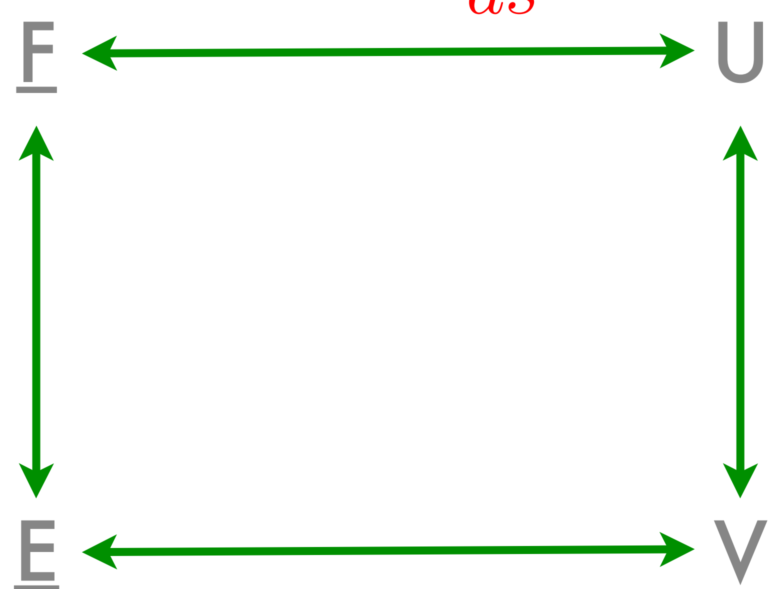
4 Electric Quantities

$$dU = -\vec{F} \cdot \vec{ds}$$

$$F_s = -\frac{dU}{ds}$$

Force

Energy



$$\vec{F} = q\vec{E}$$

$$\vec{E} = \frac{\vec{F}}{q}$$

$$U = qV$$

$$V = \frac{U}{q}$$

Field

Potential

$$dV = -\vec{E} \cdot \vec{ds}$$

$$E_s = -\frac{dV}{ds}$$

Equipotentials and Fields

Consider the relationship: $dV = -\vec{E} \cdot \vec{ds}$

What happens if we choose the path *ds* to follow an equipotential?

The change in voltage *dV* along the path is:

A) positive
B) negative
C) zero
D) can't determine

Which from the equation above means that the angle between ***E*** and ***ds*** is:

A) 0^o
B) 90^o
C) 180^o
D) can't determine

Charges and Fields PhET

Equipotentials and Fields

Consider the relationship: $dV = -\vec{E} \cdot \vec{ds}$

What happens if we choose the path *ds* to follow an equipotential?

The change in voltage *dV* along the path is:

A) positive
B) negative
C) zero
D) can't determine

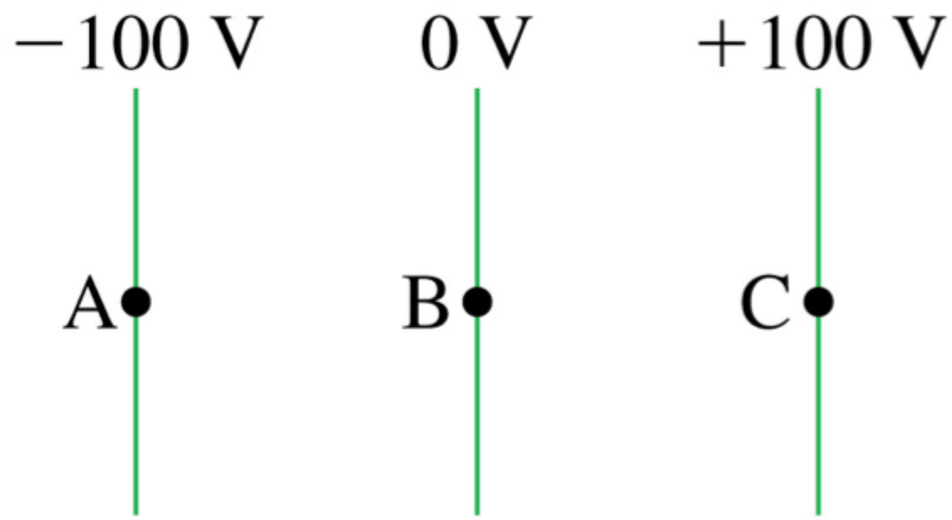
Which from the equation above means that the angle between ***E*** and ***ds*** is:

A) 0º
B) 90º
C) 180º
D) can't determine

The electric field runs downhill perpendicular to equipotential lines.

Charges and Fields PhET

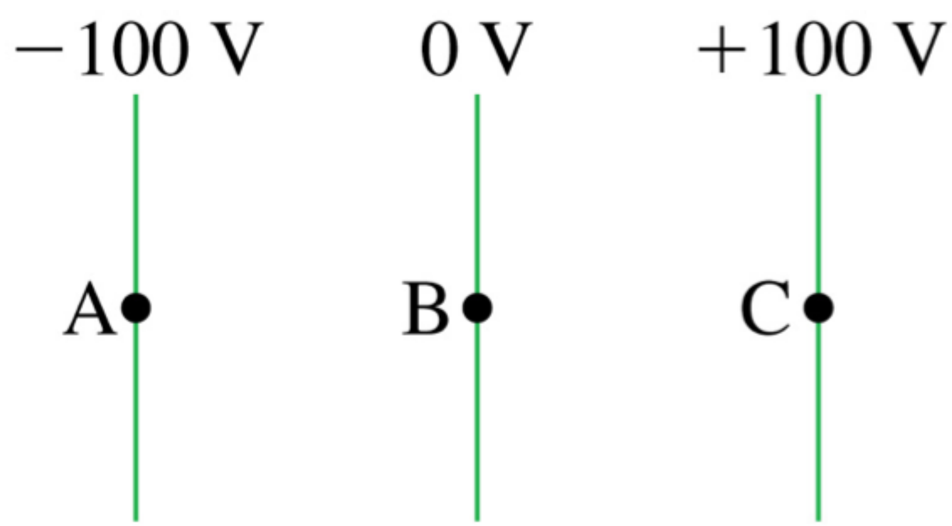
Clicker Question



An electron is released from rest at point B, where the potential is 0 V. Afterward, the electron

A) moves toward A with a steady speed.
B) moves toward A with an increasing speed.
C) moves toward C with a steady speed.
D) moves toward C with an increasing speed.
E) remains at rest at B.

Clicker Question



An electron is released from rest at point B, where the potential is 0 V. Afterward, the electron

A) moves toward A with a steady speed.
B) moves toward A with an increasing speed.
C) moves toward C with a steady speed.
D) moves toward C with an increasing speed.
E) remains at rest at B.

The **electric field** is in the **direction of decreasing potential**. Negative charges move *against electric fields* and *up potential gradient.s* That's the opposite of positively charged particles.

$$E_s = -\frac{dV}{ds}$$

Alessandro Volta

Discovered methane (1776), invented the battery (to prove Galvani wrong) and capacitance (early 1800s).

It's Alive!

Frankenstein 1931

Electromagnetism

Big Picture

1. **Understand the fundamentals** of electrostatics and magnetism.
2. **Use these fundamentals** to "build" circuit components. Learn to analyze these circuits
3. **Express these fundamentals** in the form of ***Maxwell's equations. Unification!*** See how Maxwell's equations predict something new.

