Maxwell's Equations

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$\oint \mathbf{B} \cdot d\mathbf{A} = 0$$

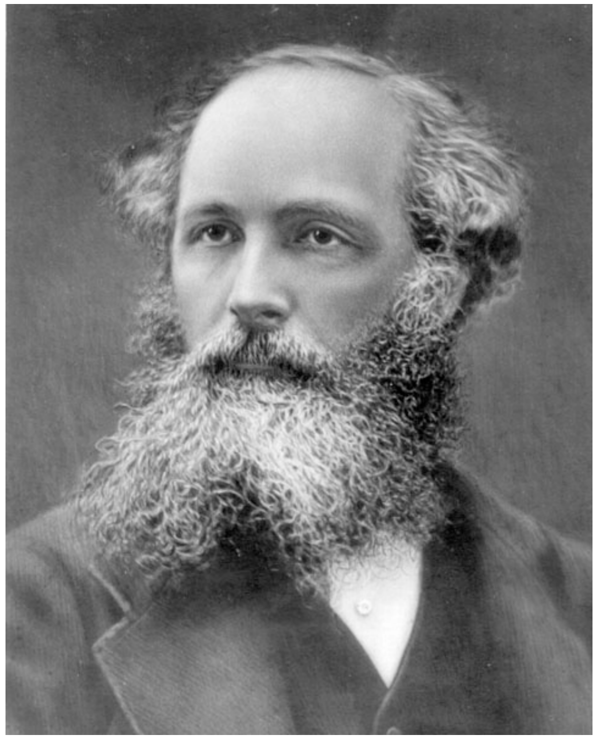
$$\oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_\mathrm{B}}{dt}$$

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 \varepsilon_0 \frac{d\Phi_\mathrm{E}}{dt} + \mu_0 i_{enc}$$

Motion of electric charge in fields: $\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$

James Clerk Maxwell

(1831-1879)



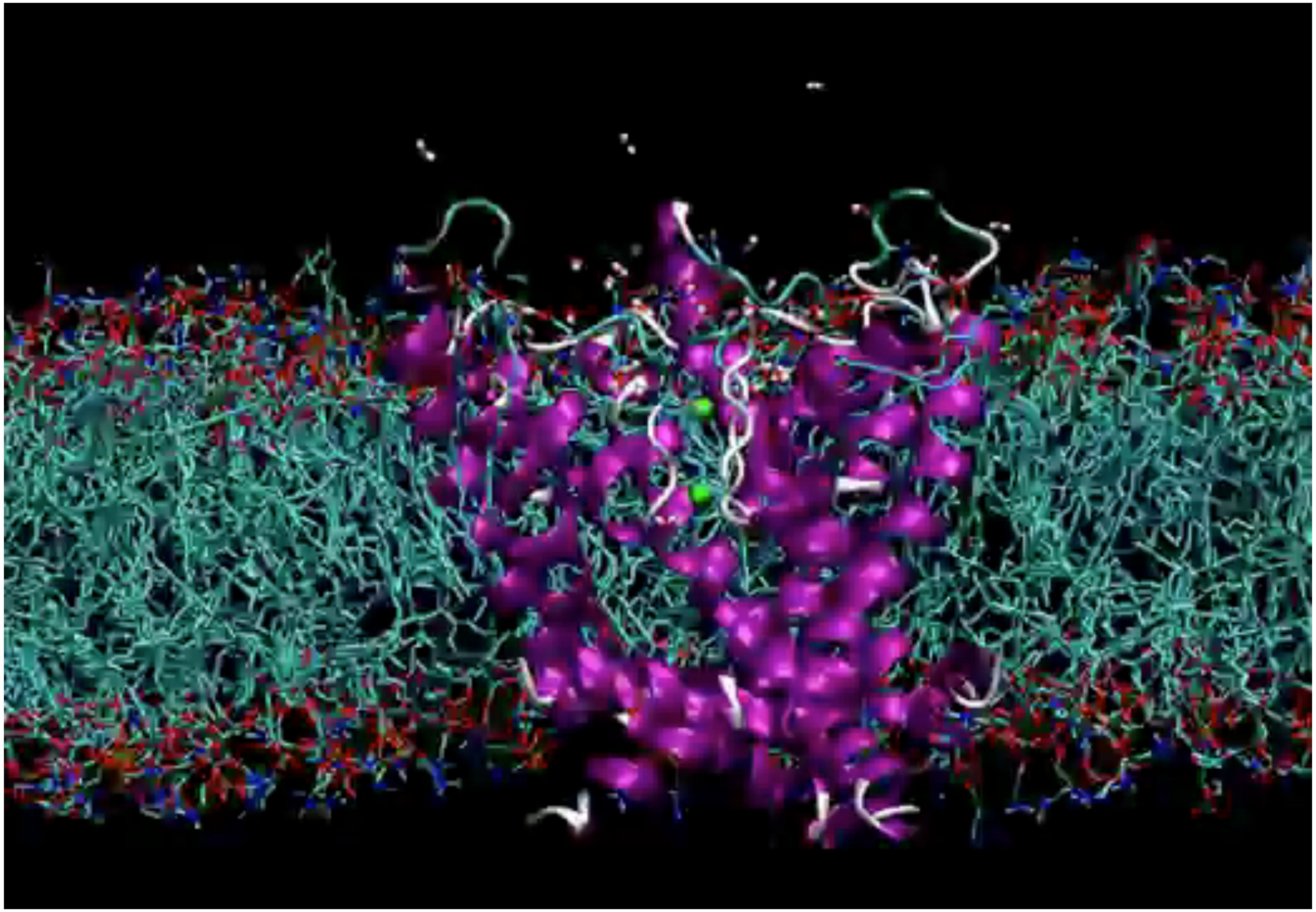
- First scientific paper at age 14 (on oval curves)
- “On Faraday’s lines of force” introduced E-M at age 24
- Professorship at age 25
- Worked out that for light R+G+B=white
- Calculated that Saturn’s rings had to be made of lumps
- Created first ever colour photograph
- Maxwell distribution and Maxwell’s Demon
- Effectively invented dimensional analysis
- Showed that light is EM radiation

Electromagnetism

Big Picture

1. **Understand the fundamentals** of electrostatics and magnetism.
2. **Use these fundamentals** to "build" circuit components. Learn to analyze these circuits
3. **Express these fundamentals** in the form of ***Maxwell's equations. Unification!*** See how Maxwell's equations predict something new.

Microscopic Current Model

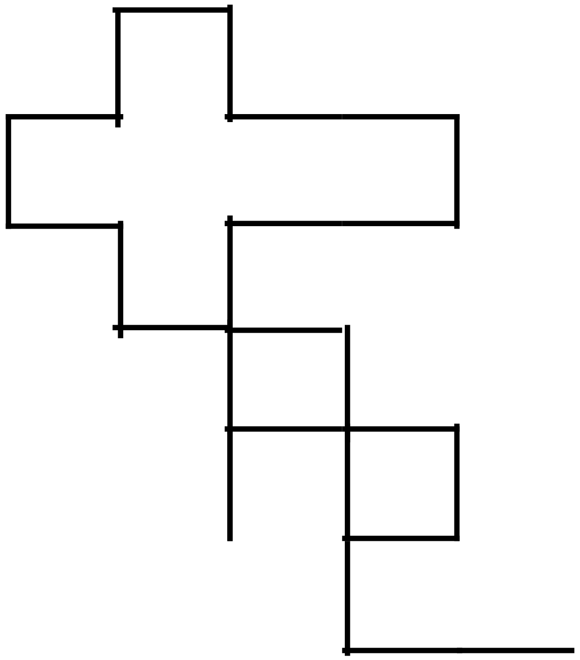


Worksheet

Ion drift through membrane channels

Q1

2D Random Walk



At every step you randomly choose to go up, down, left, or right.

Worksheet

Ion drift through membrane channels

Q2

Current

Plinko is an **exact analogy** to the current model presented in the text book. The **physics is identical**.

Plinko disc = electrons
Plinko peg = atom
gravitational field = electric field

The differences:

1. Plinkos move in 2 dimension, while electrons move in 3 dimensions
2. Plinkos move with the gravitational field, electrons move against the electric field.

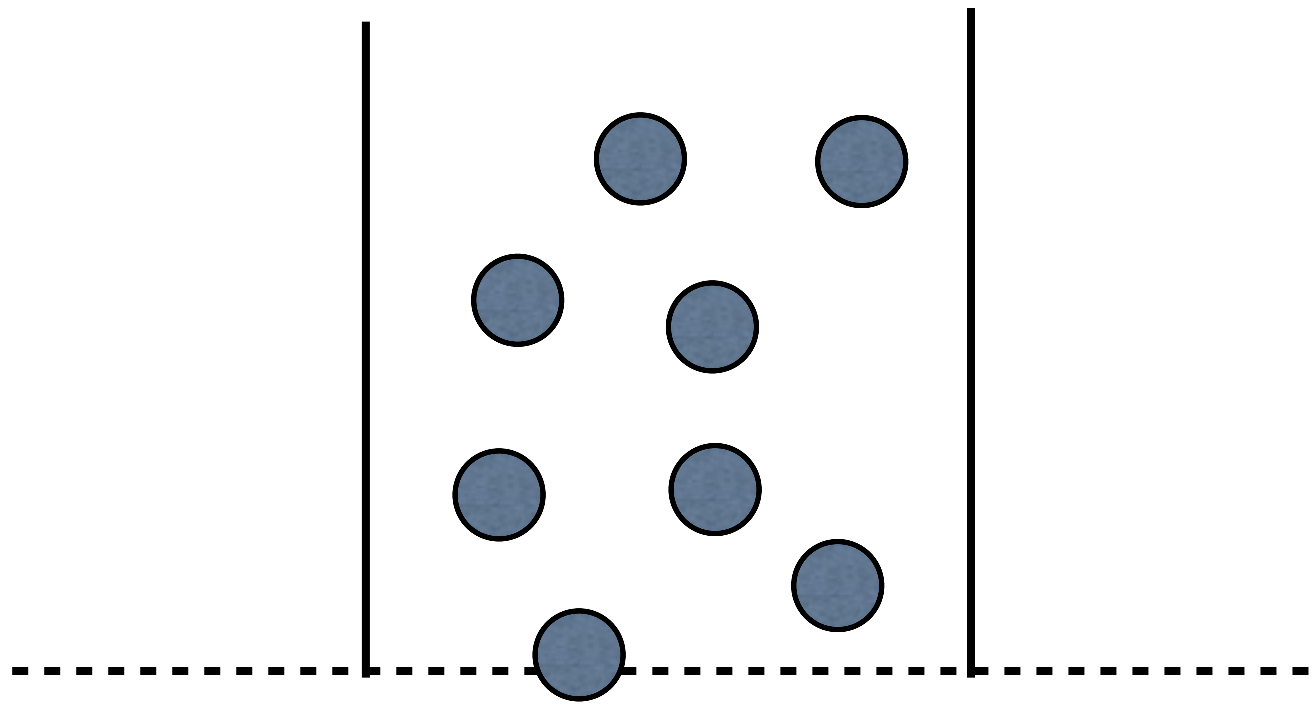
Worksheet

Ion drift through membrane channels

Q3

Current

The **number current** i_e is the number of electrons that go past the dotted line per second.

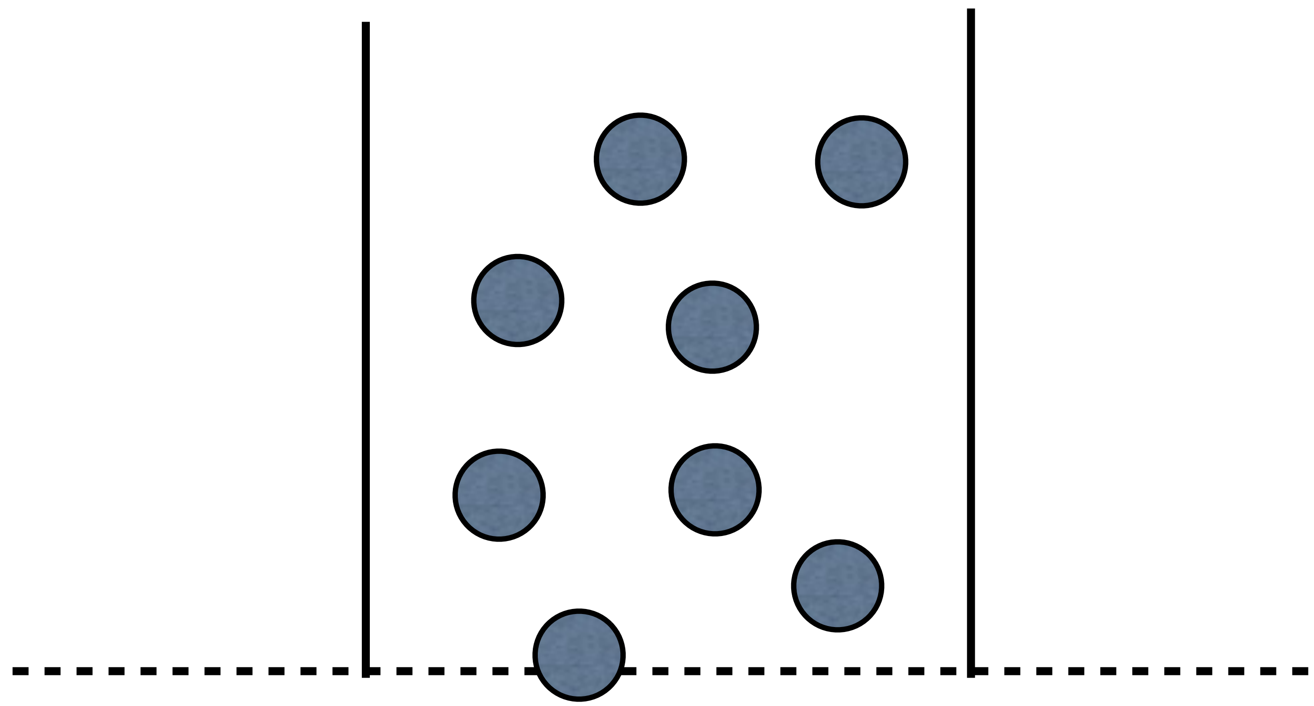


If the density of electrons increases the current i_e

a) increases
b) decreases
c) stays the same

Current

The **number current i_e** is the number of electrons that go past the dotted line per second.

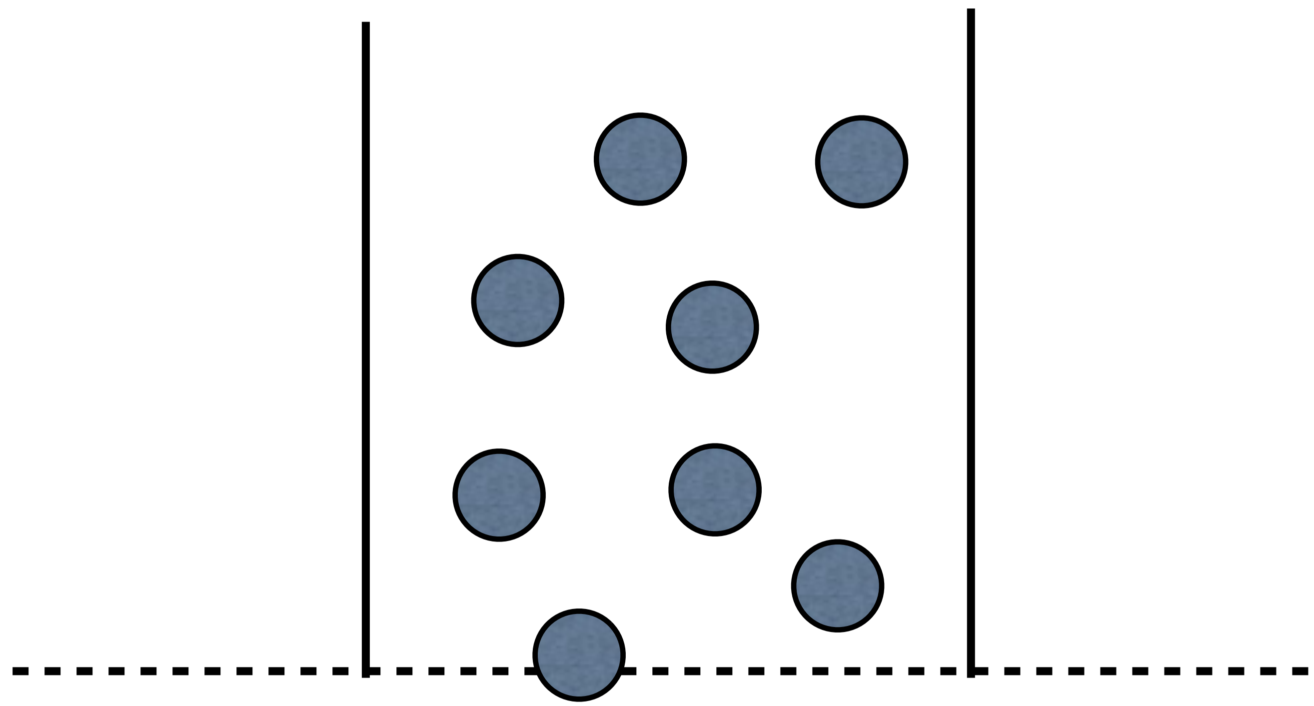


If the speed at which the electrons are moving increases, the current i_e

a) increases
b) decreases
c) stays the same

Current

The **number current** i_e is the number of electrons that go past the dotted line per second.



If the width increases and the density of electrons stays the same, the current i_e

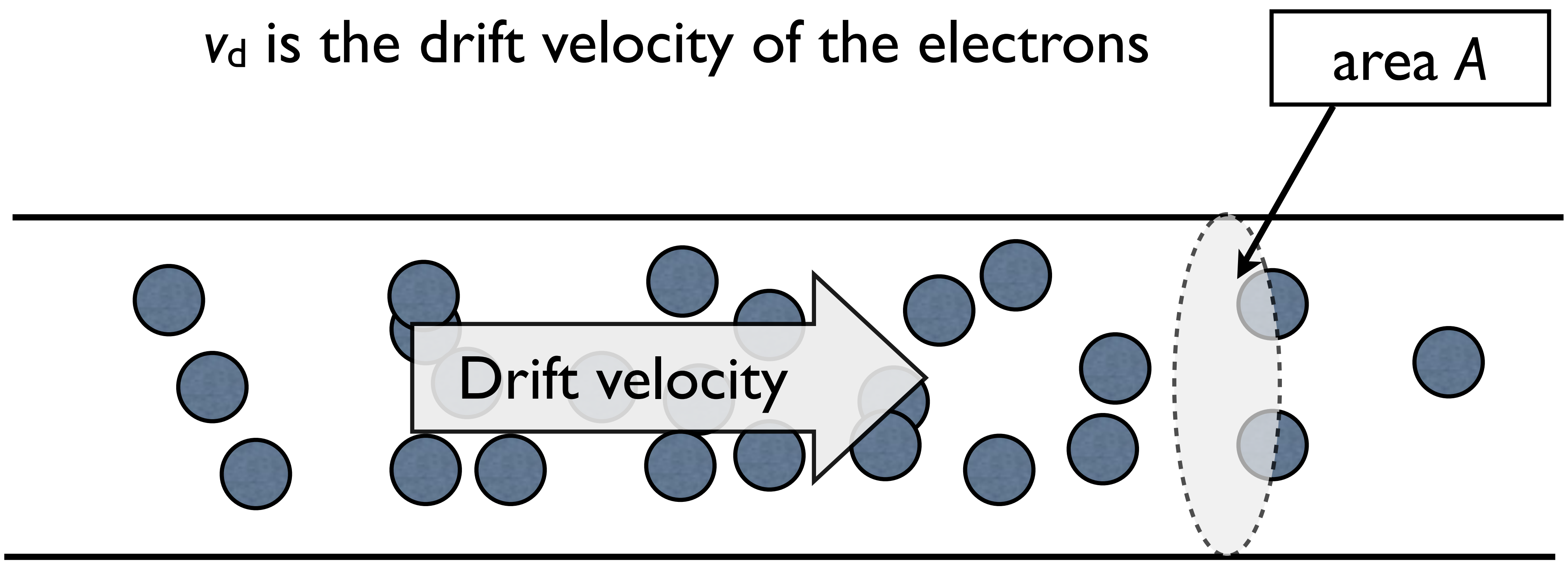
a) increases
b) decreases
c) stays the same

Current

The current, which is Coulomb's per second, is simply

$$I = e i_e = e n_e A v_{\mathrm{d}}$$

e is the charge is the electron
n_e is the density of electrons
A is the cross sectional area of the wire
v_d is the drift velocity of the electrons



Worksheet

Ion drift through membrane channels

Q4

Current and E-Field

Using the current from earlier, and the drift speed, the current is **proportional to the electric field!**

$$I = \frac{e^2 n_e \tau}{m_e} AE$$

which can be written as

$$I = \sigma AE \quad \text{where} \quad \sigma = \frac{e^2 n_e \tau}{m_e}$$

The **material constant σ** is called the **conductivity** (SI unit = Ω^{-1} m^{-1}).

