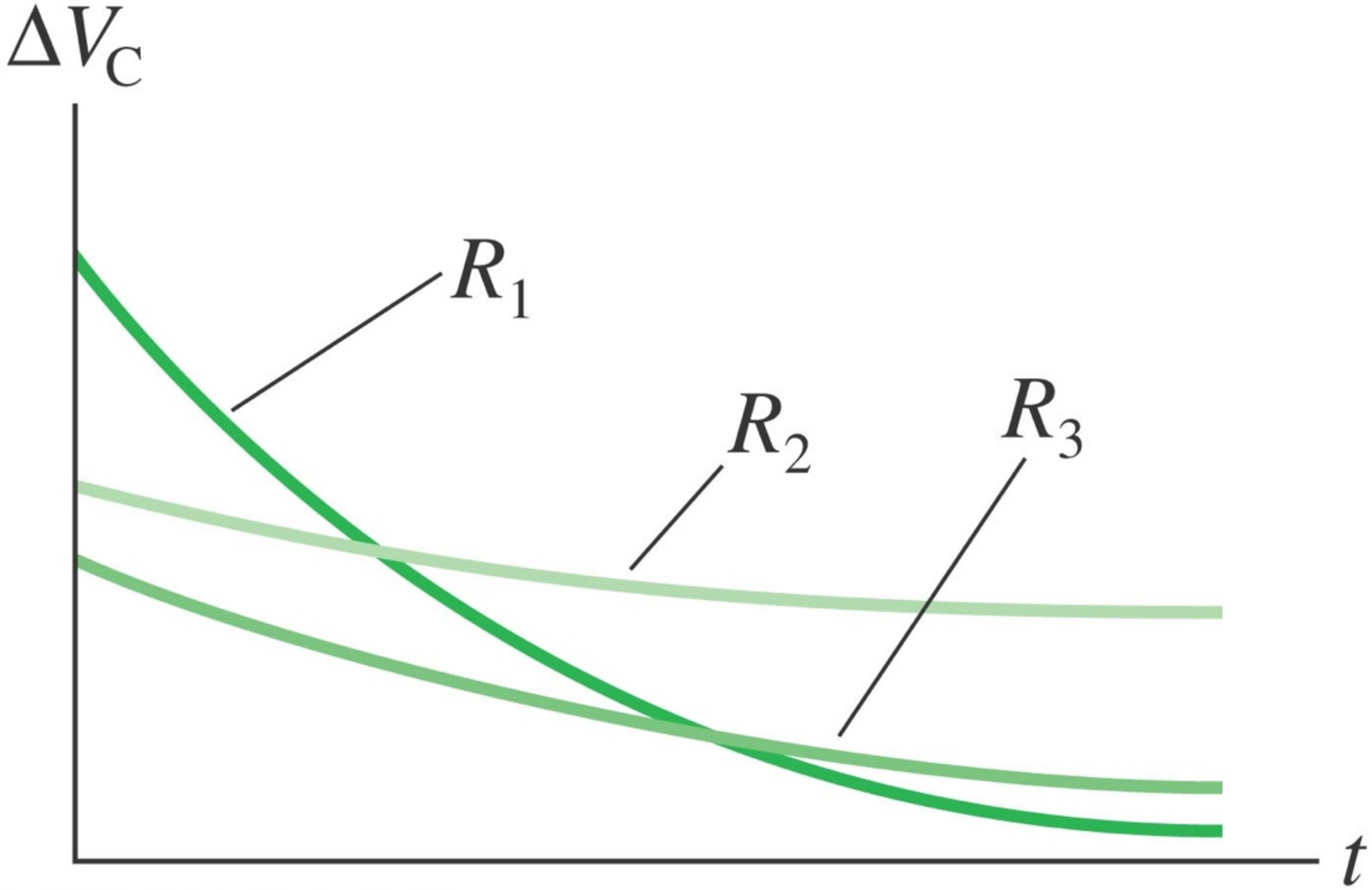

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