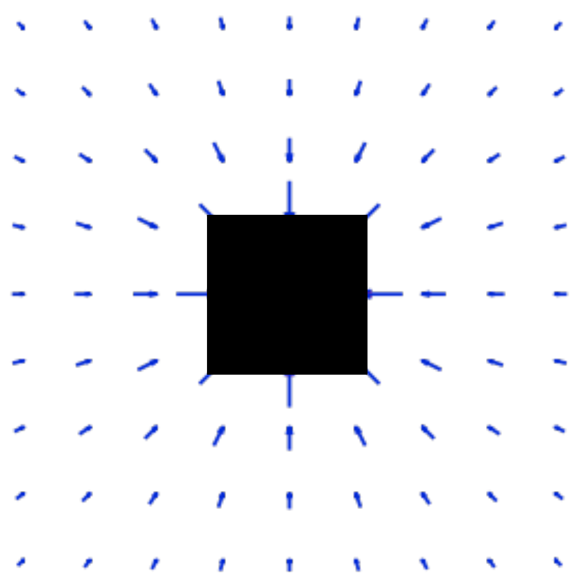
Gauss's Law


GD9674175N9
DEUTSCHE BUNDESBANK
Banknote
10
Deutsche Bundesbank
Frankfurt am Main
1.Oktober 1993
GD9674175N9
1777–1855 Carl Friedr. Gauß
10
ZEHN DEUTSCHE MARK

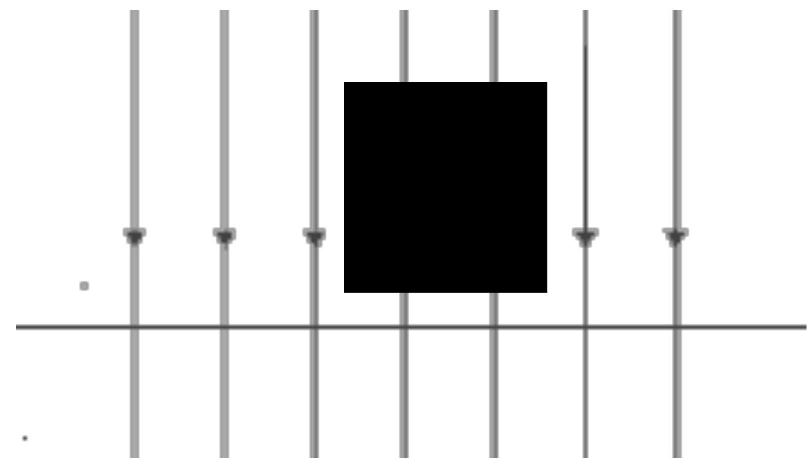
Flux and Charge

Gauss realized that the **charge configuration inside a surface** can be determined by the **field lines through the surface (flux)**.

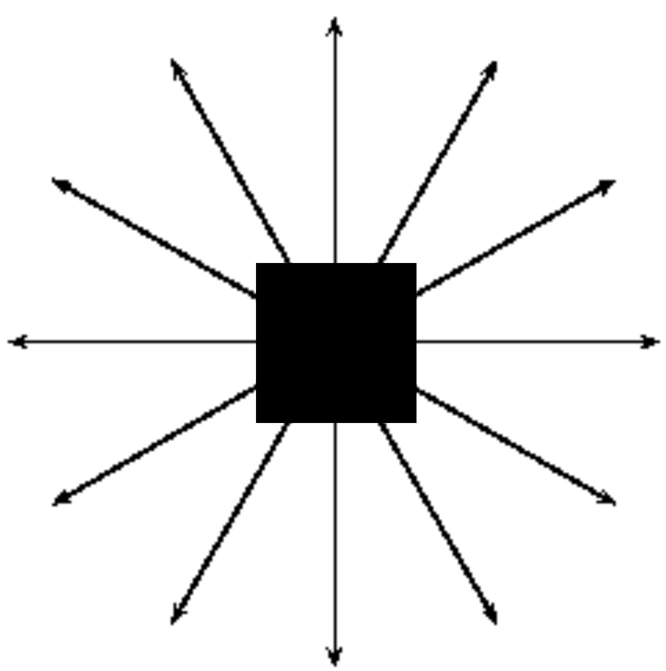
1.



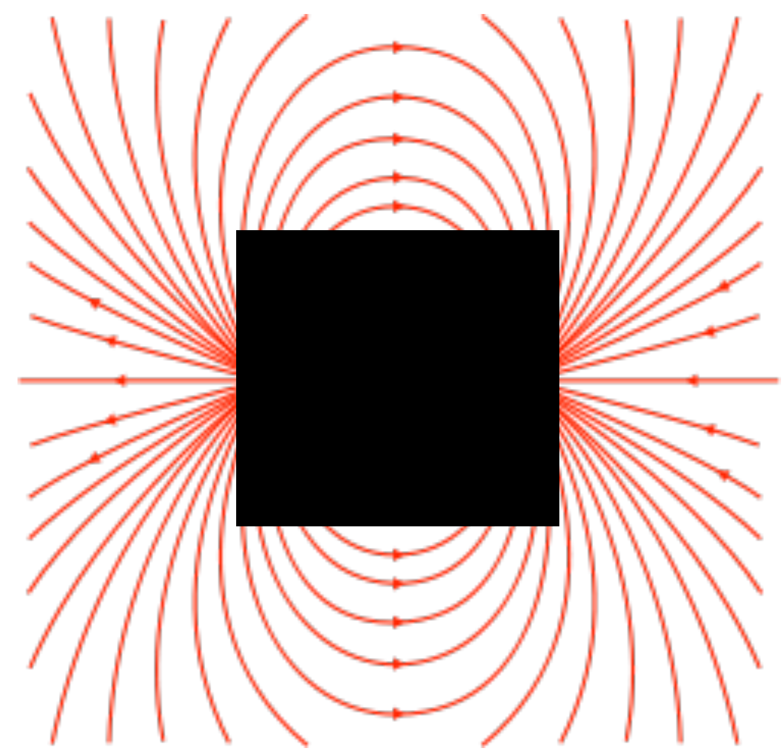
2.



3.



4.



Carl Frederich Gauss

- German (1777-1855), from a poor family
- *Disquisitiones Arithmeticae* (written 1798, 21!)
- Contributed to number theory, statistics, analysis, geometry, geophysics, astronomy, optics, electrostatics, ... (predicted orbit of Ceres)
- Hated teaching!
- Wanted a heptadecagon (17-gon) on his grave
- Large number of things named after him:

Gaussian integers, Gauss curvature, Gauss (unit), Gaussian distribution, Gaussian beam, Gaussian quadratures, Gaussian noise, ...

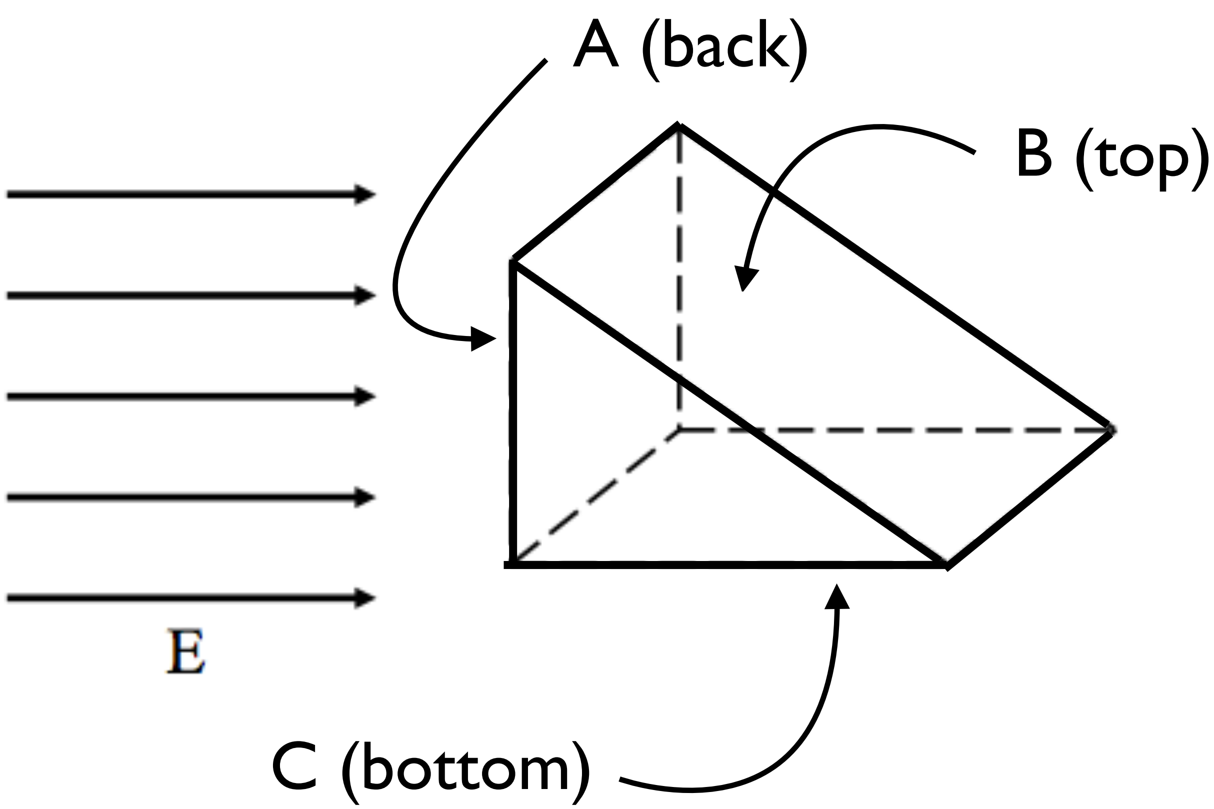
& Gauss's Law

Worksheet

Rain and Sprinklers
Q1

Clicker Question

A prism-shaped closed surface is in a constant, uniform electric field *E*.