The **inverse quantity** $\rho = \frac{1}{\sigma}$ is called the **resistivity** (SI unit: Ω m).

What's Moving?

In a metal, how do we know that electrons (negative charge) moves and not protons?

What's Moving?

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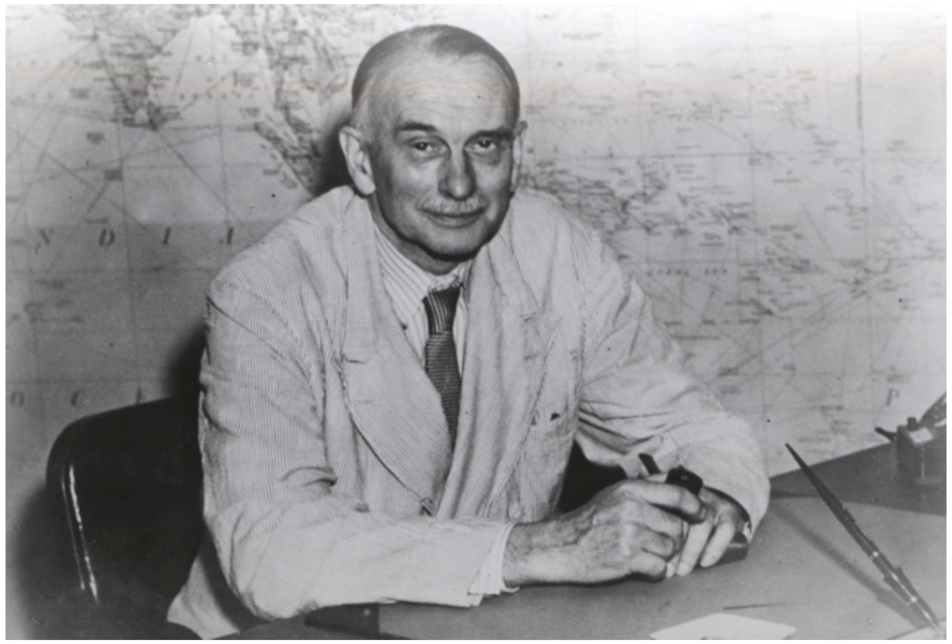
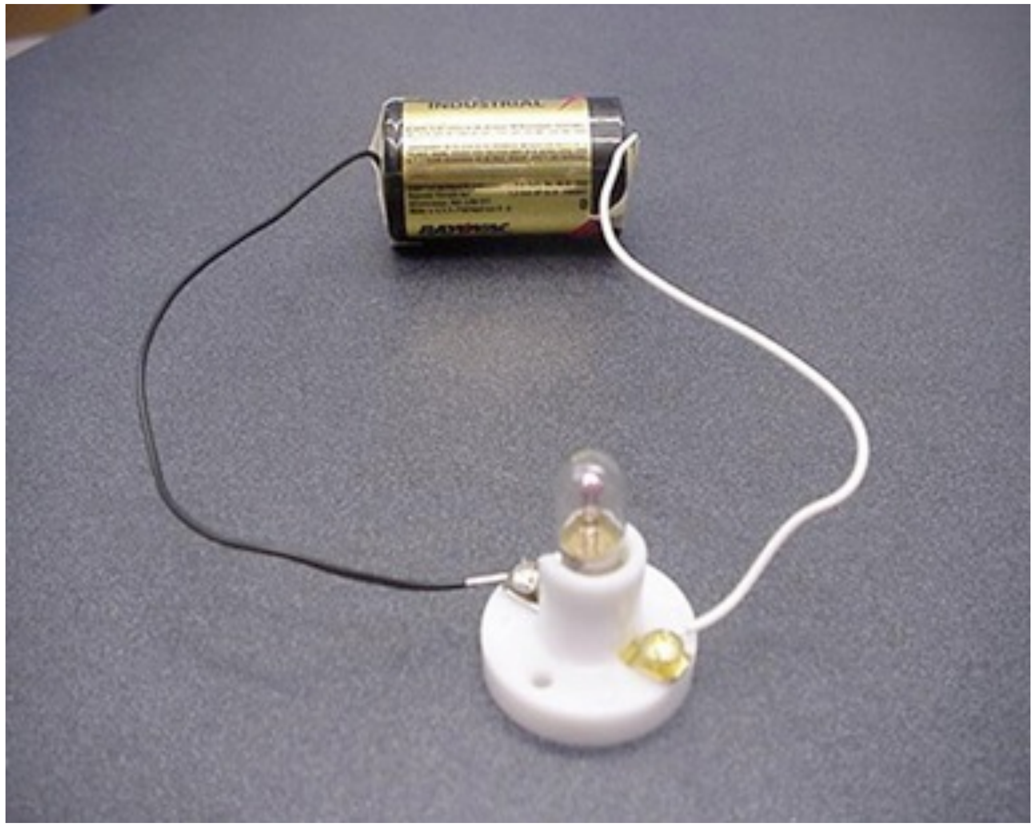


Tolman-Stewart experiment:

Accelerate a metal and whatever's moving should slosh to the back. This experiment also measured the mass of the electron.

E-Field and ΔV

The presence of the electric field in the wire implies that there's a potential difference ΔV from one end of the wire to the other.



$$dV = -\vec{E} \cdot d\vec{s}$$

E-Field and ΔV

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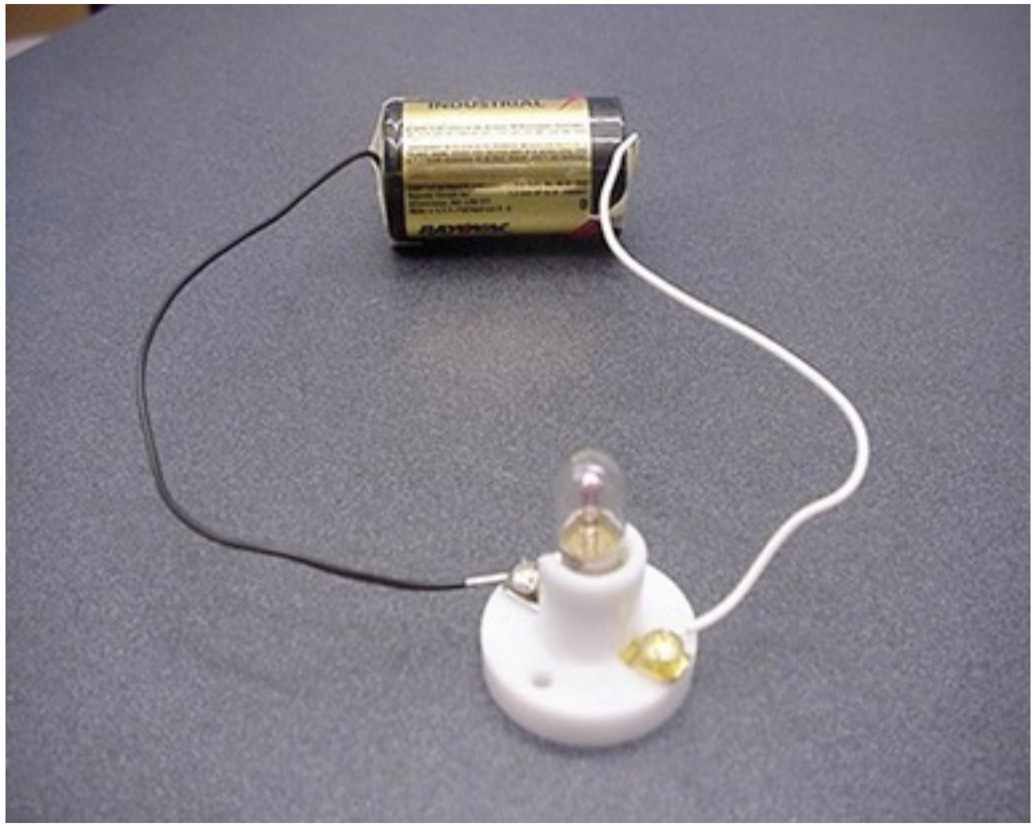


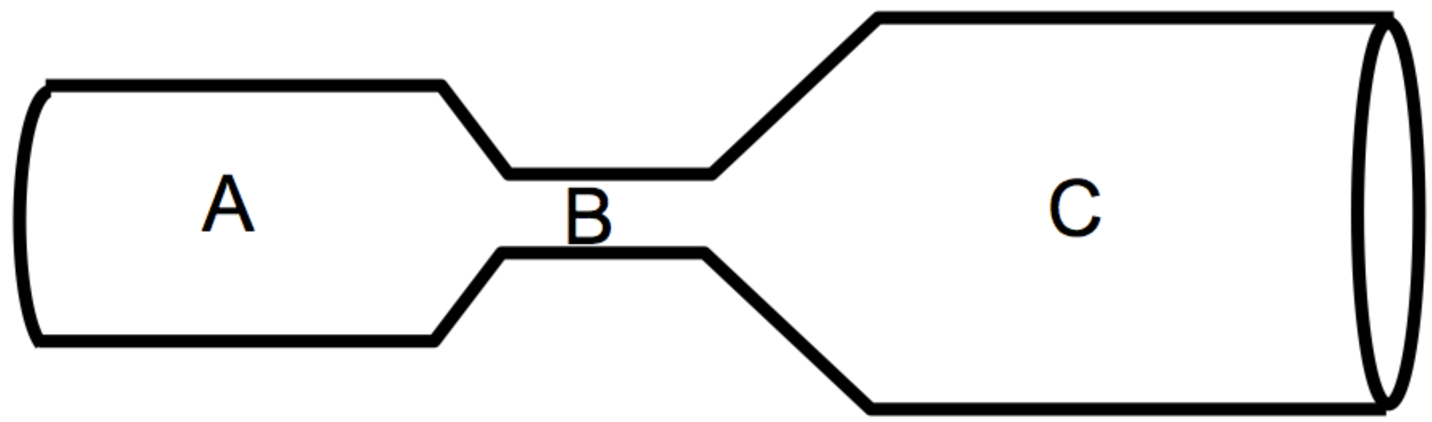
$$dV = -\vec{E} \cdot d\vec{s}$$

Wait, shouldn't E = 0 and ΔV = constant inside a conductor?

This is true for a system in static equilibrium. Current flow is a **non-equilibrium state.**
The system is trying to get into a lowest energy configuration.

Clicker Question

A copper cylinder is machined to have the following shape. The ends are connected to a battery so that a current flows through the copper.



Which region has the greatest magnitude of current, I?

a) A
b) B
c) C
d) All three are the same
e) Not sure/ not enough info

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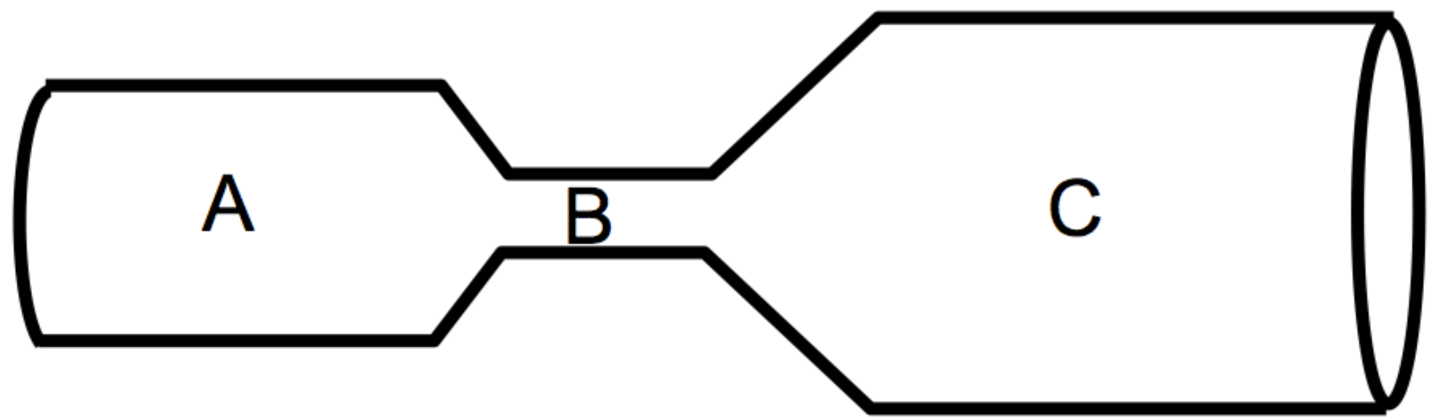


Which region has the greatest magnitude of current, I?

a) A
b) B
c) C
d) All three are the same
e) Not sure/ not enough info

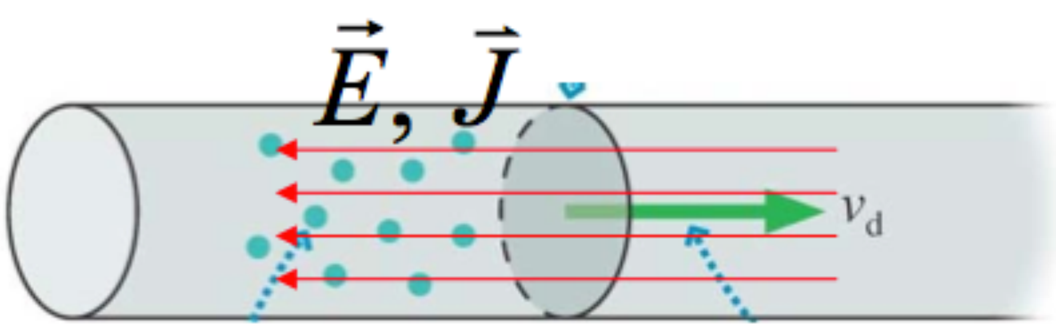
Current is conserved.
Think of water flow.

Current Density

The current divided by the cross sectional area gives us the current density,

$$J = \frac{I}{A} = \sigma E$$

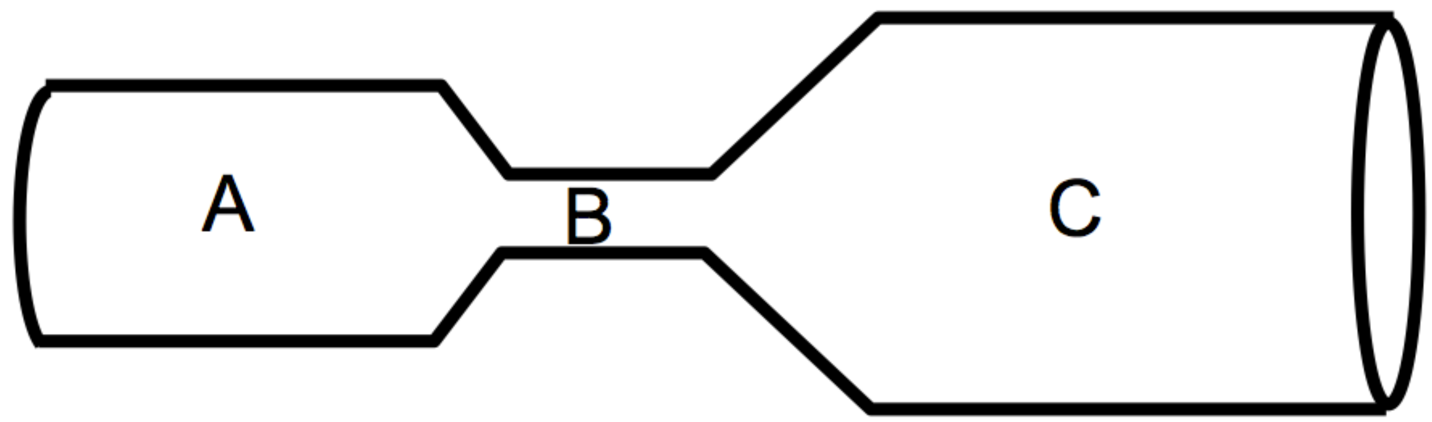
A **microscopic quantity** that is proportional to the **electric field**.



Note: The current actually flows against the electric field.

Clicker Question

A copper cylinder is machined to have the following shape. The ends are connected to a battery so that a current flows through the copper.



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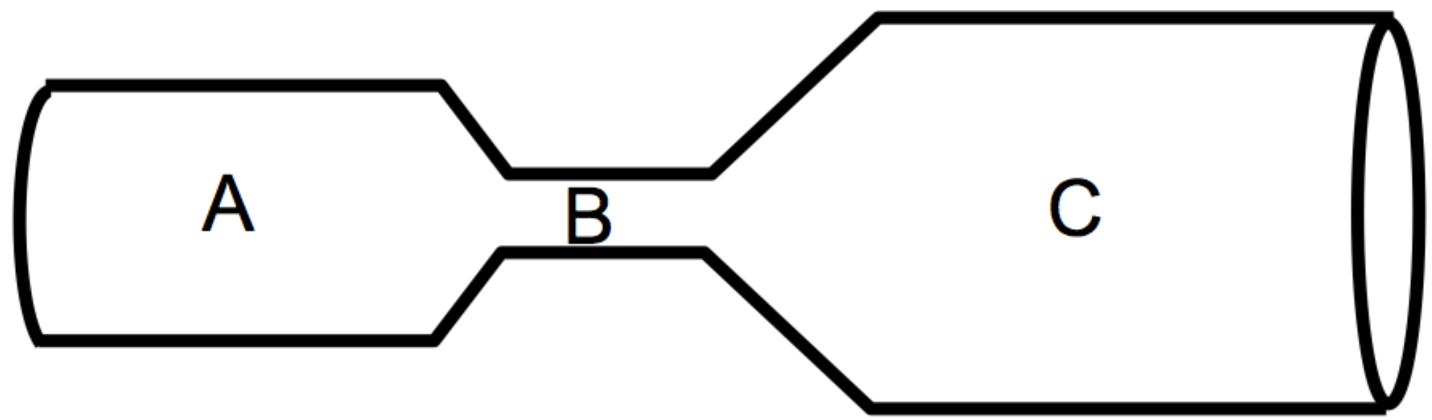


Which region has the greatest current density |J|?

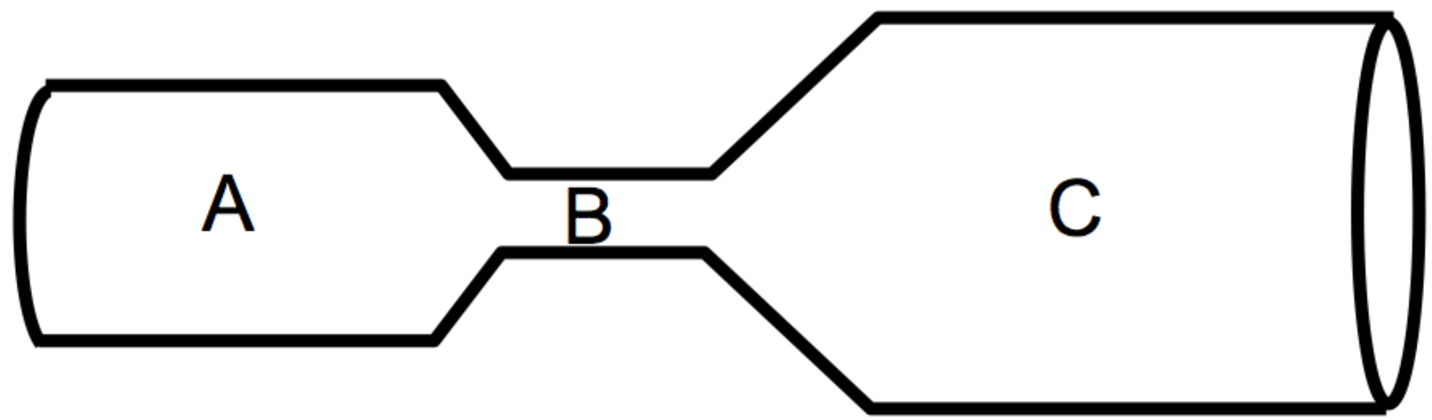
a) A
b) B
c) C
d) All three are the same
e) Not sure/ not enough info

Current is conserved so all the electrons must flow faster in the narrower part.

This also means that B has **greatest electric field (question from PIAZZA)!**

Clicker Question

A copper cylinder is machined to have the following shape. The ends are connected to a battery so that a current flows through the copper.

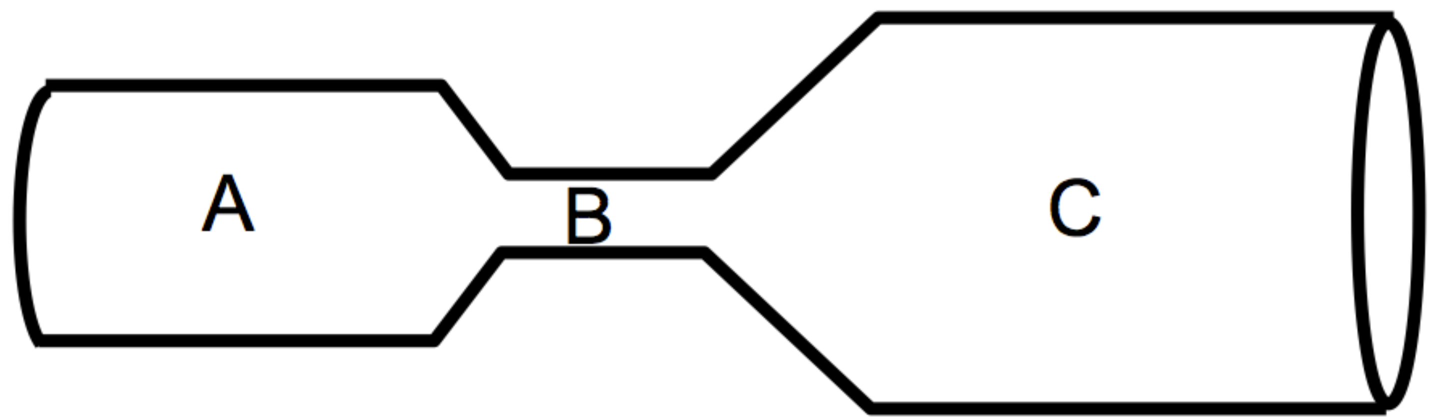


Which region has the greatest conductivity σ?

a) A
b) B
c) C
d) All three are the same
e) Not sure/ not enough info

Clicker Question

A copper cylinder is machined to have the following shape. The ends are connected to a battery so that a current flows through the copper.



Which region has the greatest conductivity σ?

a) A
b) B
c) C
d) All three are the same
e) Not sure/ not enough info

Conductivity and resistivity are **microscopic properties** of the material.

Fuses

- Current density is a microscopic property related to the conductivity of the material.
- It is also related to how much current a material can handle. For example, a builder has to use wires of a minimum diameter ('gauge') for household wiring.
- The maximum current in household wires is limited by breakers or fuses for good reason.

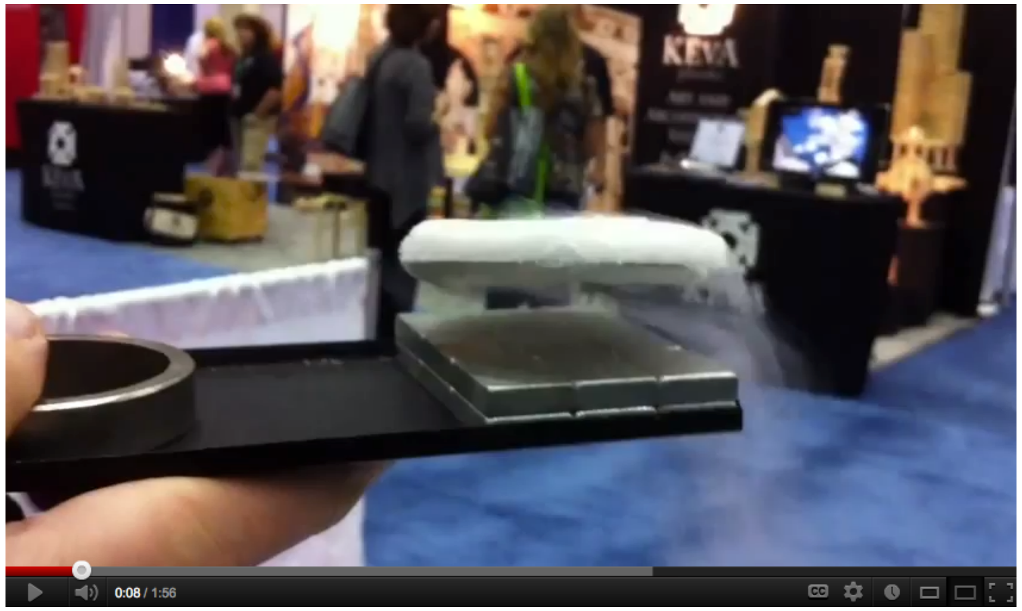
Nail Burner Demo

Superconductivity

A superconductor has a **resistivity of zero**!

Superconducting Magnets

- currents flow without resistance creating giant magnetic fields
- used in particle accelerators, MRI, mass spectrometers

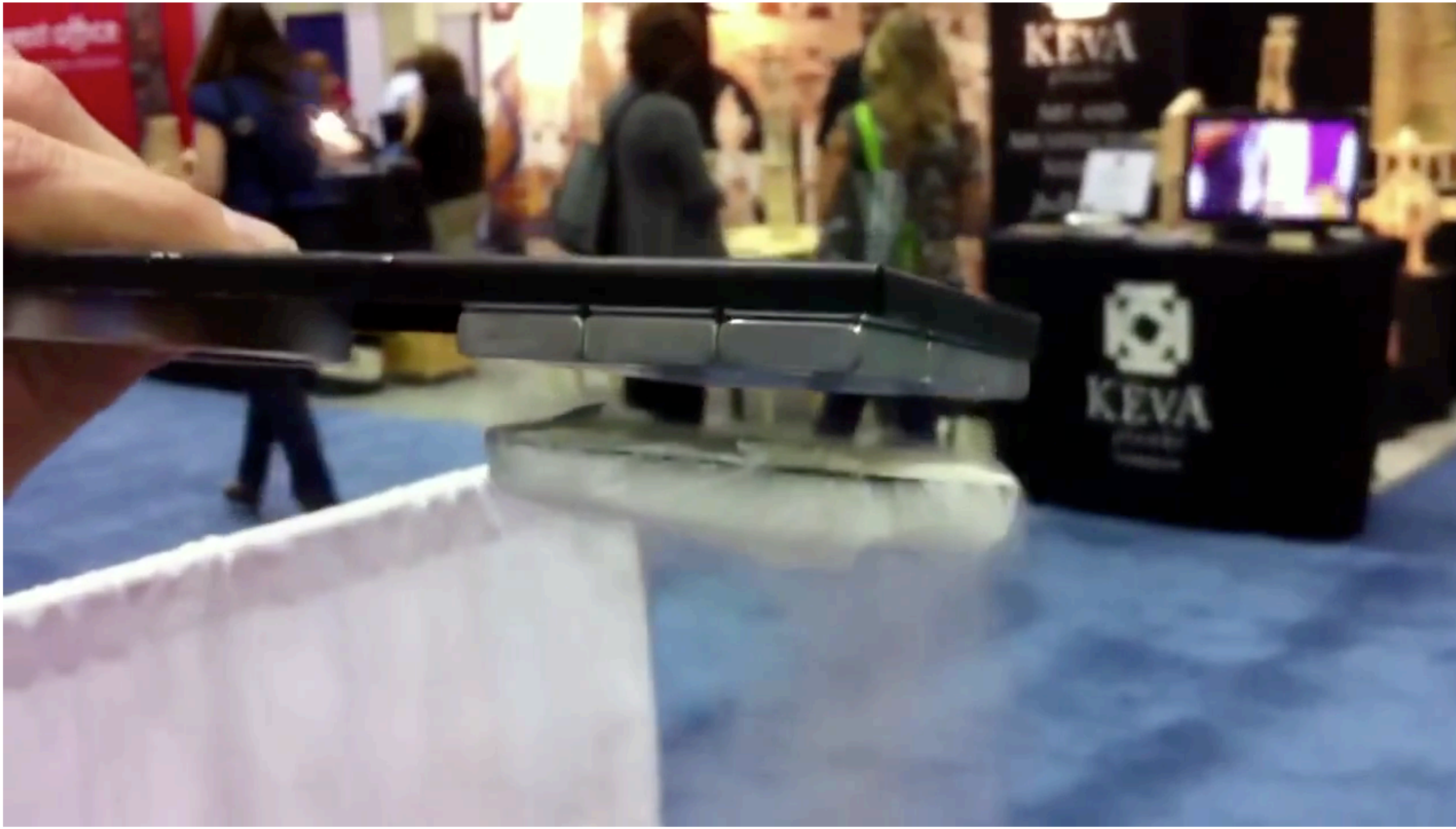


SQUIDs (**Superconducting QUantum Interference Devices**)

- detect magnetic fields of **10^{-18} Tesla!**
- animals have fields of 10^{-9} to 10^{-6} Tesla
- used in medical imaging (MEG, MRI,...)
- oil prospecting, detecting land mines
- Gravity Probe B to verify general relativity

levitating a superconductor (video)

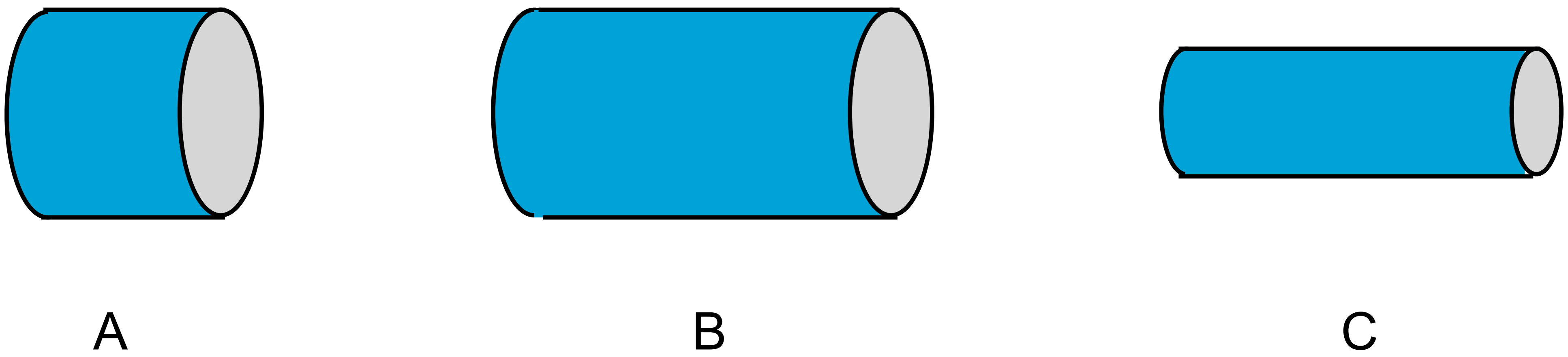
Superconductivity (ρ = 0)



levitating a superconductor (video)

Clicker Question

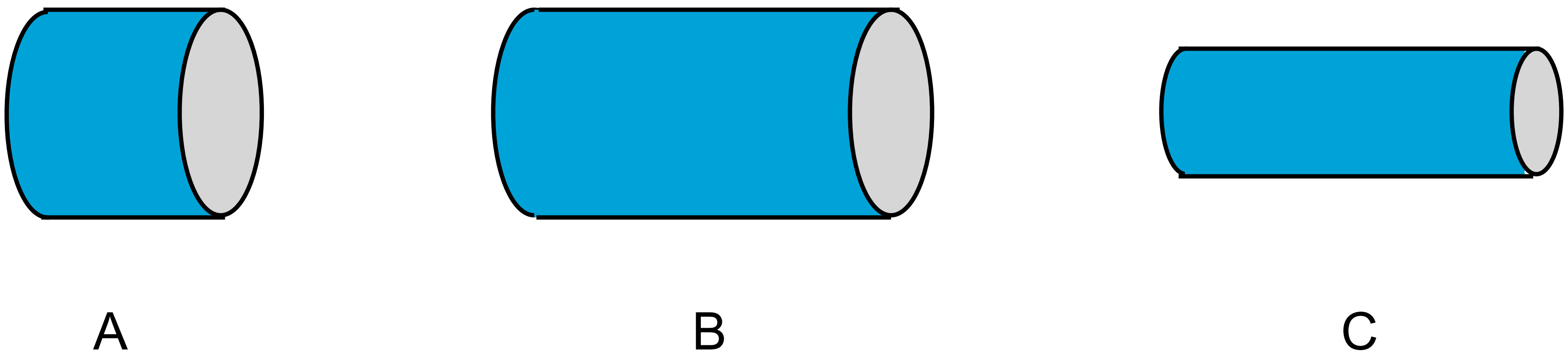
Rank these tubes by how hard it would be to push stuff (say towels) through them at the same rate.



a) A harder than B harder than C
b) C harder than B harder than A
c) B harder than C harder than A
d) A harder than C harder than B
e) C harder than A harder than B

Clicker Question

Rank these tubes by how hard it would be to push stuff (say towels) through them at the same rate.



a) A harder than B harder than C

b) C harder than B harder than A

c) B harder than C harder than A

d) A harder than C harder than B

e) C harder than A harder than B

Resistance and Resistivity

We can combine $I = \sigma A E$ and $V = Ed$ (but we're going to rename *d* to *L*) to get

$$I = \sigma \frac{A}{L} V$$

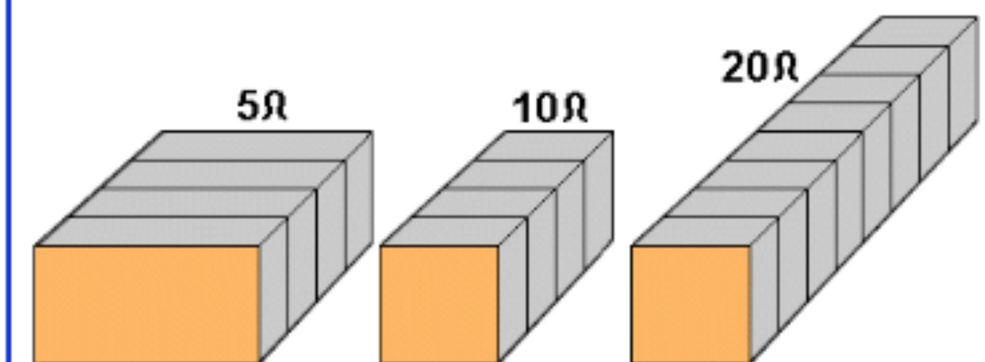
Ohm's Law !

$$\Delta V = IR$$

The **resistance R** of a real wire depends on the cross sectional area A and its length L:

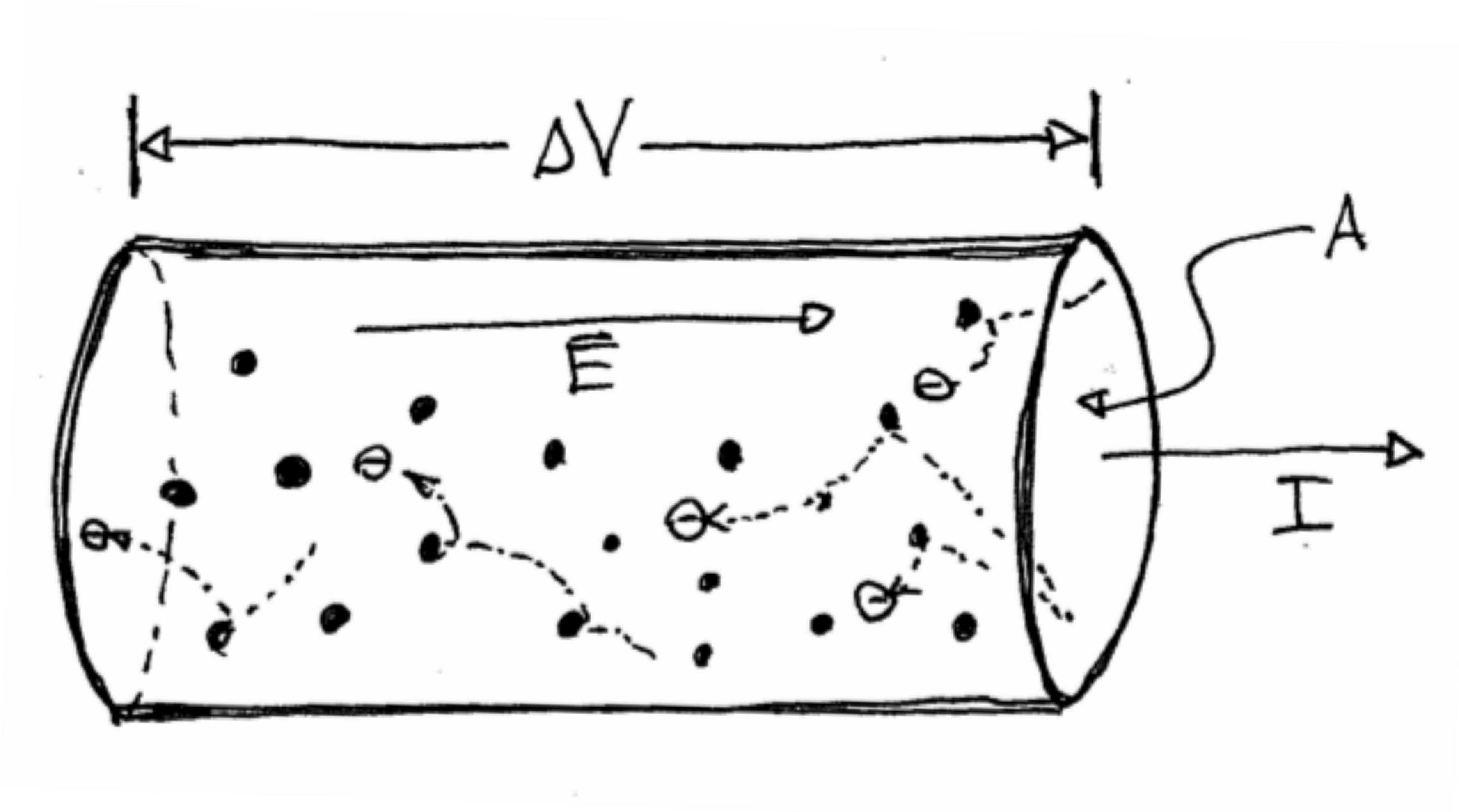
$$R = \rho \frac{L}{A}$$

A macroscopic property of the whole wire depending on the length and cross sectional area.



resistance of wire drops if wire has larger cross-sectional area but increases with length

Resistance



Higher voltage leads to a **larger electric field** and an increase in drift speed and and **increase in current**.