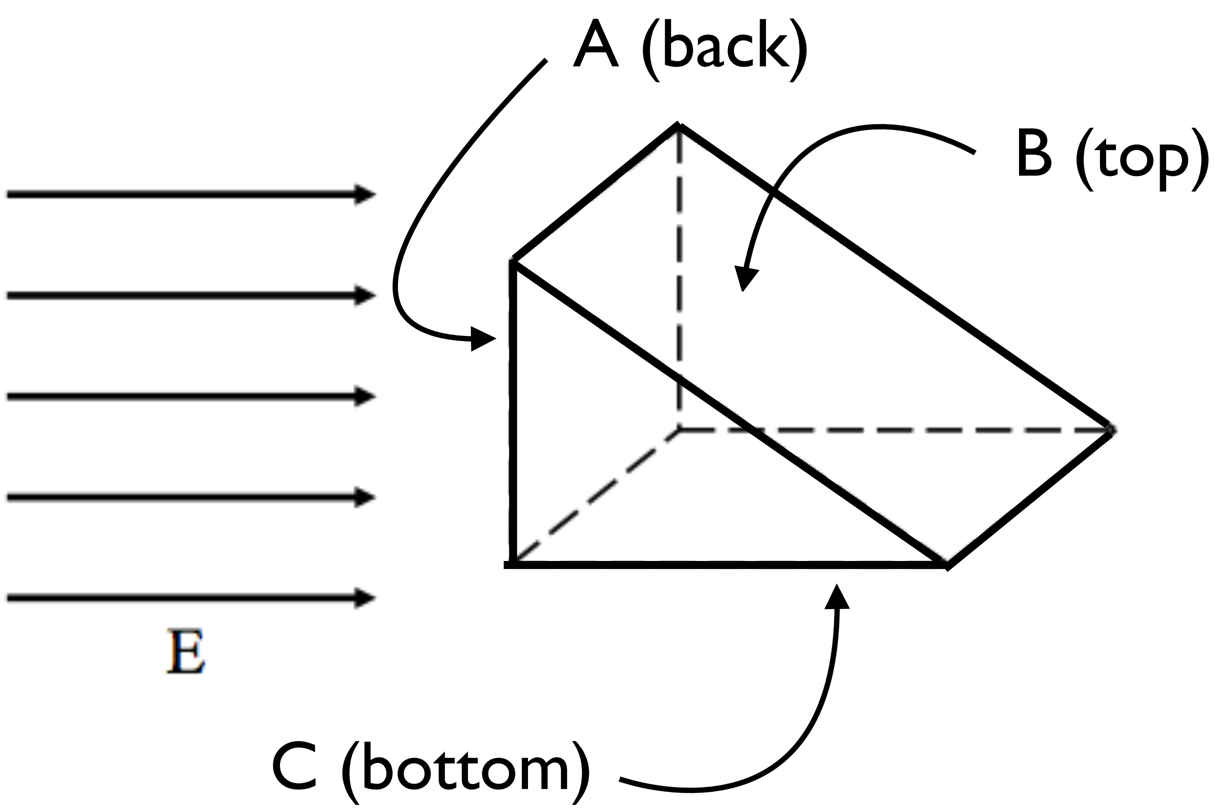


Which face has the largest magnitude of electric flux through it?

A) A
B) B
C) C
D) A = B

Clicker Question

A prism-shaped closed surface is in a constant, uniform electric field *E*.

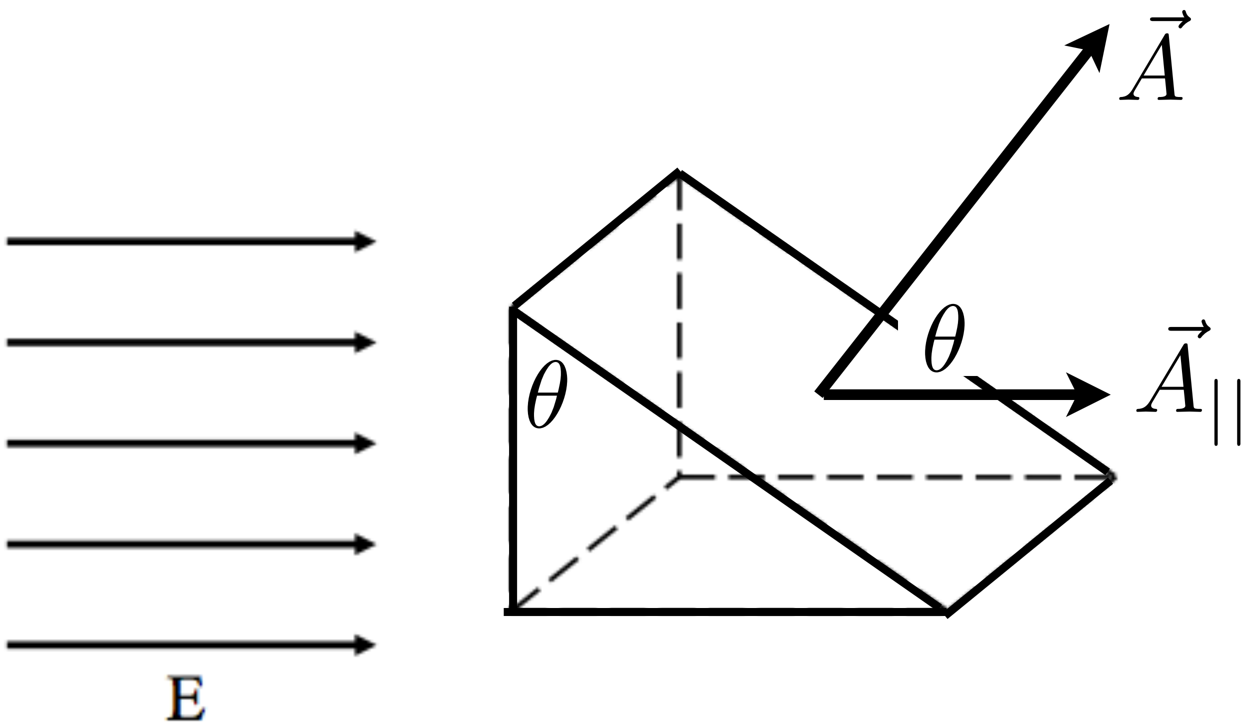


Which face has the largest magnitude of electric flux through it?

A) A
B) B
C) C
D) A = B

Even though B is bigger than A, B is tilted compared to A. So, A and B have the same number of field lines going through them.

Flux and Angle



Flux Transparencies

The tilted surface and surface || to the electric field have the **same flux** through them.

These areas are related by the angle between their area vectors.

$$A_{||} = A\cos(\theta)$$

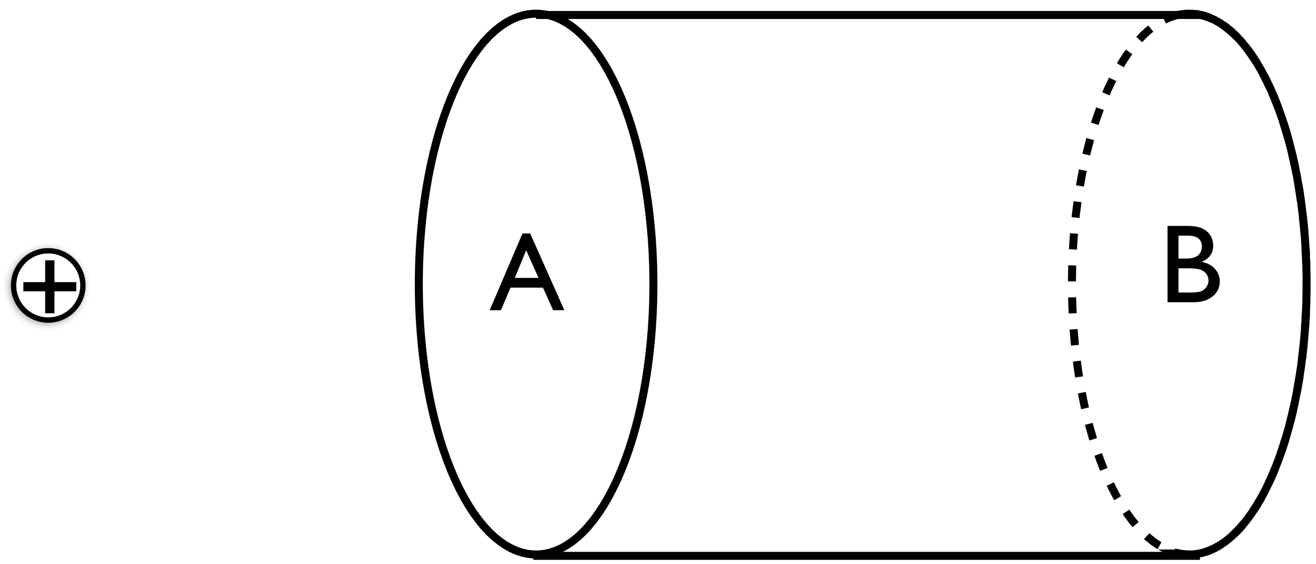
The **flux though the tilted area** can be written in terms of the flux through the perpendicular area:

$$\Phi = EA_{||} = EA\cos(\theta) = \vec{E} \cdot \vec{A}$$

Worksheet

Rain and Sprinklers
Q2 and Q3

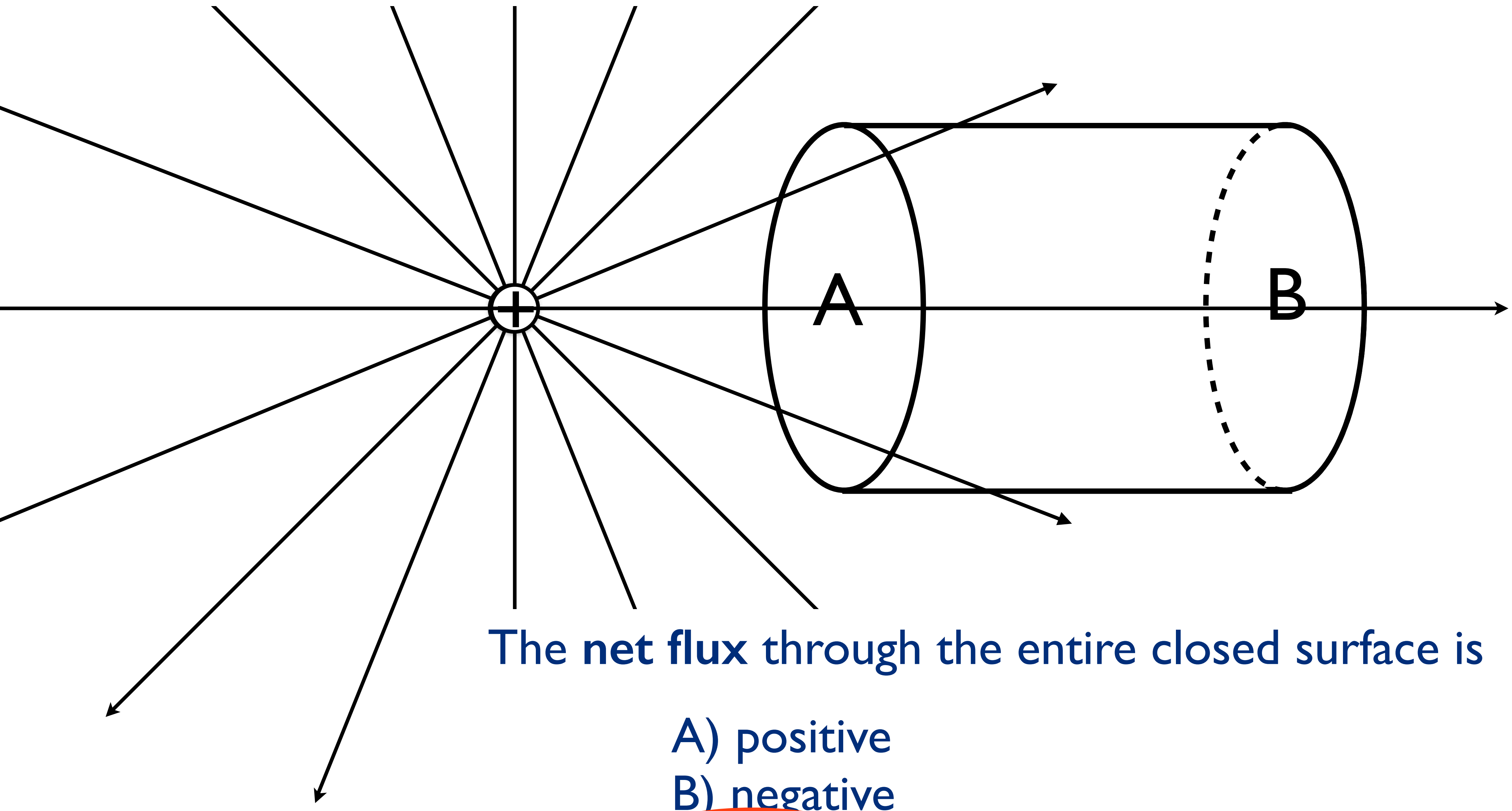
Clicker Question



The **net flux** through the entire closed surface is

A) positive
B) negative
C) zero

Clicker Question



The **net flux** through the entire closed surface is

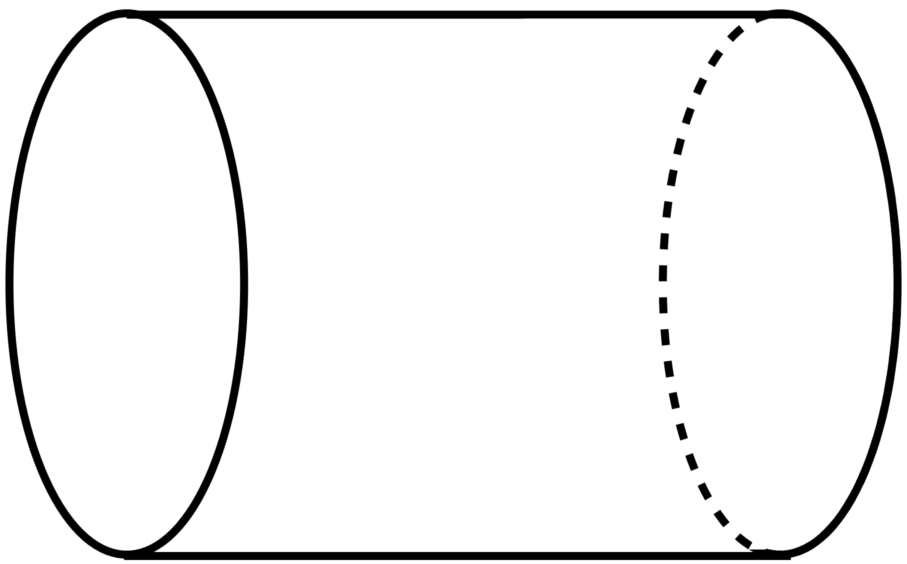
A) positive
B) negative
C) zero

Every field line **going in** to the surface also **goes out** of the surface.

Where must the charge be located for the flux to be positive?

Non-zero net flux

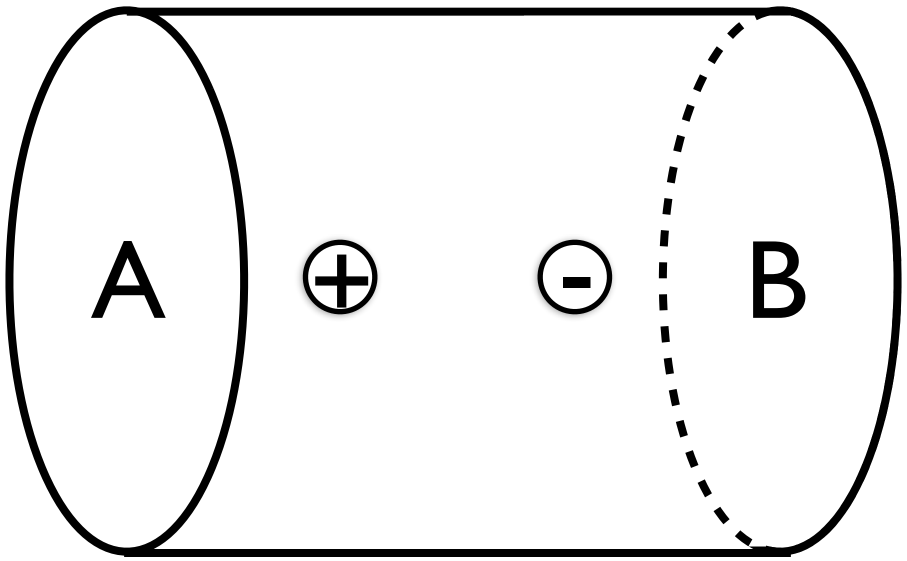
Where must the charge be located for the net flux to be non-zero?