This leads to more collisions, more vibrations, and more energy loss (heat, light).

Battery Resistor Circuit

Power Dissipated in a Resistor

The **drop in voltage across a resistor** means that energy is being lost:

$$\Delta U = q\Delta V$$

The **rate of energy lost** is the power:

$$P = \frac{dU}{dt} = \frac{dq}{dt}\Delta V = I\Delta V$$

Using Ohm's Law:

$$P = I^2 R = \frac{V^2}{R}$$

This is the power dissipated in a component with resistance *R* and current *I* flowing through it.

Electromagnetism

Big Picture

1. **Understand the fundamentals** of electrostatics and magnetism.
   - Coulomb's law, the electric field, electric potential, force on a charge

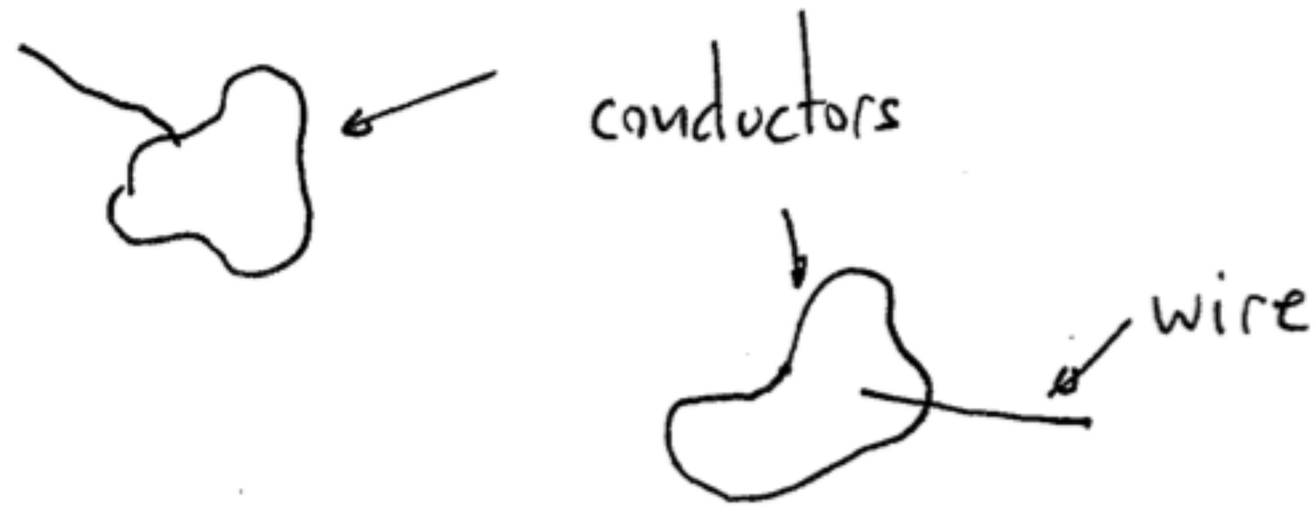
2. **Use these fundamentals** to "build" circuit components. Learn to analyze these circuits
   - capacitor, current, resistor, loop law, circuit analysis

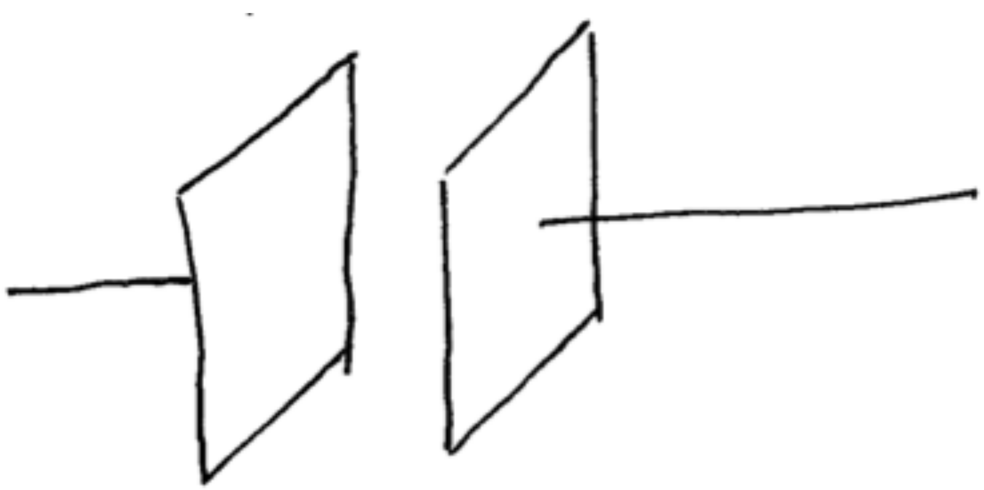
3. **Express these fundamentals** in the form of Maxwell's equations. See how Maxwell's equations predict something new.
   - Gauss's Law
   - Where are Maxwell's other three equations?

Capacitance

What is a Capacitor?

Two conductors separated by some distance.



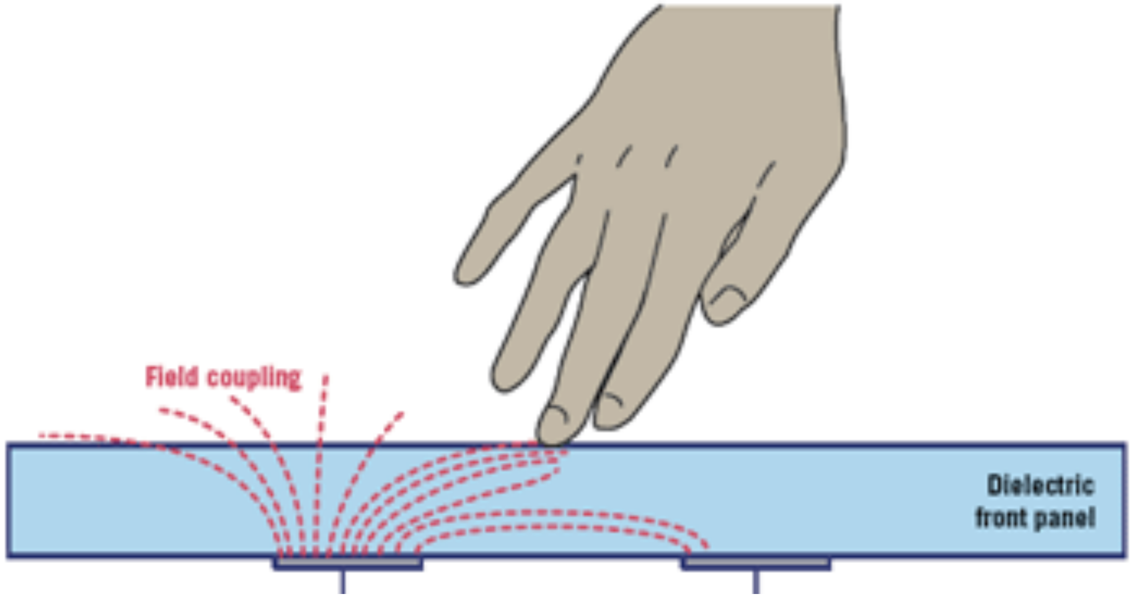


Most common configuration is the parallel plate capacitor.

The capacitor is a **physical break in the circuit**!

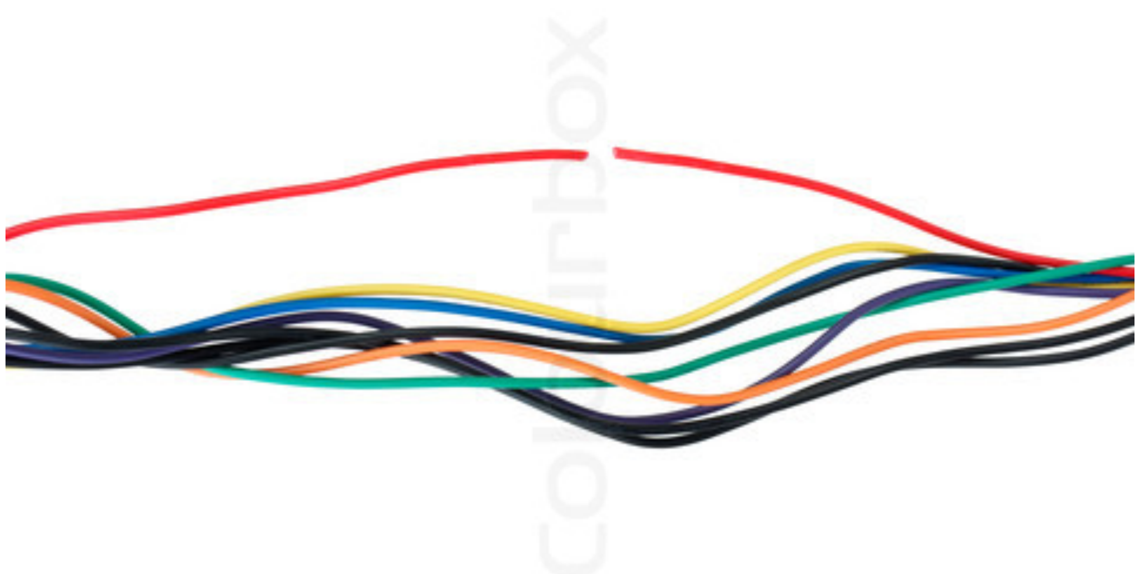
Examples of Capacitors

Examples of Capacitors

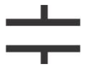


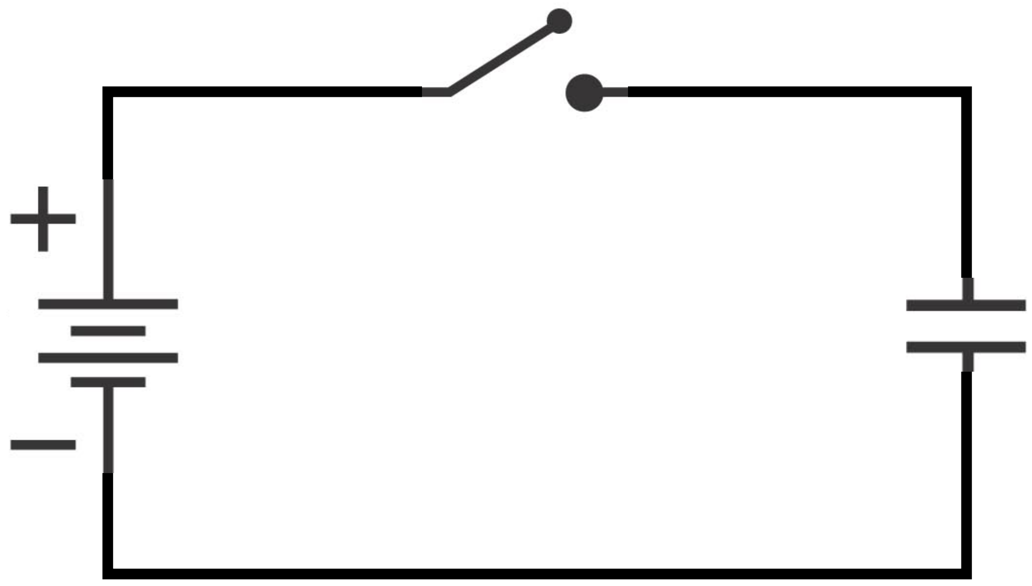
Conductive finger and conductive layer in a *touch screen* form a capacitor.

A **break** in a wire or a bad connection forms a capacitor.



Clicker Question

As described, the capacitor is essentially a break in the circuit (it's even drawn that way: ).



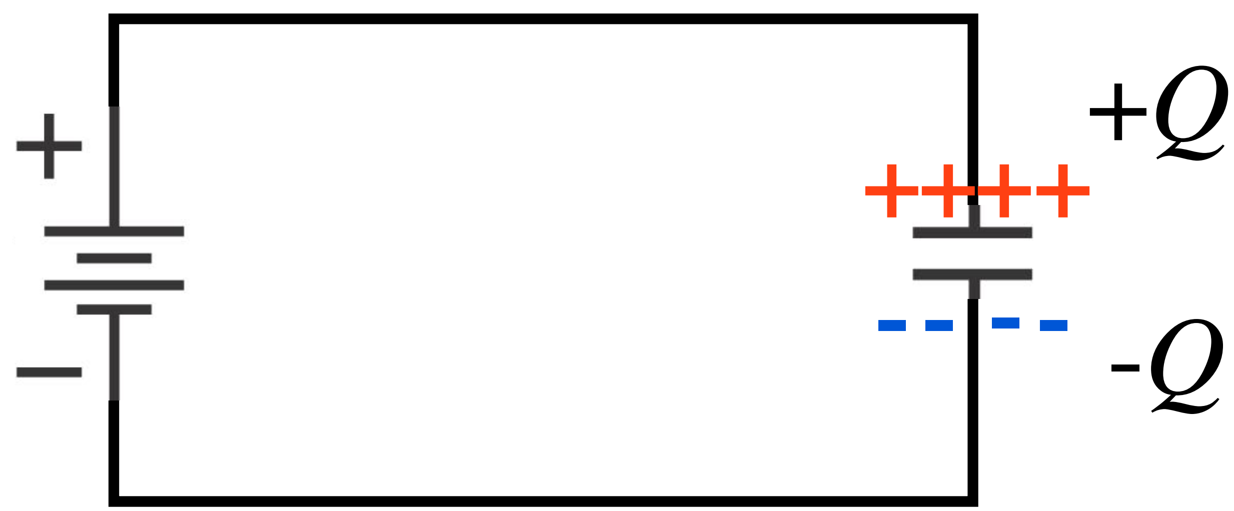
Does current flow when the switch is closed?

a) yes
b) sort of
c) no

AC/DC Circuit PhET #1 and #2

Clicker Question

The current **flows**, then **stops**.



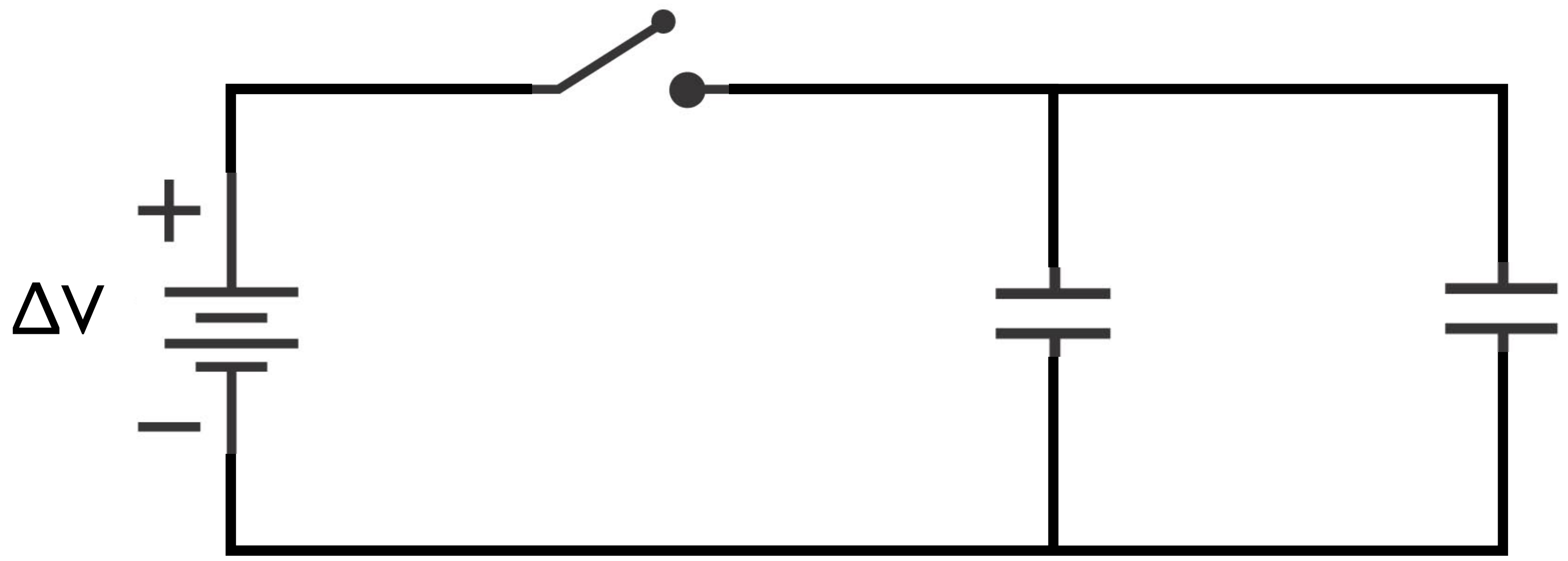
Current stops because the **battery can't do any more work**.

This is called "charging" the capacitor with **charge Q**, even though the capacitor's **total charge is zero**.

AC/DC Circuit PhET #1 and #2

Observation

Watch closely. What happens now when we close the switch?