Flux Transparencies

Clicker Question

A positive and negative charge rest inside the cylinder.
They're equal distance away from the ends of the cylinder.

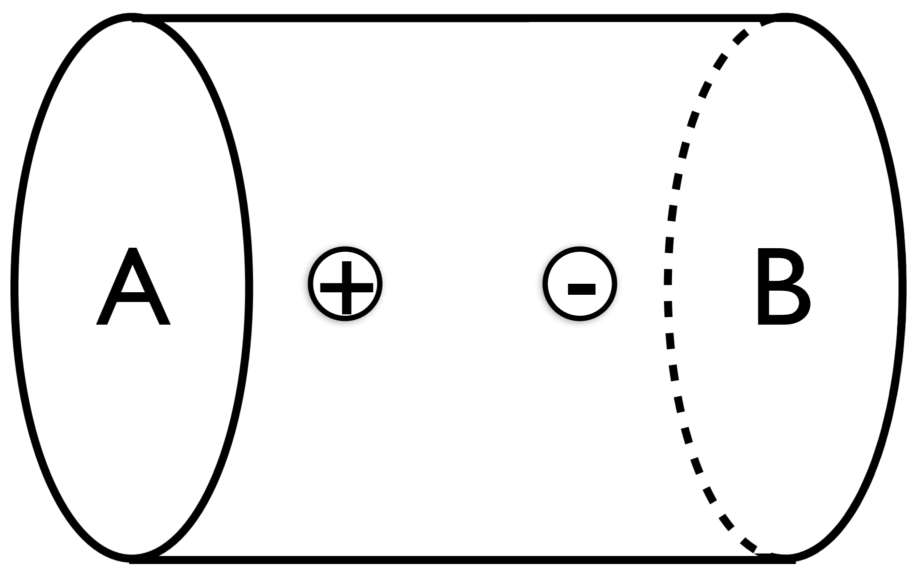


What is the net flux through the closed surface?

A) positive
B) negative
C) zero

Clicker Question

A positive and negative charge rest inside the cylinder. They're equal distance away from the ends of the cylinder.



What is the net flux through the closed surface?

A) positive
B) negative
C) zero

The flux through A is positive. The flux trough B is negative, but the same magnitude. The flux through the curved side is zero. So the net flux is zero.

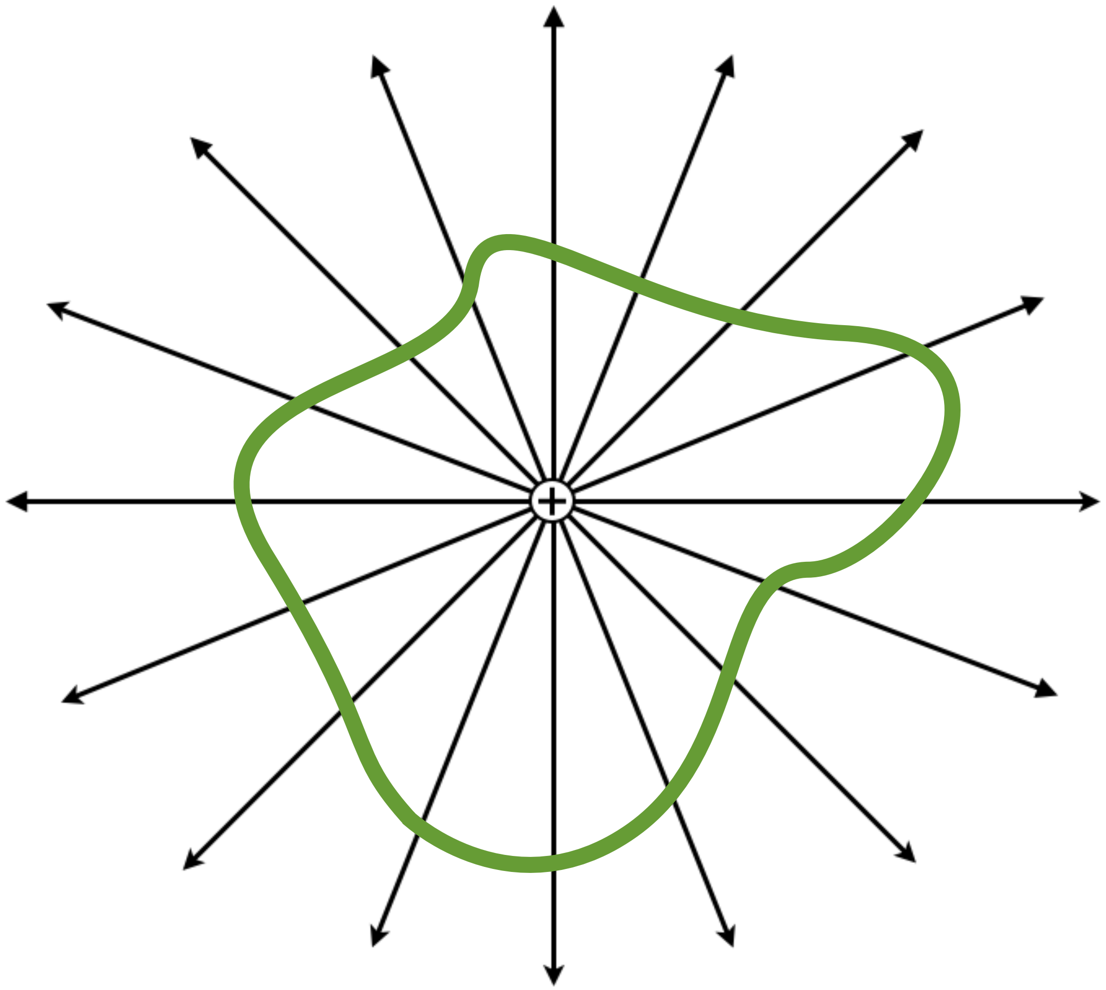
It's not enough that the charges are inside the surface, but they can't add to zero.

Worksheet

Rain and Sprinklers
Q4 - Q7

Gauss's Law

The net flux through a closed surface (Gaussian surface) is proportional to the **net charge inside the surface.** It does not depend on the shape of the surface.



Gauss's Law:

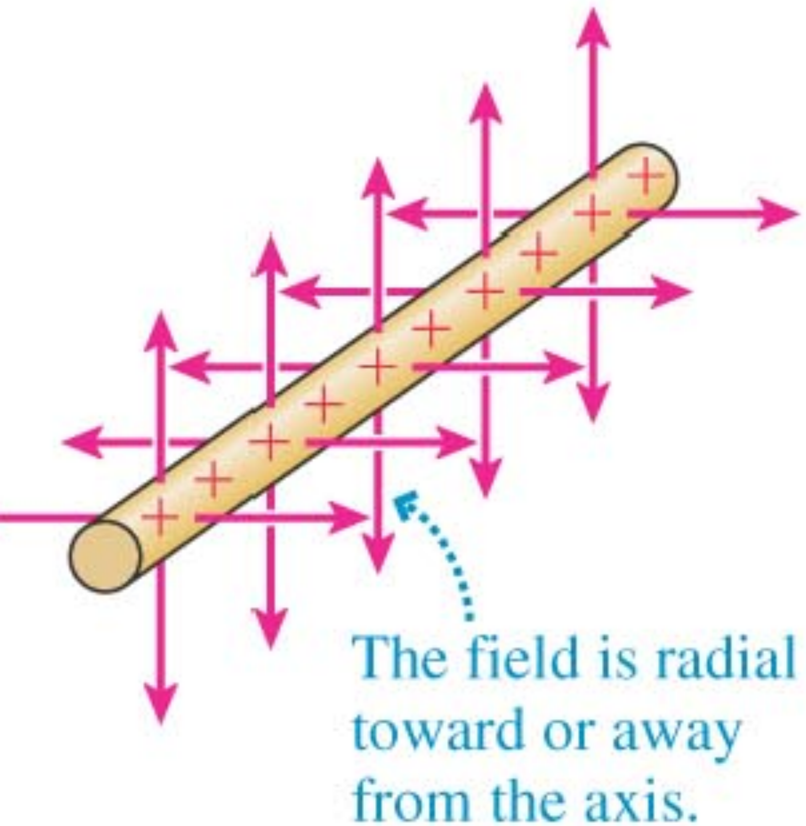
$$\Phi_{\text{net}} = \frac{Q_{\text{enclosed}}}{\epsilon_0}$$

Flux Transparencies

The flux **out** of the closed Gaussian surface is **positive**. Flux flowing **in to** the closed Gaussian surface is **negative**.