AC/DC Circuit PhET #1 and #2

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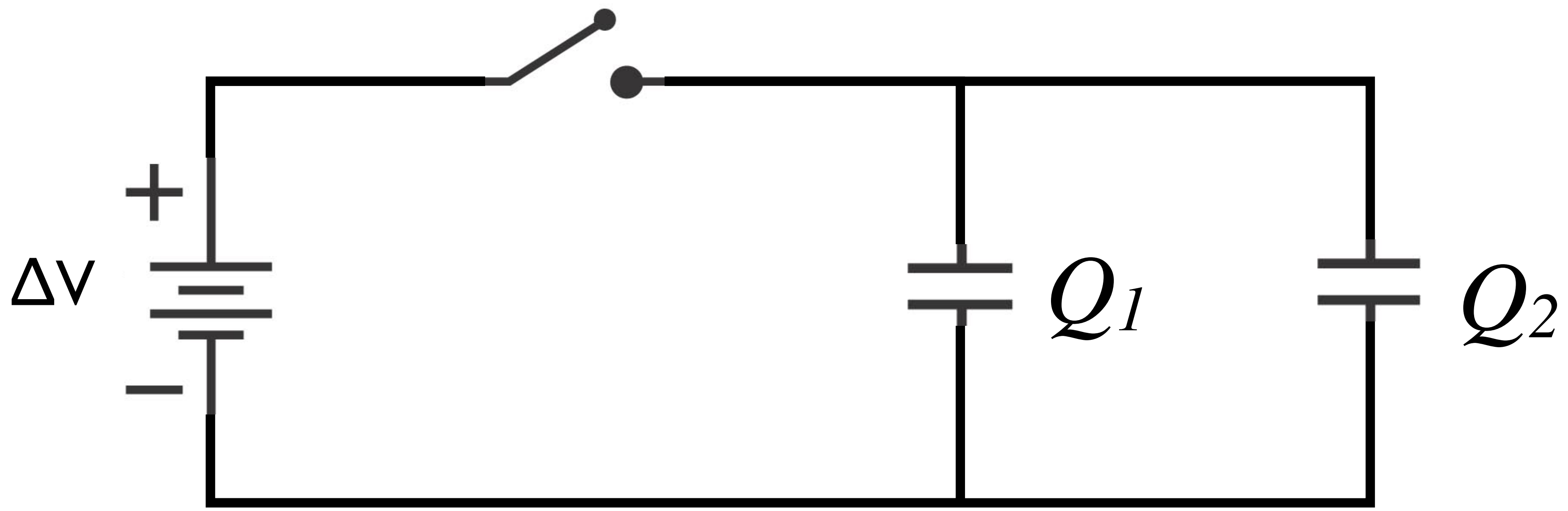


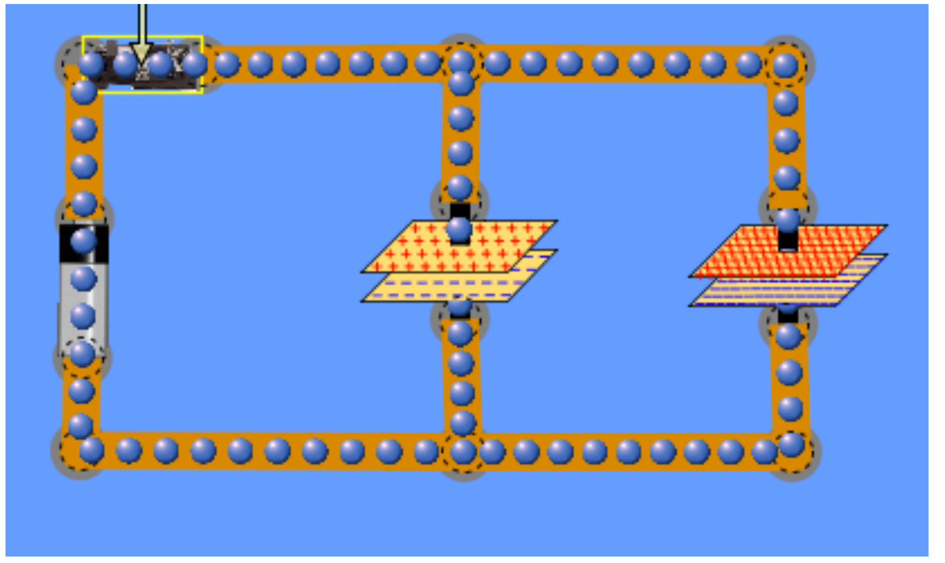
1. The capacitors have **different charges Q_1 and Q_2**.
2. The capacitors have the **same voltage ΔV** across them.
3. The ratio **Q**/**ΔV** is different for each capacitor.

AC/DC Circuit PhET #1 and #2

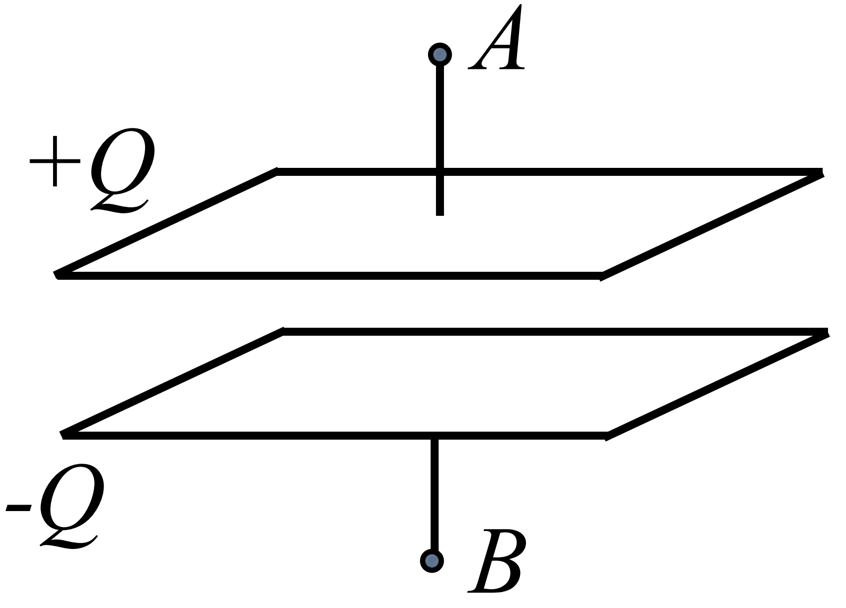
Capacitance

The ability for a capacitor to hold charge for a given voltage.

$$C = \frac{Q}{V}$$



where V is the voltage across the capacitor and $+Q$ and $-Q$ are the charges on the plates.



$$V = |\Delta V_{AB}|$$

Uses of Capacitors

Particle Accelerators

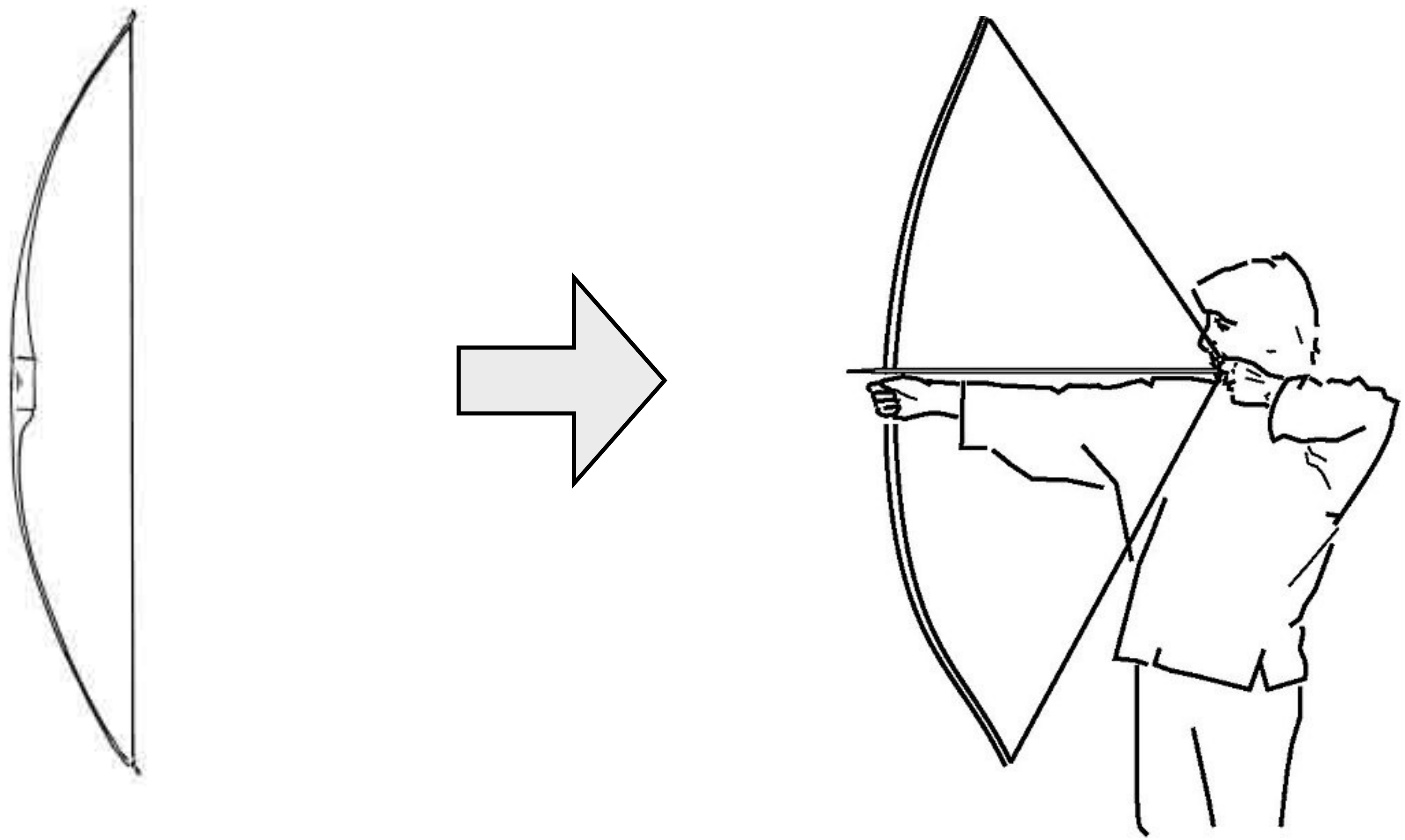
Camera Flashes

RAM

Defibrillators

Storing Potential Energy

Pulling on a bow string increases potential energy



Shoots arrows faster than we can throw them.

Storing Potential Energy

Lifting a clam increases potential energy

Birds drop clams to break them open.

Storing Potential Energy

The capacitor is great for delivering charge in a short amount of time.

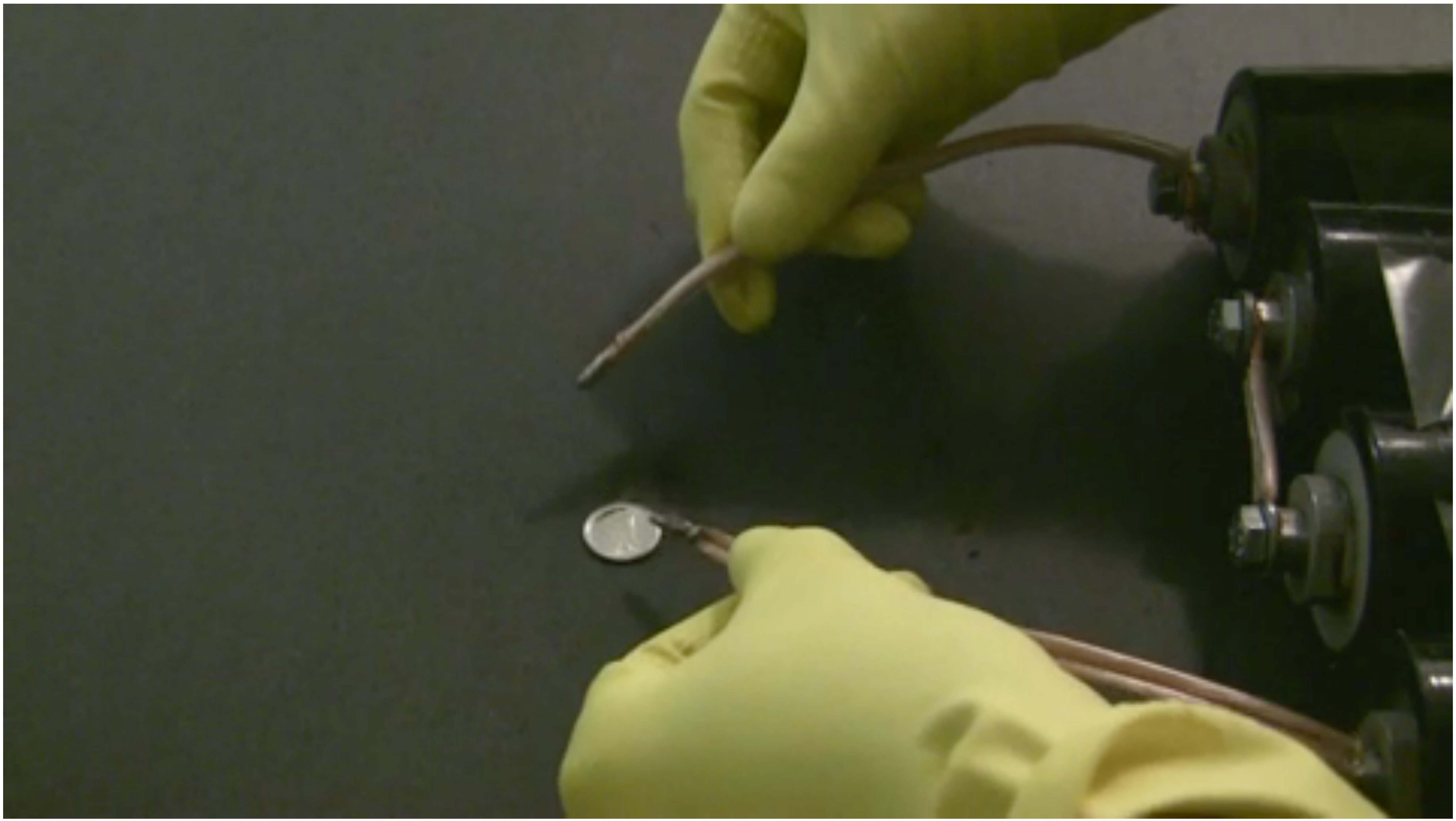
Defibrillators

However, the addition of a resistor let's you carefully control how quickly the charge leaves the capacitor.

Used in high-pass and low-pass filters. Can also integrate and differentiate voltages!

Capacitors Release Energy Quickly

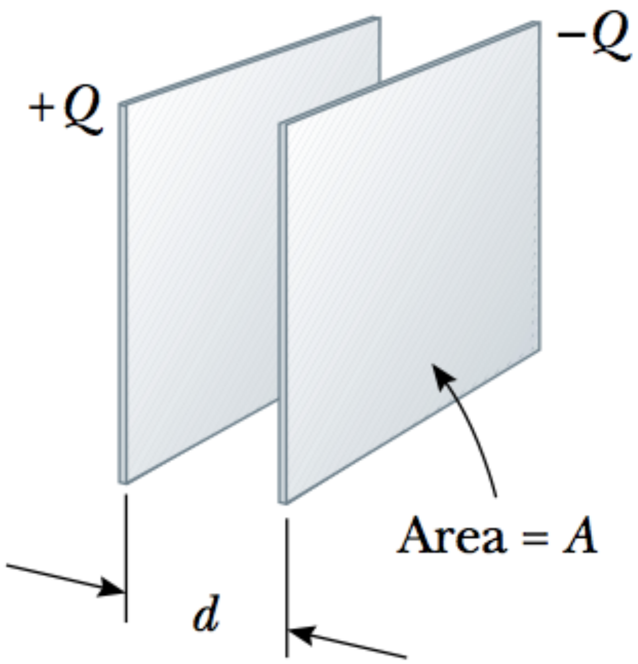
A 9V battery can't do this.



fun with ultacapacitors (video)

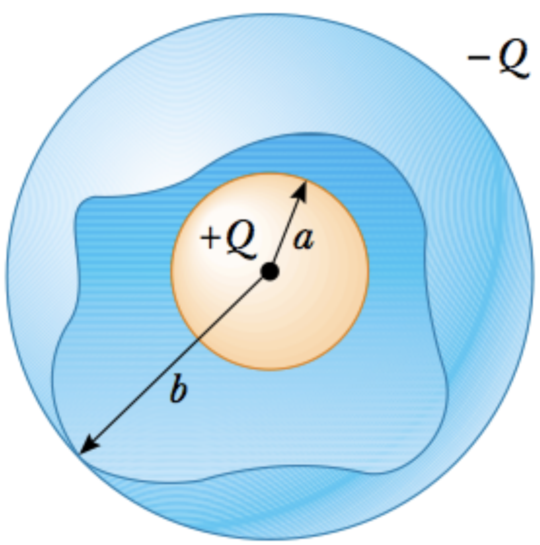
Capacitance is Geometric

Parallel Plate (from assignment)



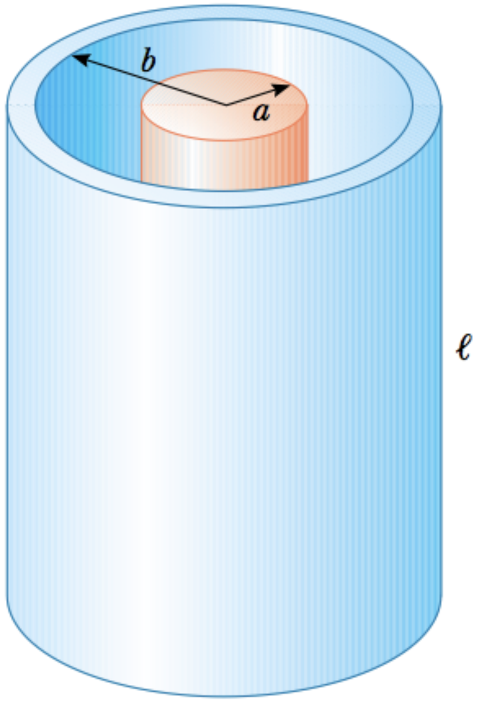
$$C = \frac{\epsilon_0 A}{d}$$

Spherical



$$C = 4\pi\epsilon_0 \left(\frac{ab}{b-a}\right)$$

Cylindrical



$$C = 2\pi\epsilon_0 \frac{\ell}{\ln\left(b/a\right)}$$

Clicker Question

A parallel-plate capacitor is ***charged*** by a 12V battery and then ***disconnected from it***. Pressing on the capacitor, i.e., reducing the gap between the plates, leads to

	C	ΔV	Q
A	+	+	+
B	+	+	-
C	+	=	+
D	+	-	=
E	=	=	=

+ increase
- decrease
= no change

Clicker Question

A parallel-plate capacitor is ***charged*** by a 12V battery and then ***disconnected from it***. Pressing on the capacitor, i.e., reducing the gap between the plates, leads to

	C	ΔV	Q
A	+	+	+
B	+	+	-
C	+	=	+
D	+	-	=
E	=	=	=

+ increase
- decrease
= no change

This is how a pressure plate sensor and some microphones work.

Capacitors Store Energy

The work required to move a little bit of charge is

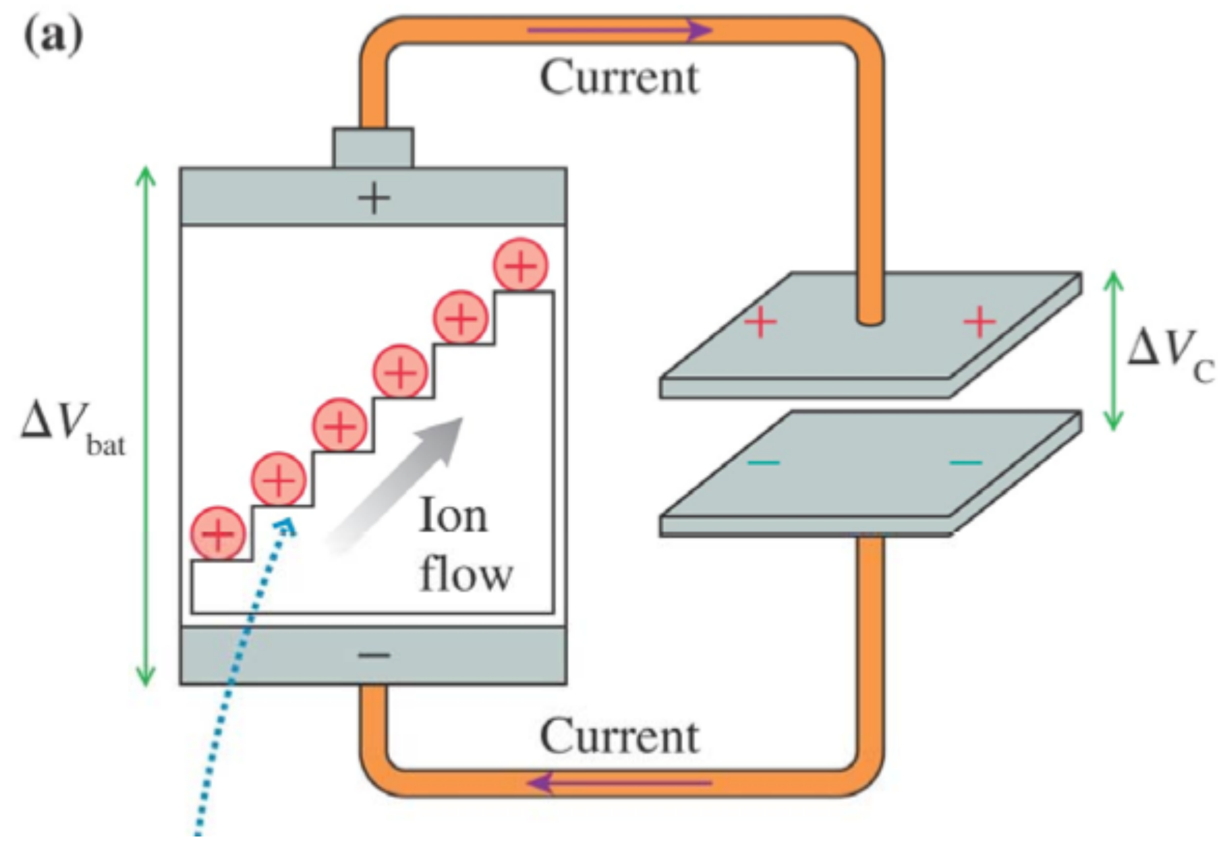
$$dW = dU = V\,dq = \frac{q}{C}dq$$

which gives the integral

$$U = \int_0^Q \frac{q}{C}dq$$

The potential energy in a capacitor is given by.