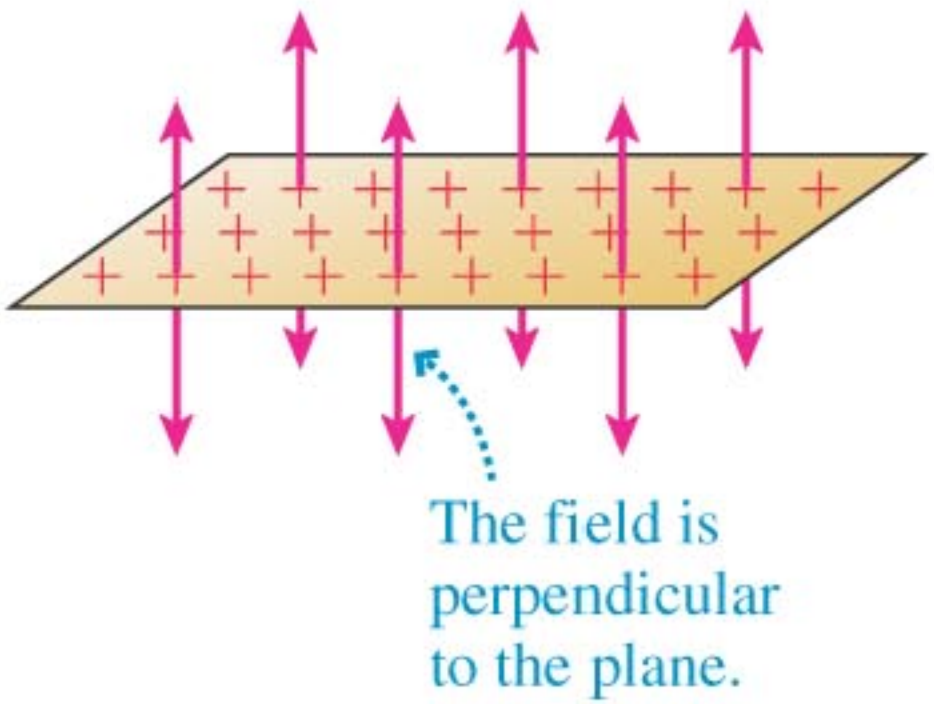
Three Basic Charge Distributions

infinite line of charge



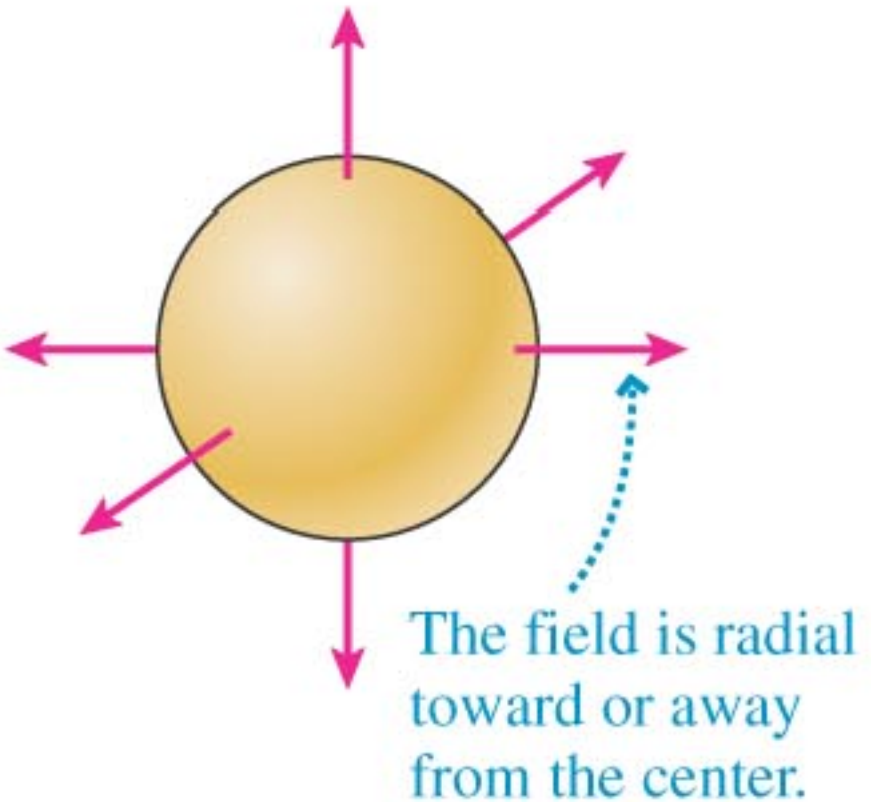
$$E_{\text{line}} = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{r}$$

infinite plane of charge



$$E_{\text{plane}} = \frac{\eta}{2\epsilon_0}$$

sphere of charge (same as point charge)



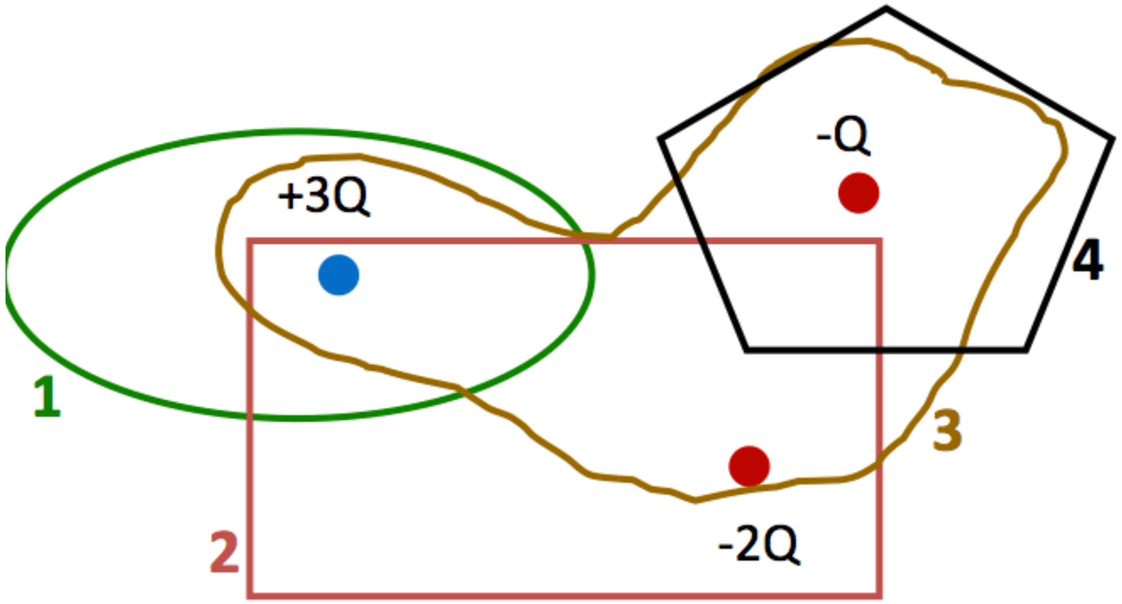
$$E_{\text{sphere}} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$$

$$\lambda = \frac{Q}{\ell} = \text{ linear charge density}$$

$$\eta = \frac{Q}{A} = \text{ surface charge density}$$

Clicker Question

Which surface has the largest **magnitude** of electric flux?



A) 1 = 2 = 3 = 4

B) 1 > 2 = 4 > 3

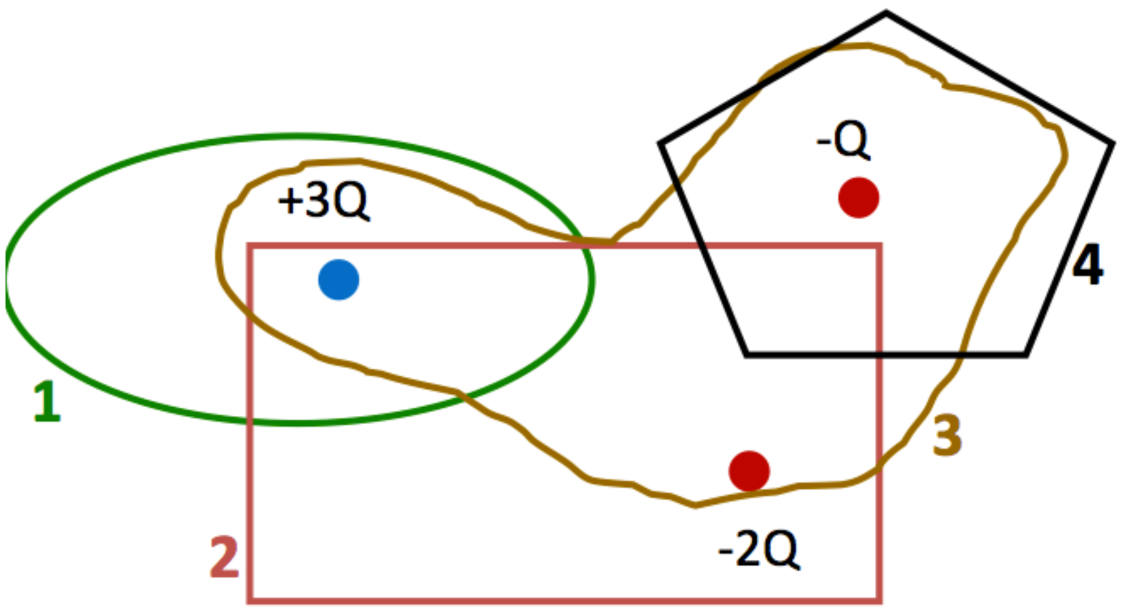
C) 3 > 2 > 1 > 4

D) 3 > 2 > 1 = 4

E) None of the above

Clicker Question

Which surface has the largest **magnitude** of electric flux?



A) 1 = 2 = 3 = 4

B) 1 > 2 = 4 > 3

C) 3 > 2 > 1 > 4

D) 3 > 2 > 1 = 4

E) None of the above

Region 1: Net charge is +3Q

Region 2: Net charge is +Q

Region 3: Net charge is 0

Region 4: Net charge is -Q

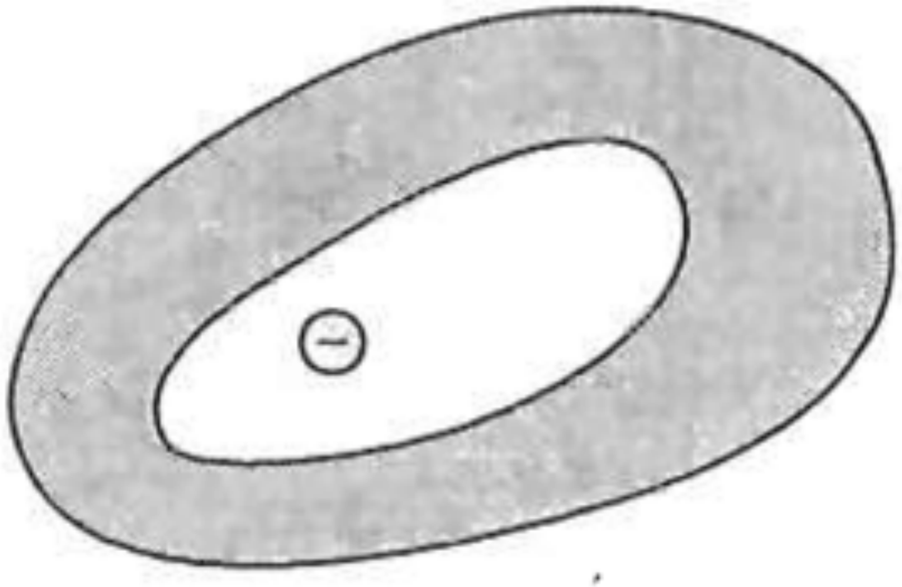
Here, we are worried about the magnitude of the flux, so the flux thru region 2 & 4 are the same. The flux thru region 2 is outward (+), while the flux thru region 4 is inward (-).

Again, electric flux is equal to "$Q_{enclosed}/\varepsilon_0$"

Clicker Question

A -10nC charge sits inside a conductor. Use Gauss's Law and the fact that E = 0 in a conductor to find the total charge on the **inside surface** of the conductor.

A) -20 nC
B) -10 nC
C) 0 nC
D) 10 nC
E) 20 nC



Clicker Question

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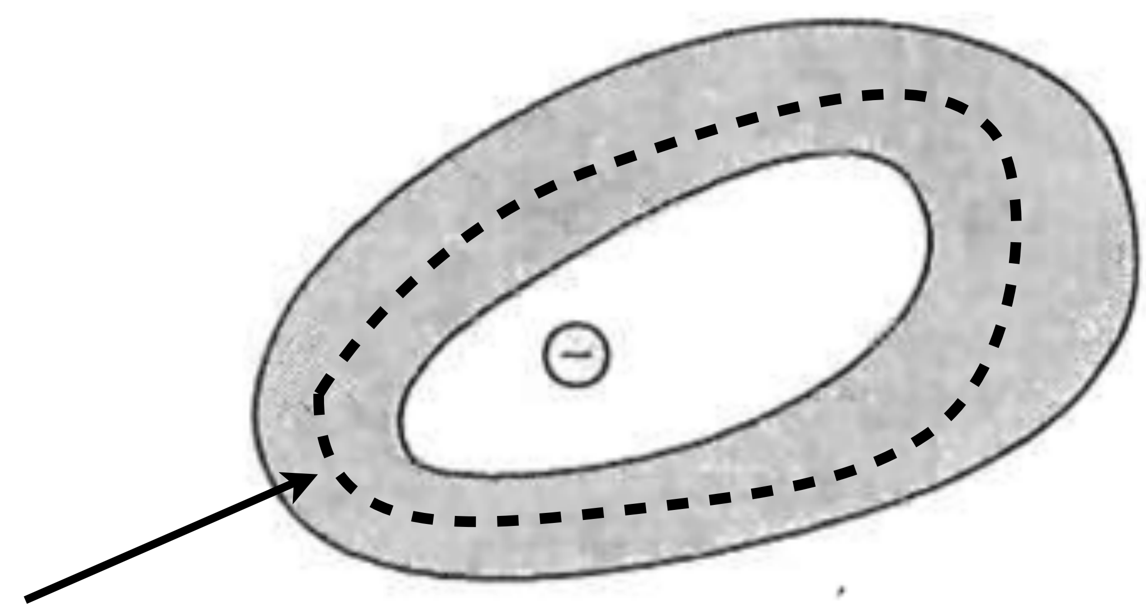
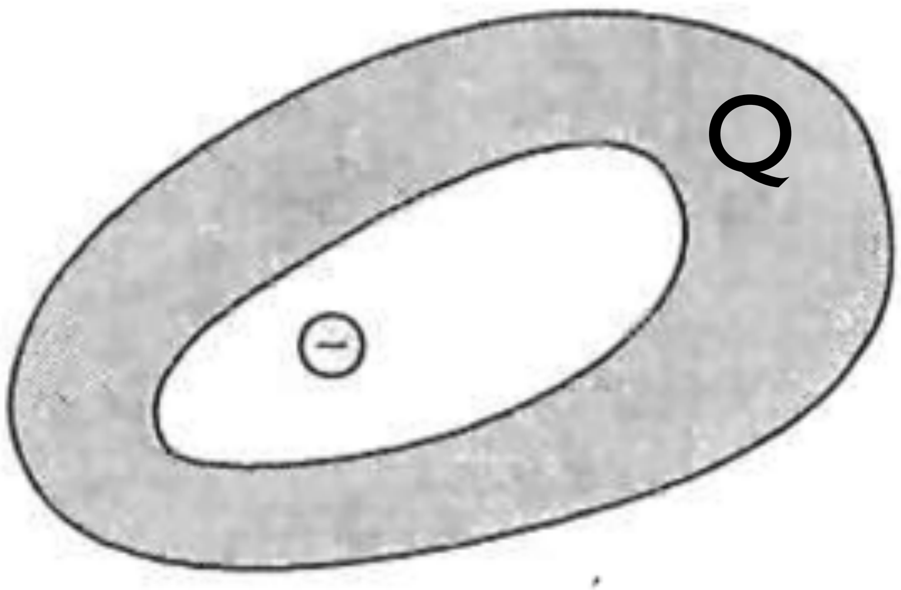


This surface has zero flux through it, which means that all the charge inside adds up to zero.

$$\Phi = \frac{Q_{\text{enclosed}}}{\epsilon_0} \quad \text{implies} \quad -10\ \text{nC} + q_{\text{surface}} = 0$$

Clicker Question

A -10nC charge sits inside a conductor.
The conductor itself has a charge of Q = 5 nC.