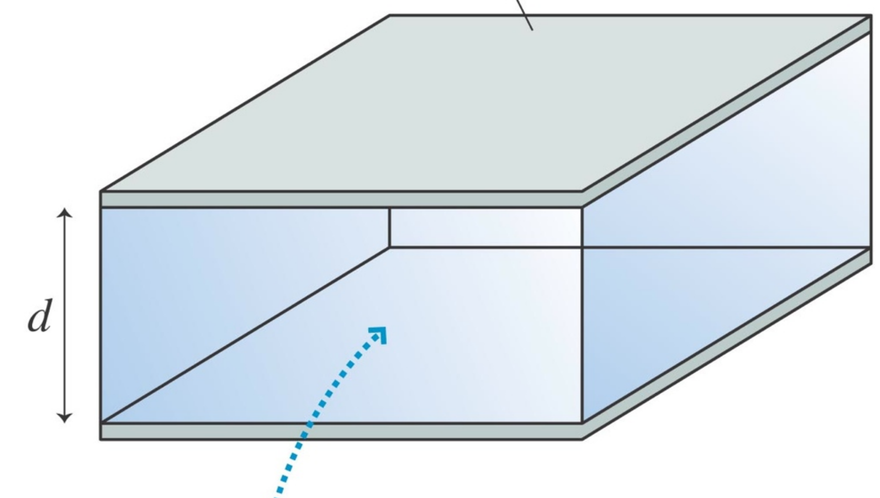
$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2$$



Electric Fields Store Energy

For a parallel plate capacitor we find that

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\frac{\epsilon_0 A}{d}(Ed)^2 = \frac{\epsilon_0}{2} \times \text{Volume}$$



The capacitor's energy is stored in the electric field in volume Ad between the plates.

The energy density of the electric field is

$$u = \frac{\epsilon_0}{2}E^2$$

This energy density a **general result**. All electric fields have this energy density.

Supplimentary

Putting a dielectric in a capacitor lowers the electric field, which **increases the capacitance.**

There are two pictures:

1. The **dielectric changes epsilon_0 to epsilon**, which changes how well the electric field can permeate space.

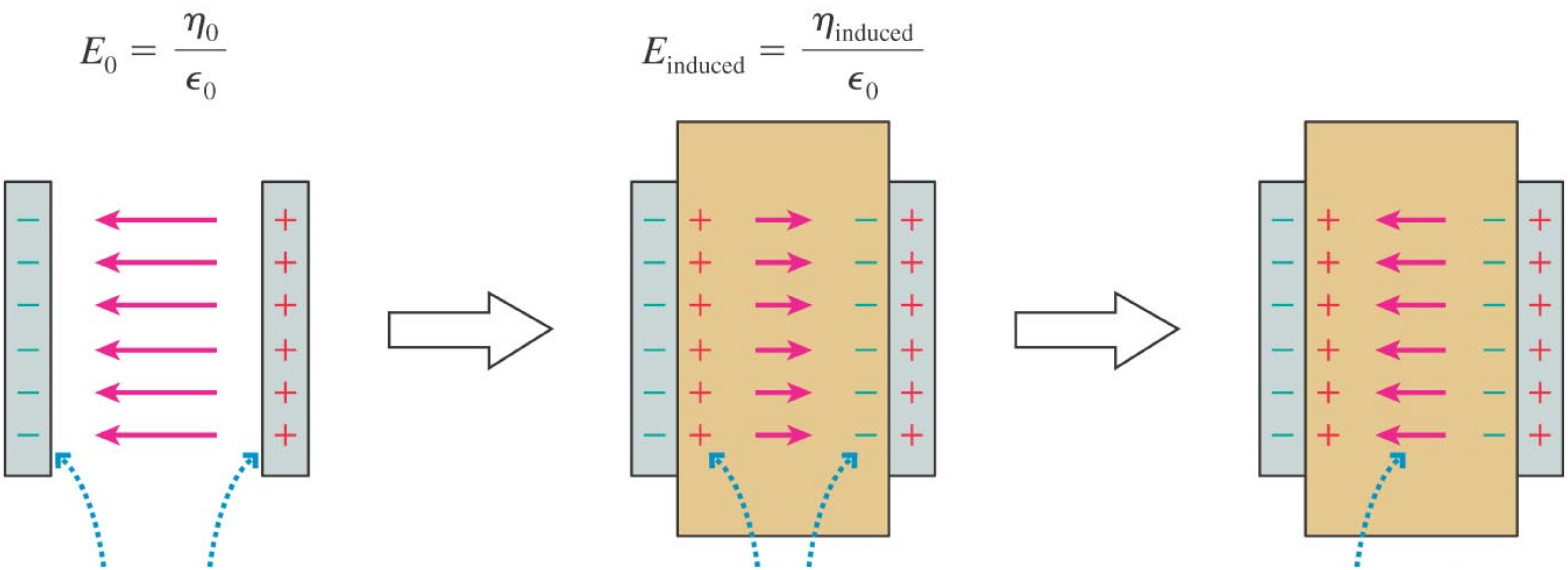
$$E_{\text{without dielectric}} = \frac{\eta}{\epsilon_0} \longrightarrow E_{\text{with dielectric}} = \frac{\eta}{\epsilon} = \frac{\eta}{\kappa\epsilon_0}$$

Because \kappa is positive, E decreases, and the capacitance increases.

2. **Superposition of the electric fields.** The electric field polarizes the dielectric. The polarization actually creates an electric field that opposes the original field, reducing the net field.

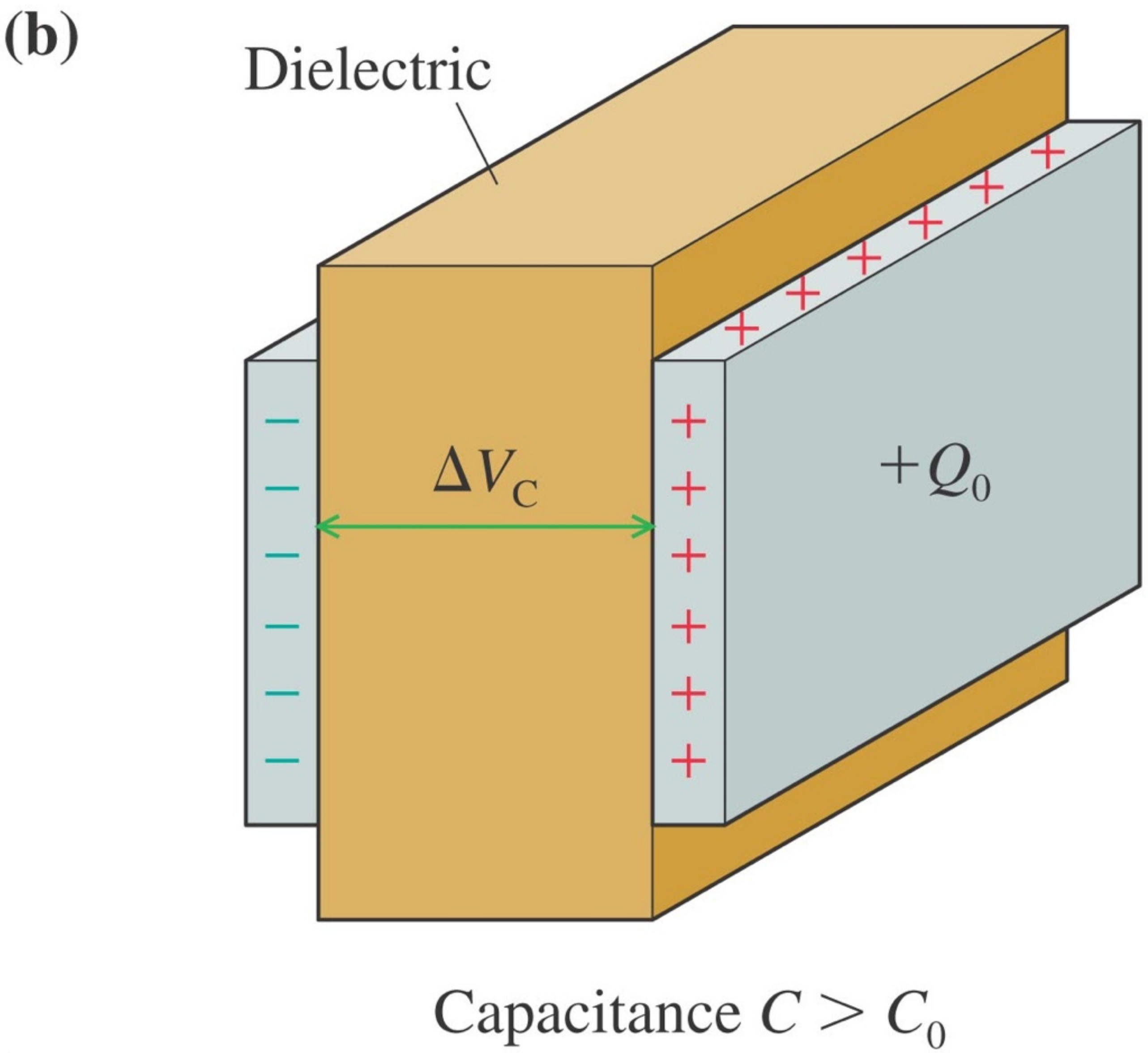
Dielectrics (in tutorial)

The polarization of an insulator in between the plates decreases the net electric field.



Lowering E increases capacitance: $C = \frac{Q}{V} = \frac{Q}{Ed}$

Dielectrics



The capacitance with the dielectric C is **greater than** the capacitance in a vacuum C_0.

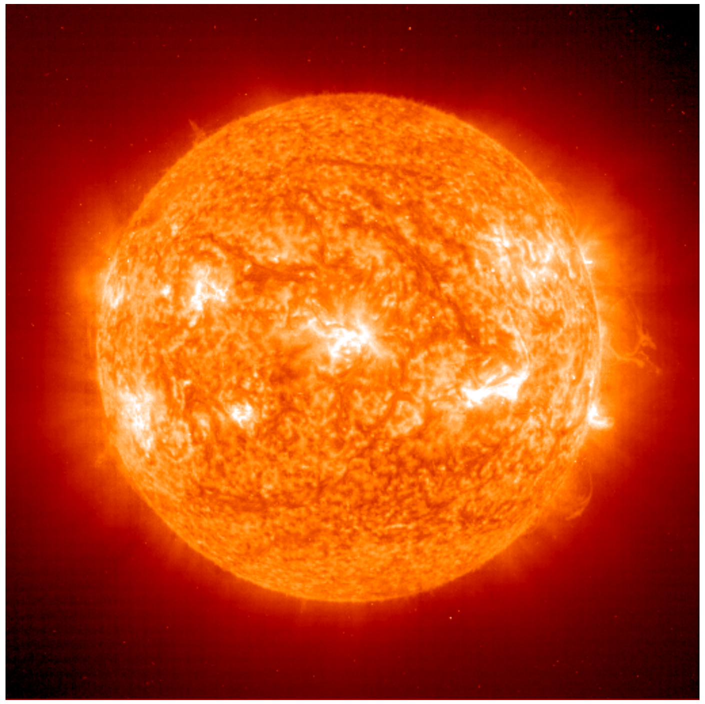
The **dielectric constant** kappa κ is the ratio of the new capacitance to the capacitance in a vacuum.

$$C = \kappa C_0$$

"Empty Space" Has Energy

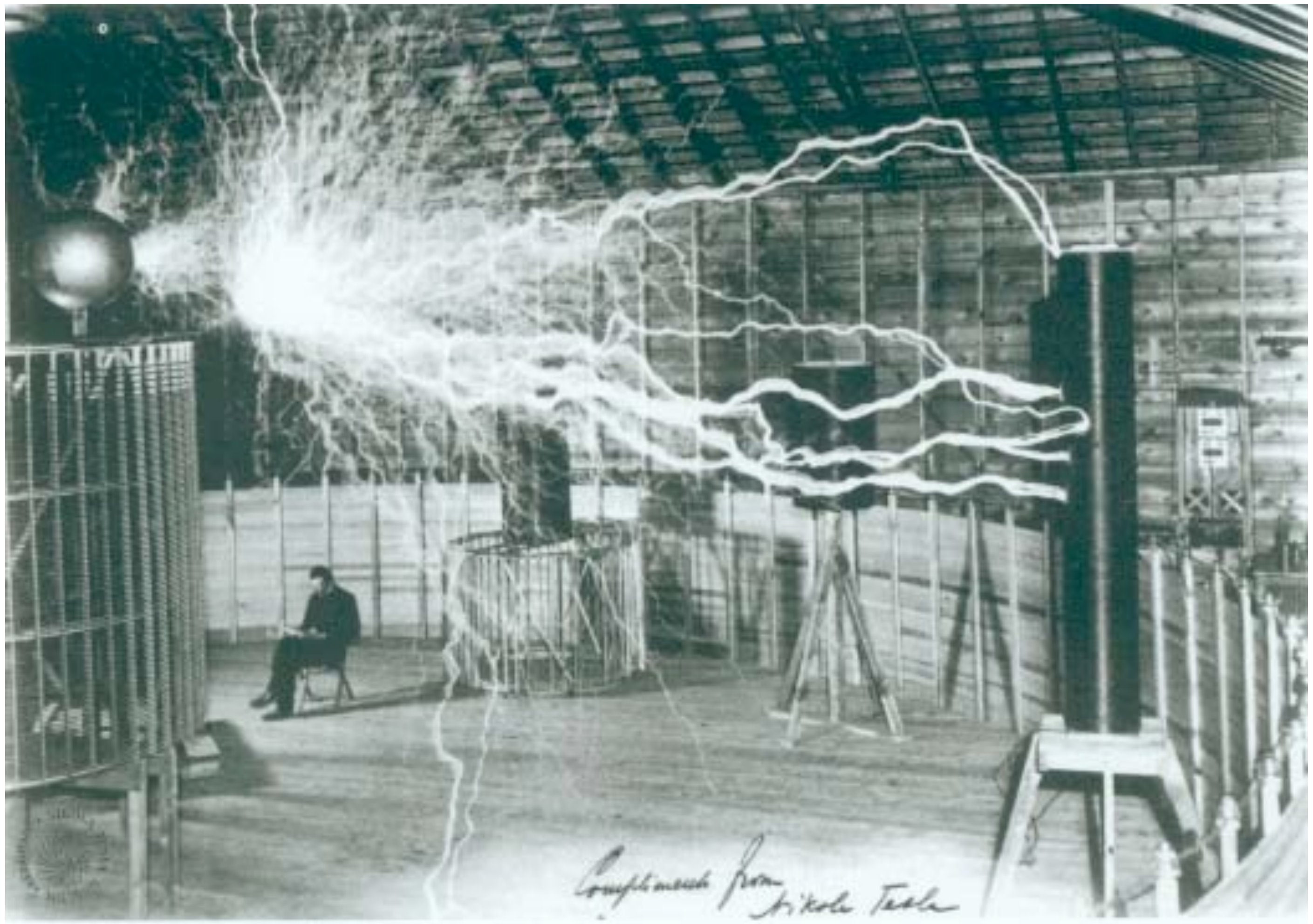
But James, the energy is stored in the charge configuration, not the field. You're playing shell games with us.

The electric field can exist in the absence of charge. The fields wiggle in symbiosis with magnetic fields to make light.



The smallest unit of wiggling electric fields is called the **photon** (discovered separately from radiation).

Circuit Analysis



Kirchhoff's Loop law

A foundation of circuit analysis.

The loop law comes from path independence:

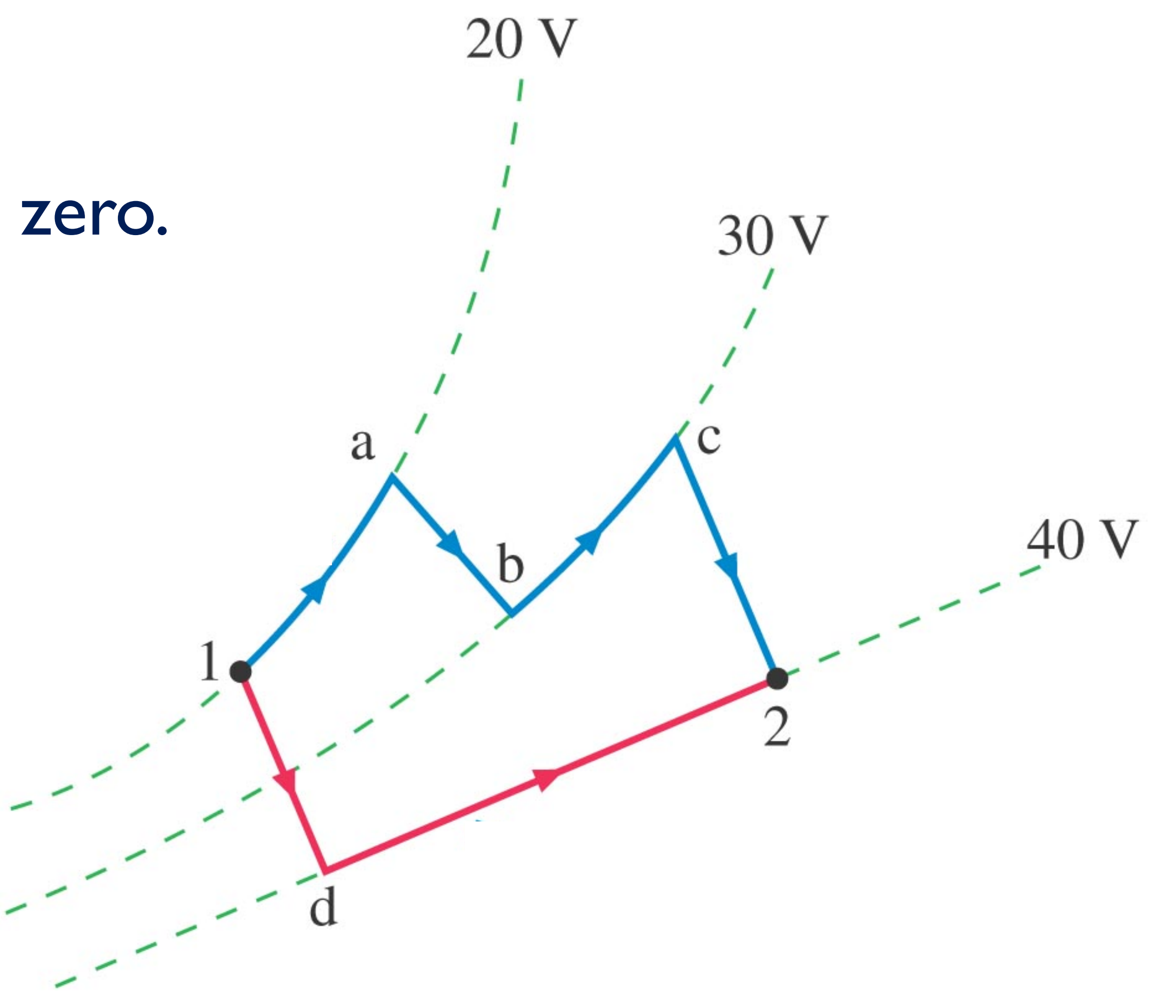
$$\Delta U = -\oint \vec{F} \cdot d\vec{s} = -q \oint \vec{E} \cdot d\vec{s} = 0$$

The change in energy around a closed path is zero.

We then know that $U = qV$ gives

$$\Delta V_{\text{loop}} = 0$$

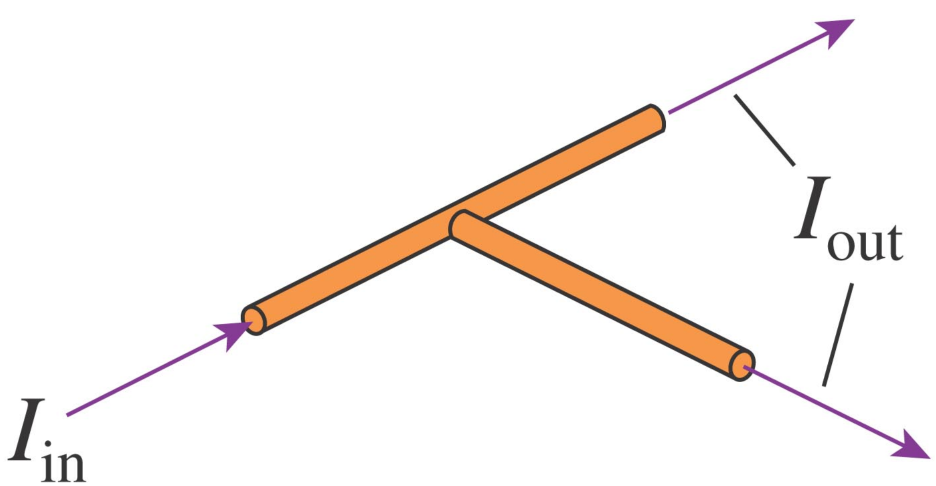
Which is the loop law.



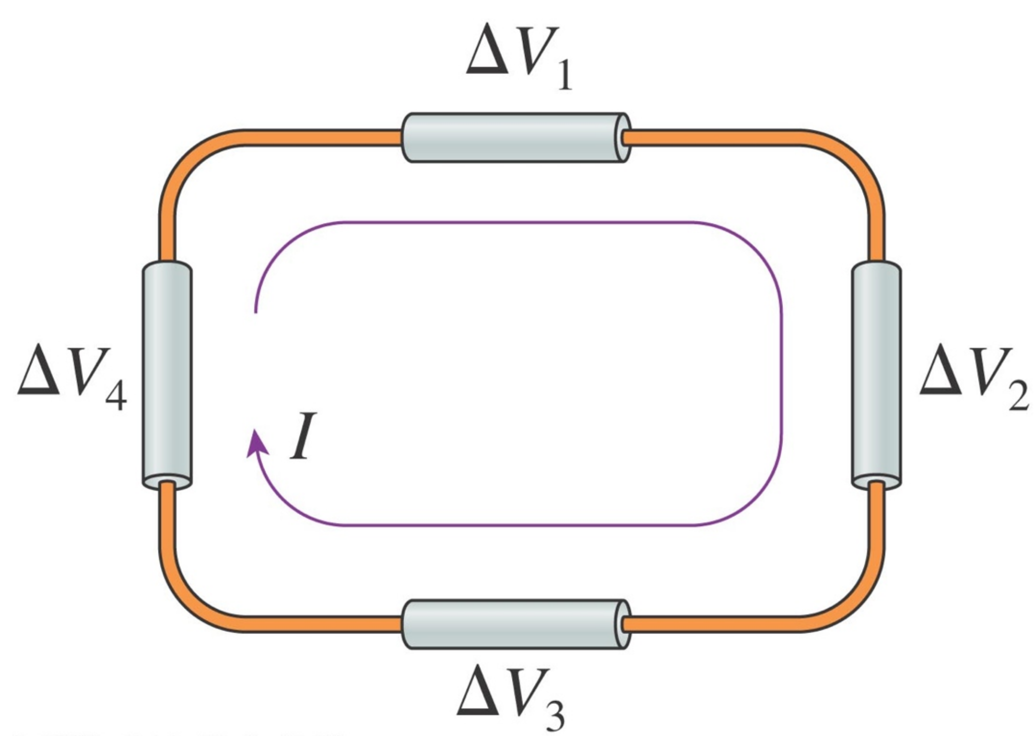
Kirchhoff's Laws

Kirchhoff's Laws are summarized as:

$$\sum I_{in} = \sum I_{out} \quad \text{at a junction}$$



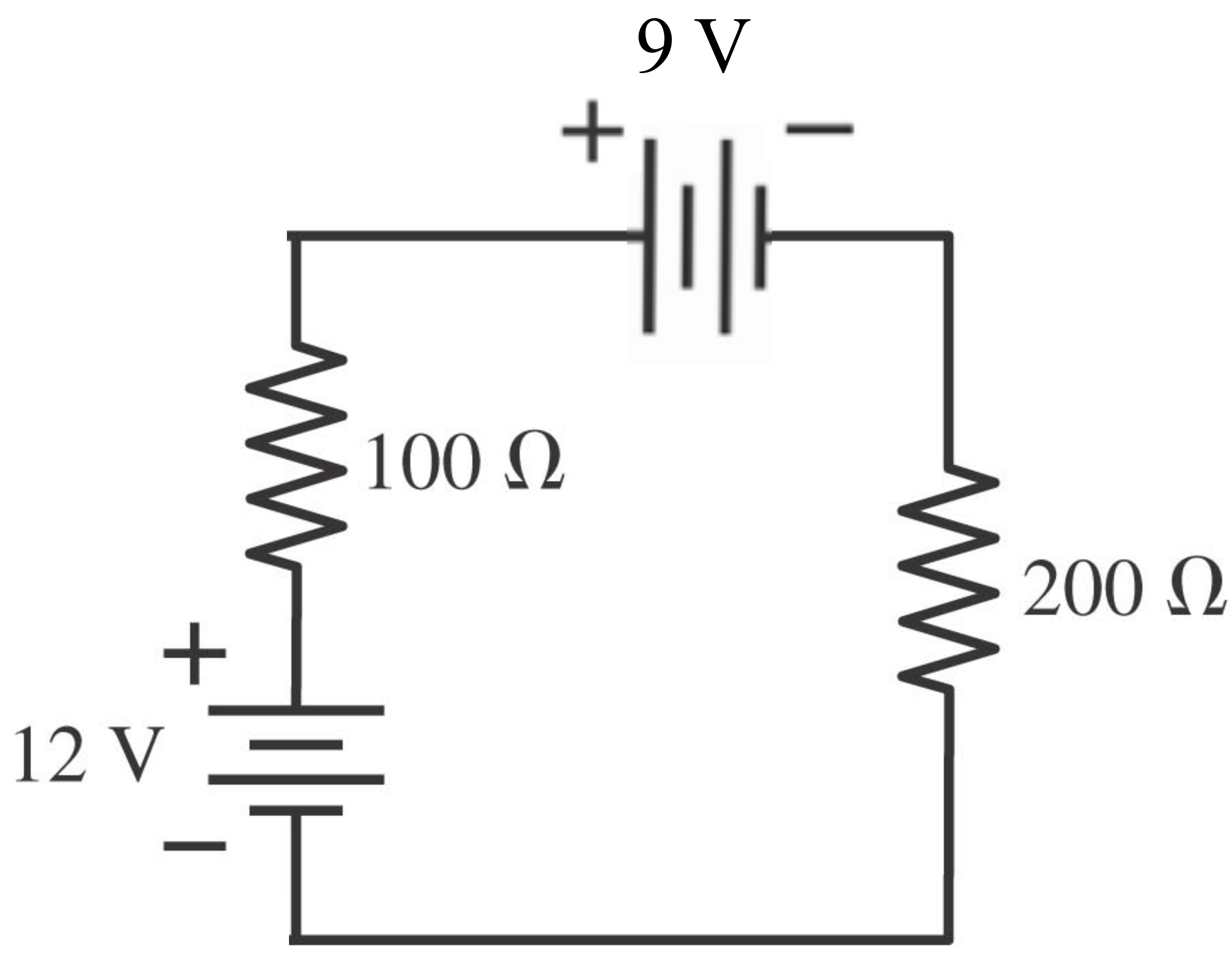
$$\sum_i \Delta V_i = 0 \quad \text{around closed loop}$$



$$\Delta V_1 + \Delta V_2 + \Delta V_3 + \Delta V_4 = 0$$

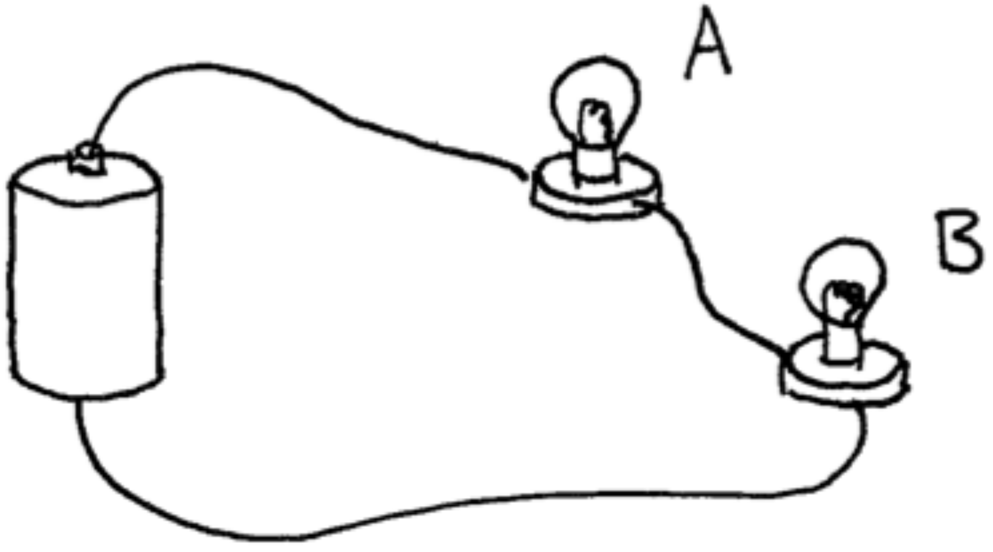
The Loop Law in Action

Find the current running through the circuit.



Clicker Question

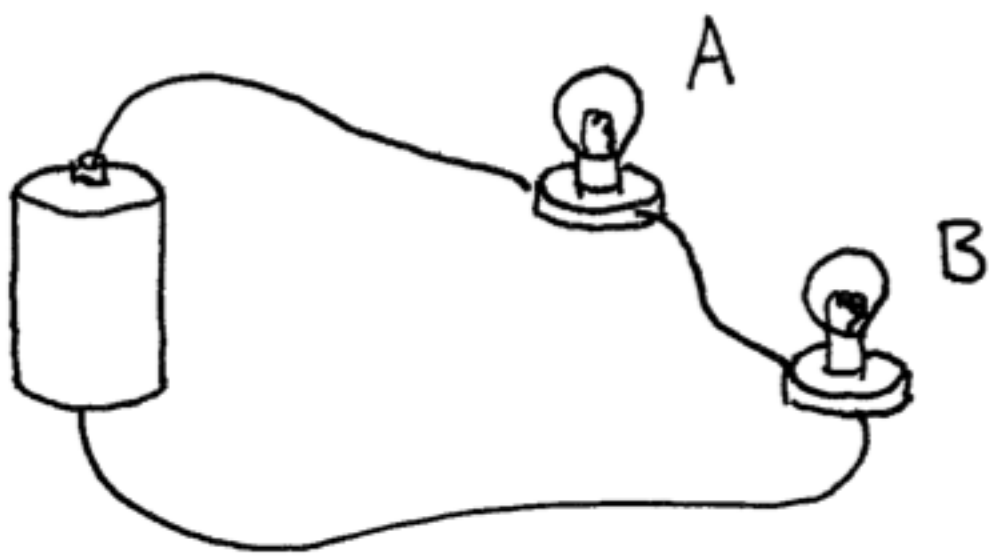
Which light bulb is brighter?



a) A
b) B
c) both the same

Clicker Question

Which light bulb is brighter?



a) A

b) B

c) both the same

Clicker Question

Which lightbulbs are brighter?

a) the bulbs in the circuit on the left

b) the bulb in the circuit on the right

c) the left circuit has one bulb brighter and one bulb dimmer than the bulb in the right circuit.

d) they're all the same brightness

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Which lightbulbs are brighter?

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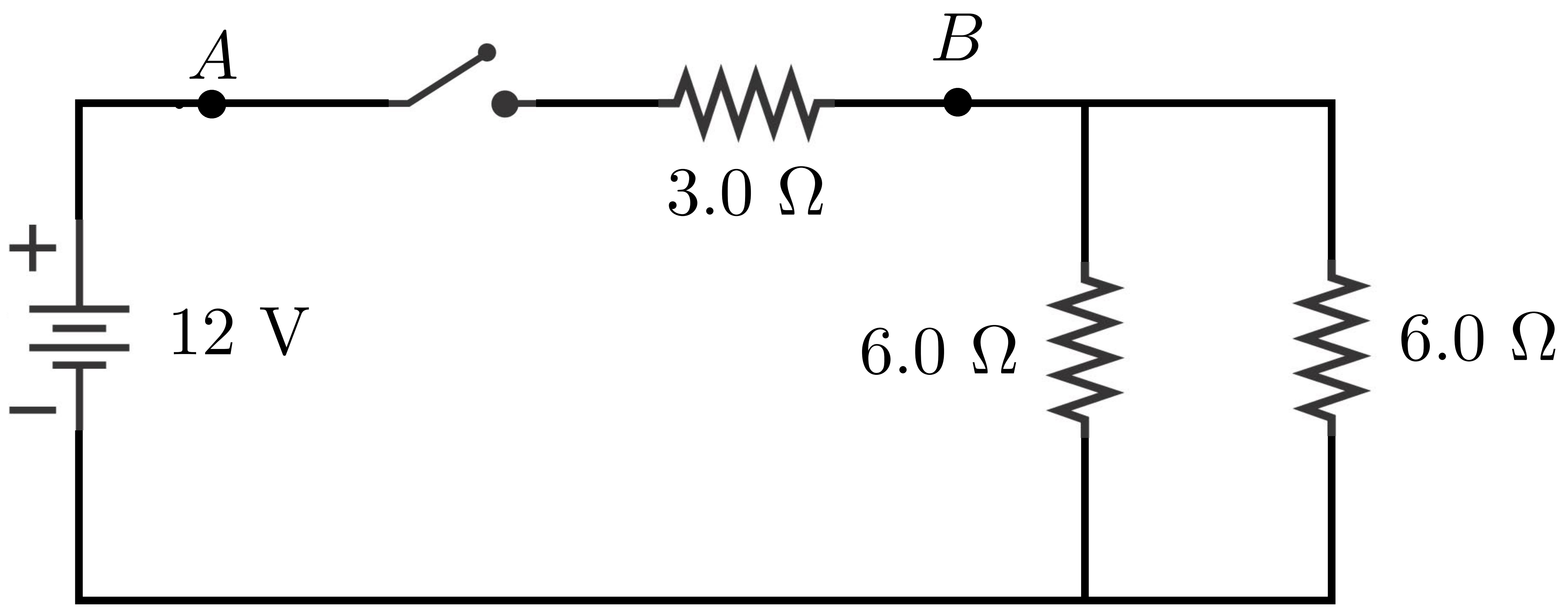
Clicker Question

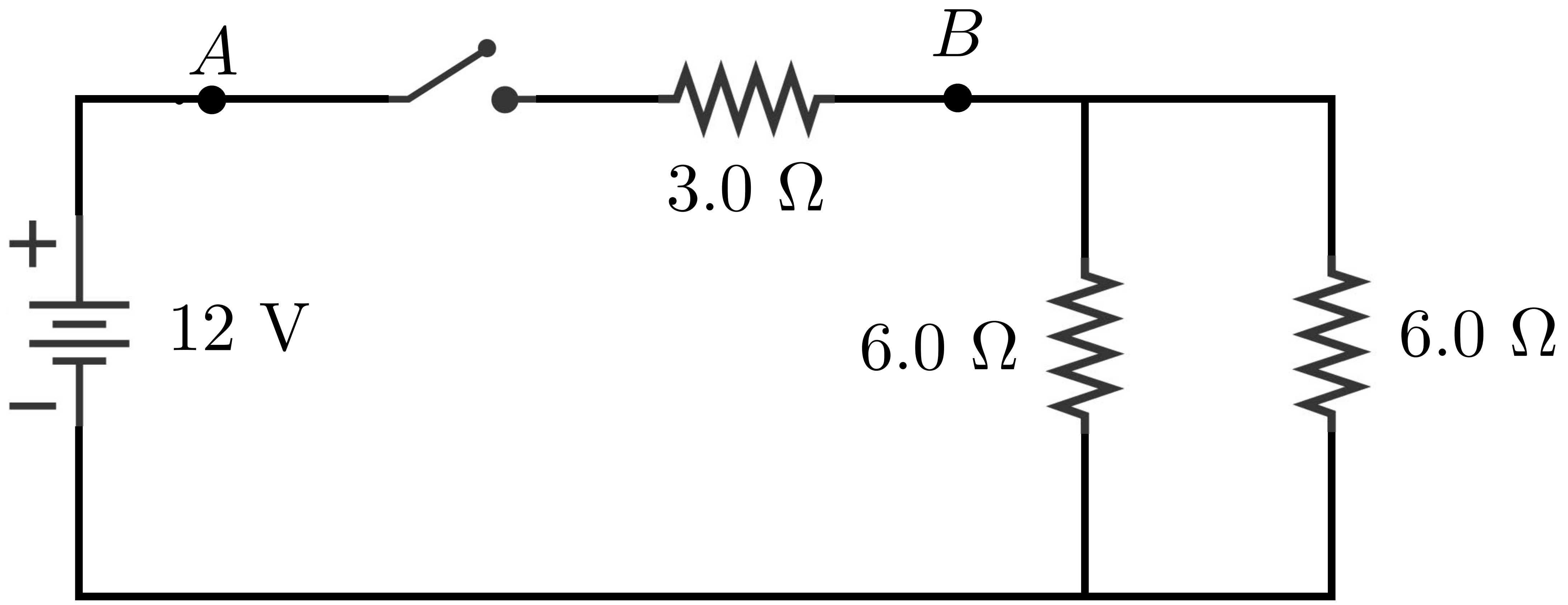
The switch is open. What is the potential difference between point *A* and *B?*



a) 0V
b) 3V
c) 6V
d) 9V
e) 12V

Clicker Question

The switch is open. What is the potential difference between point *A* and *B?*



a) 0V

b) 3V

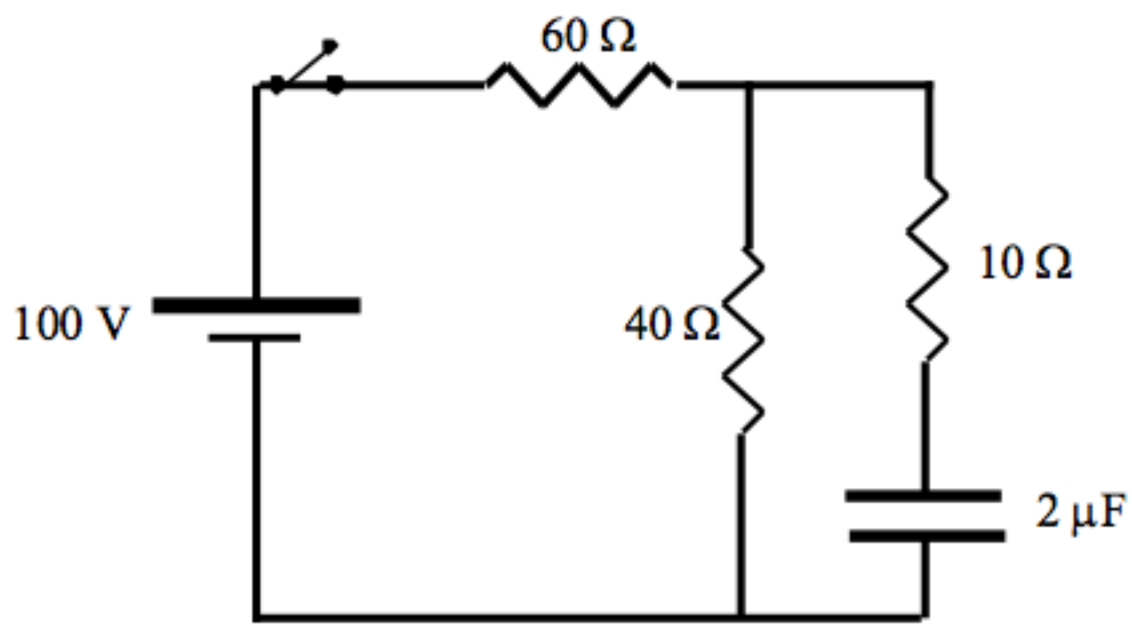
c) 6V

d) 9V

e) 12V

Clicker Question

We close the switch. What is the voltage across the capacitor once the circuit has run for a while?



a) 0 V
b) 11.8 V
c) 40 V
d) 60 V
e) Unable to determine.

Clicker Question

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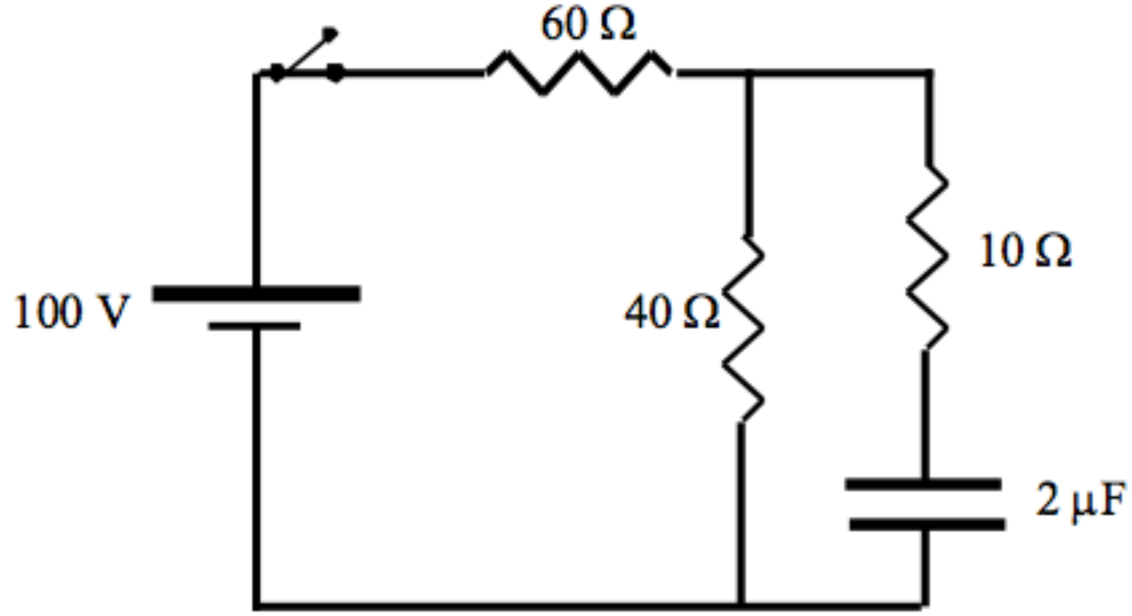

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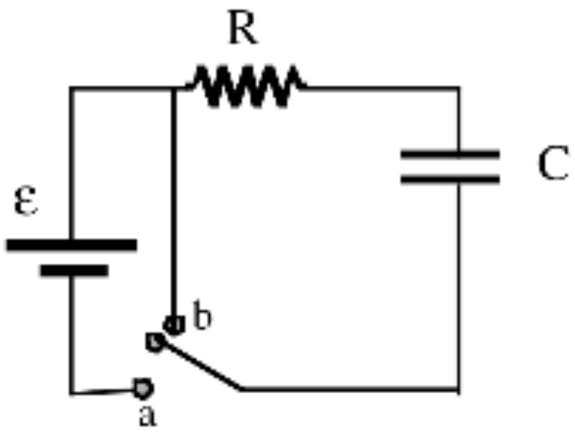
RC Circuits

Charging and Discharging Capacitors



ε=100 V
R=1000 Ω
C=1000 μF

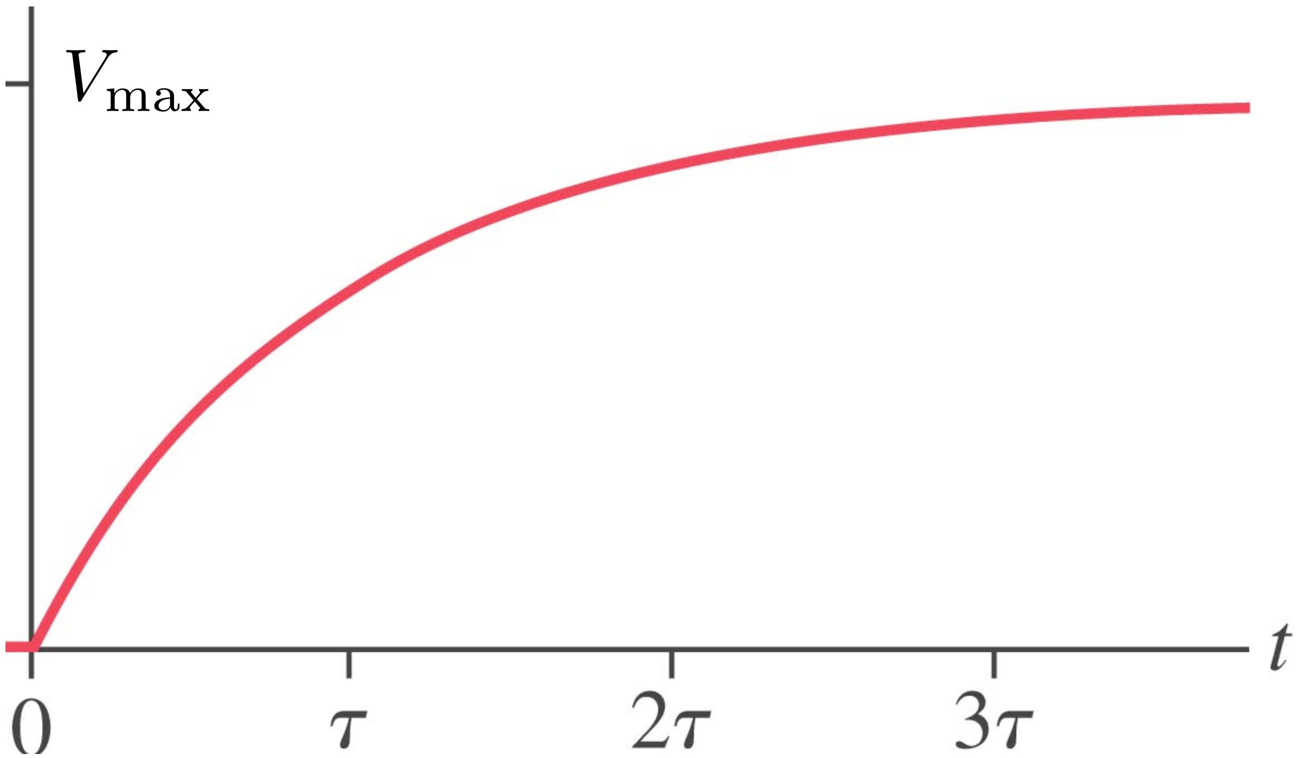
Switch to a $\Rightarrow$ charge $\Rightarrow$ long time $\Rightarrow$ switch to b $\Rightarrow$ discharge

		Charge		Discharge	
Quantity	max value	$t = 0$	$t = \infty$	$t = 0$	$t = \infty$
q					
ΔV_C					
I					
ΔV_R					

RC Circuits

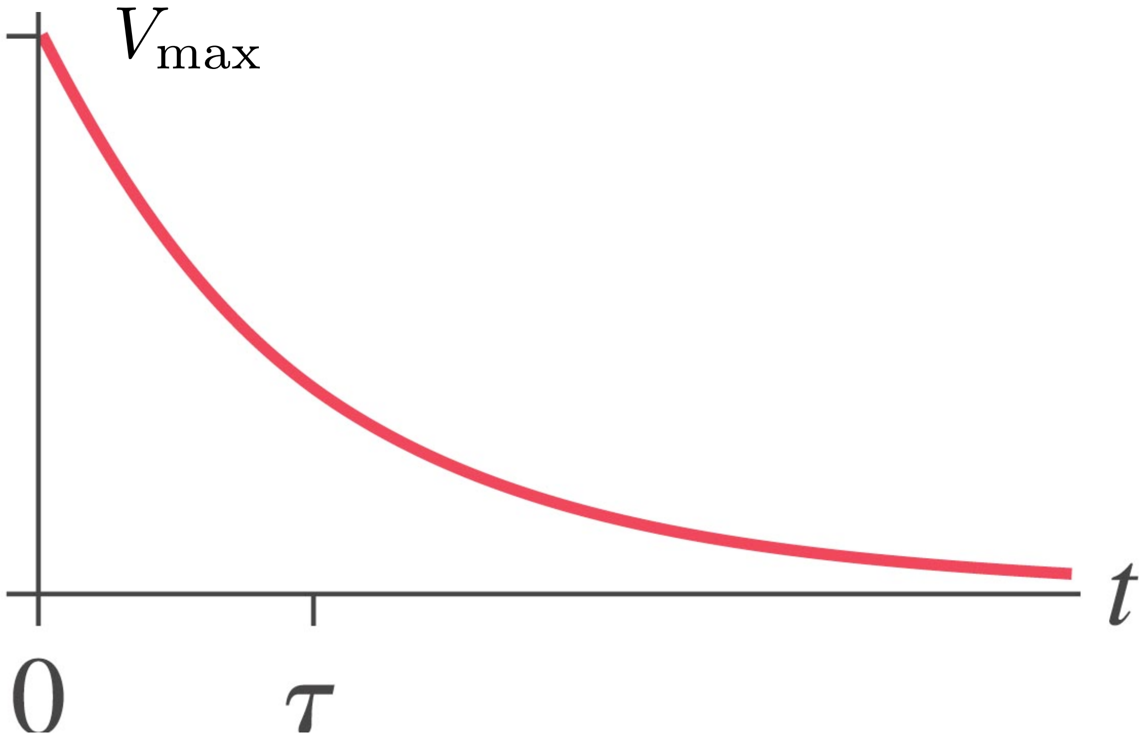
Charging and discharging a capacitor is governed by exponential laws.

charging:



$$V_C = V_{max}\left(1 - e^{-t/RC}\right)$$

discharging:



$$V_C = V_{max}\, e^{-t/RC}$$

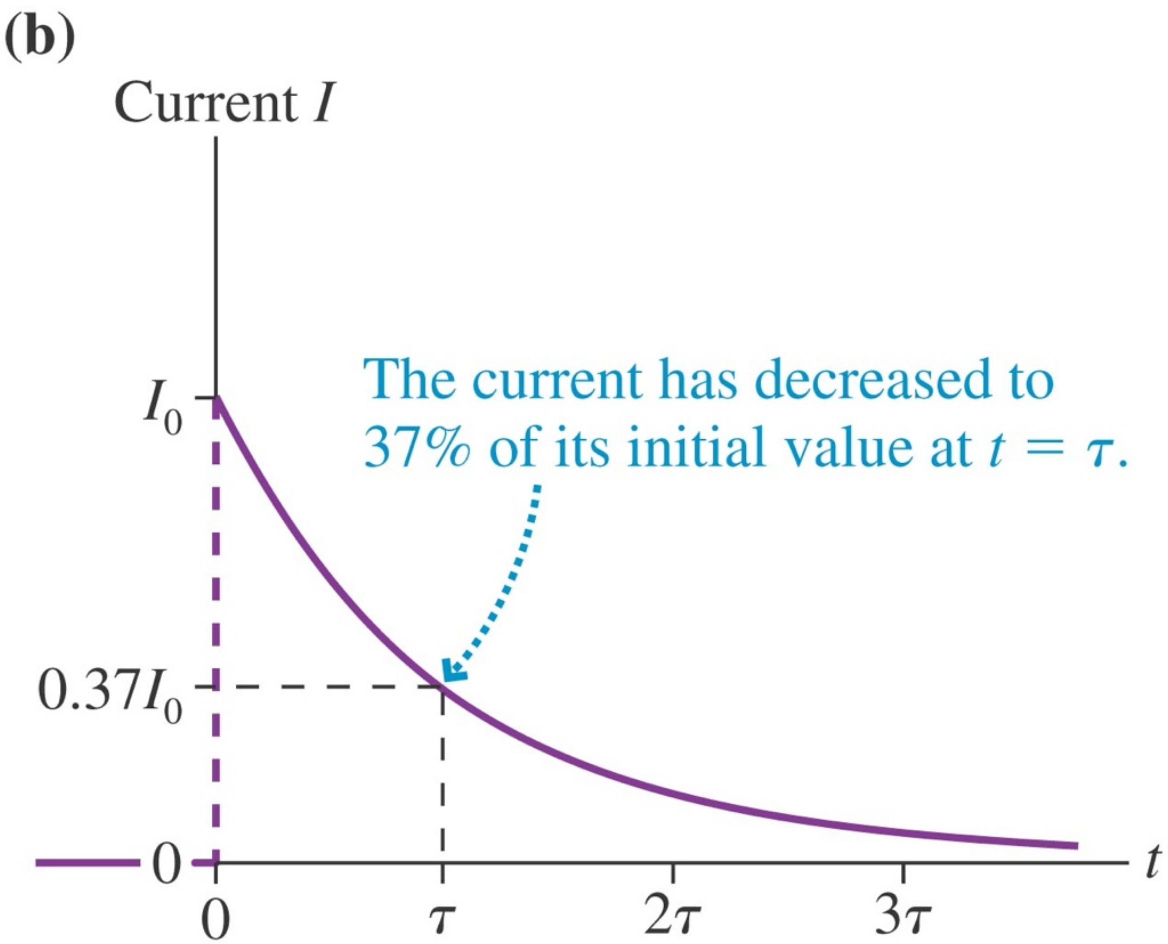
Circuit Construction Kit PhET

RC Circuits

We can define a time constant to characterize the exponential decay

$$\tau = RC$$

It's mathematically identical to the lifetime in radioactive decay.



Clicker Question

This graph shows V_C of a capacitor that is separately discharged through three difference resistors.

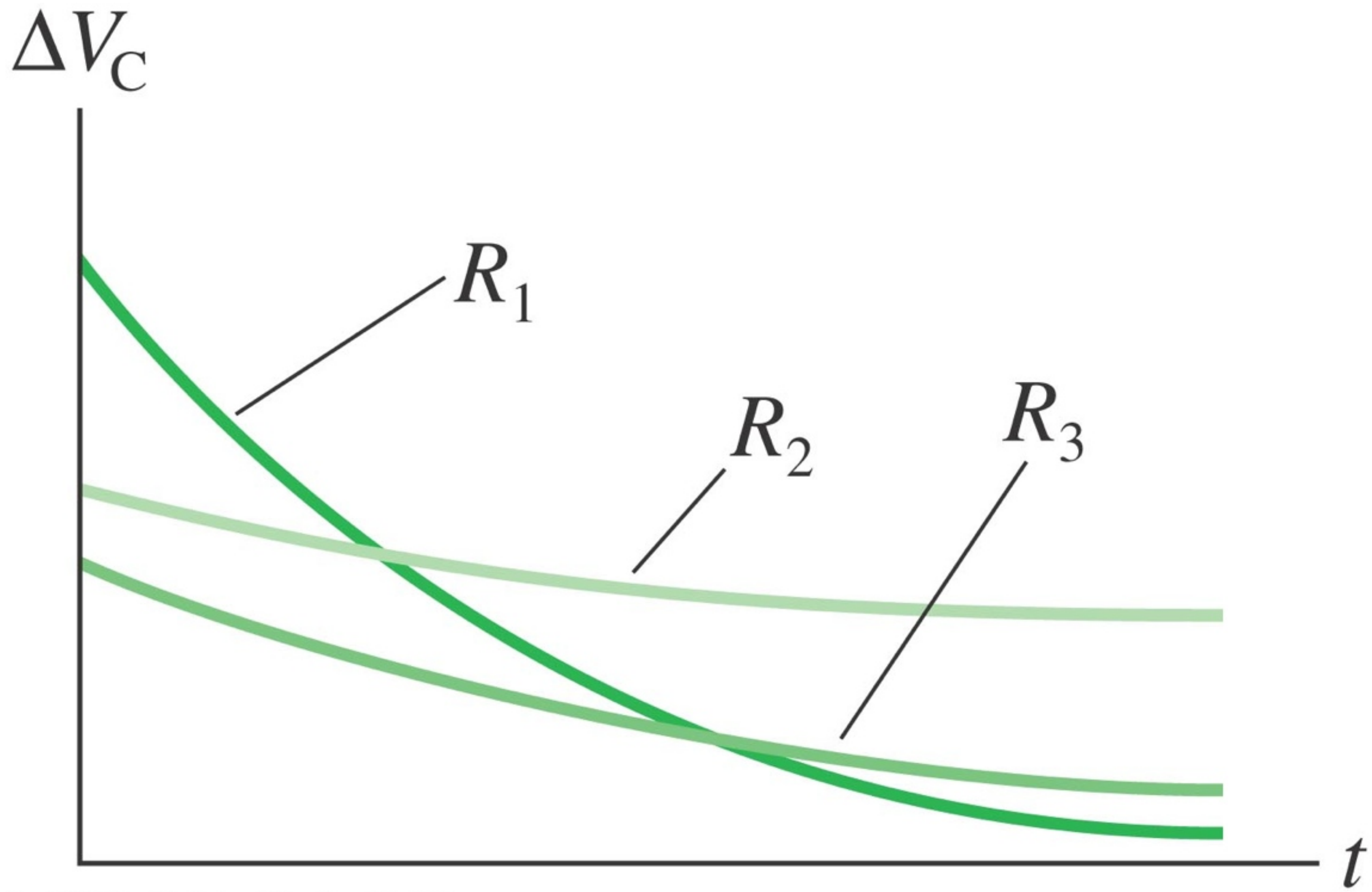
Rank the value of the resistance from smallest to largest.

a) $R_1 > R_2 > R_3$

b) $R_3 > R_2 > R_1$

c) $R_1 > R_3 > R_2$

d) $R_2 > R_3 > R_1$



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