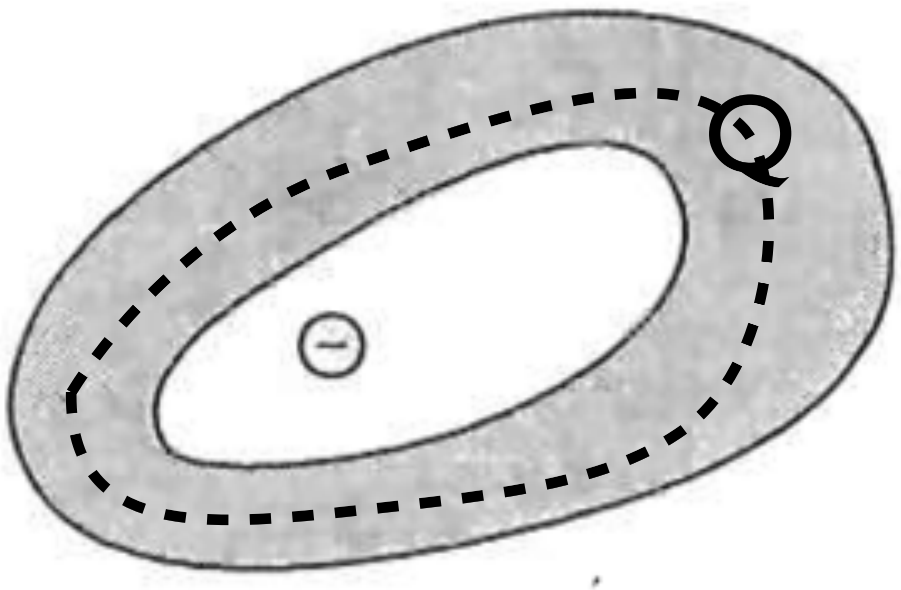


Find the total charge on the **outside surface** of the conductor.

A) -10 nC
B) -5 nC
C) 0 nC
D) 5 nC
E) 10 nC

Clicker Question

A -10nC charge sits inside a conductor.
The conductor itself has a charge of Q = 5 nC.



Find the total charge on the **outside surface** of the conductor.

A) -10 nC
B) -5 nC
C) 0 nC
D) 5 nC
E) 10 nC

E = 0 inside conductor, so the inside surface still has -10 nC.

The total conductor has a charge equal to the sum of the charges on each surface:

$$Q = q_{\text{outside}} + q_{\text{inside}}$$

which means that

$$q_{\text{outside}} = Q - q_{\text{inside}} = 5\ \text{nC} - 10\ \text{nC} = -5\ \text{nC}$$

Maxwell's Equations

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$\oint \mathbf{B} \cdot d\mathbf{A} = 0$$

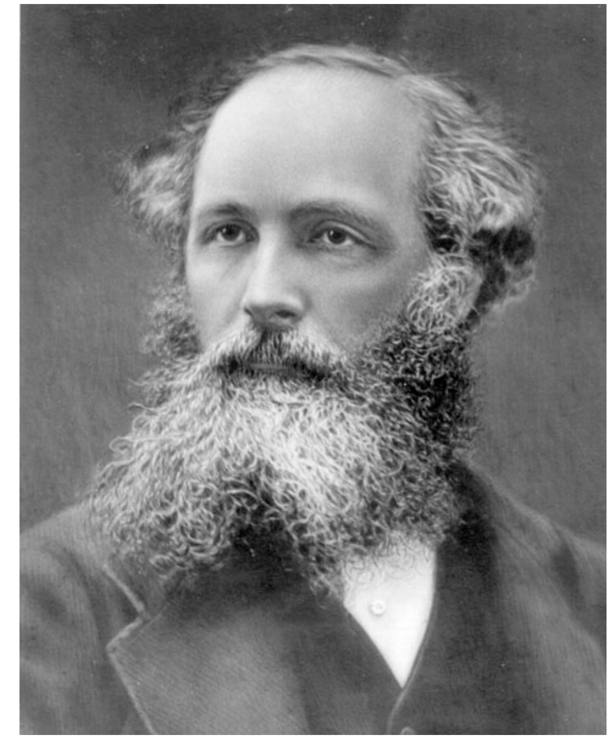
$$\oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_{\mathrm{B}}}{d\mathrm{t}}$$

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 \varepsilon_0 \frac{d\Phi_{\mathrm{E}}}{d\mathrm{t}} + \mu_0 i_{enc}$$

Motion of electric charge in fields: $\vec{F} = q\vec{E}$

James Clerk Maxwell

(1831-1879)

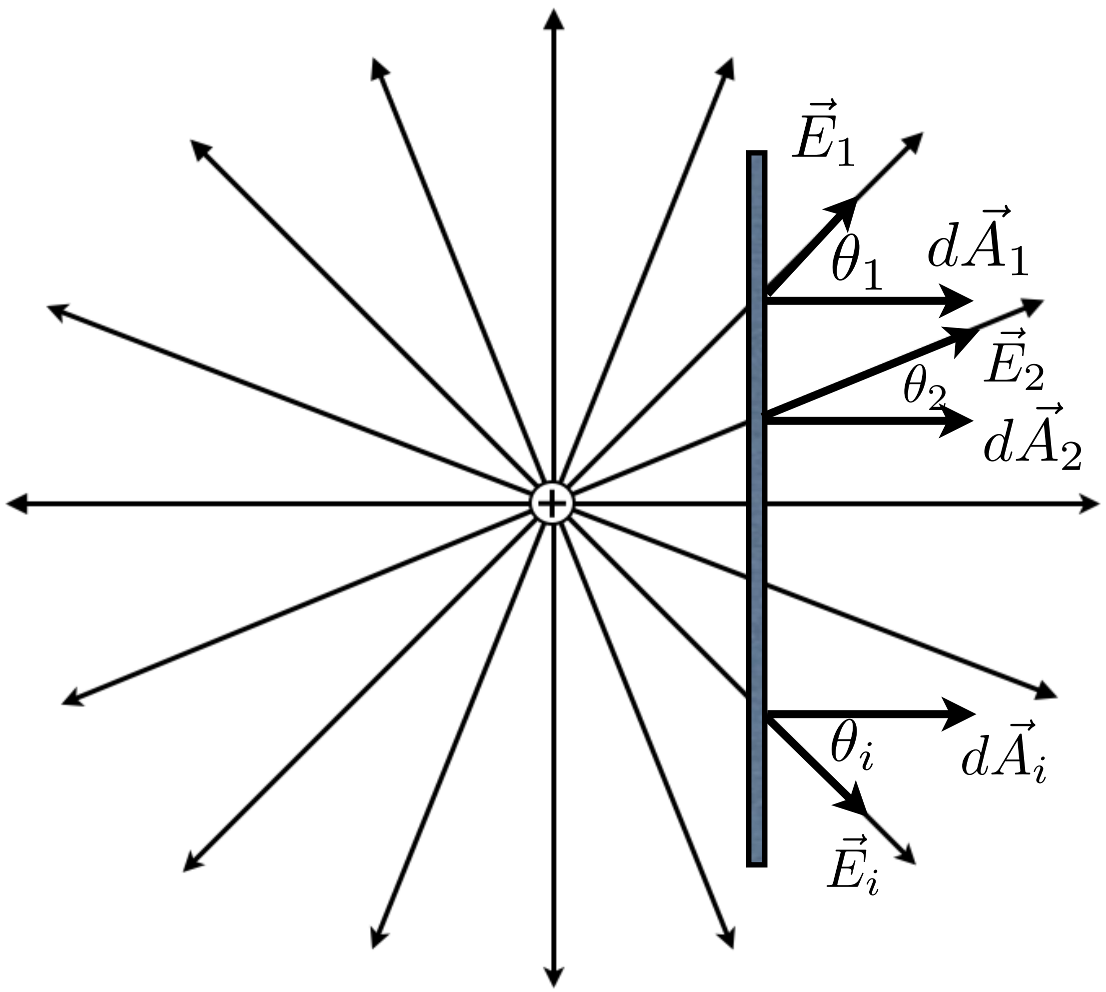


- First scientific paper at age 14 (on oval curves)
- “On Faraday’s lines of force” introduced E-M at age 24
- Professorship at age 25
- Worked out that for light R+G+B=white
- Calculated that Saturn’s rings had to be made of lumps
- Created first ever colour photograph
- Maxwell distribution and Maxwell’s Demon
- Effectively invented dimensional analysis
- Showed that light is EM radiation

Extra Stuff

Flux and Non-Uniform Fields

The **angle** between the surface and the field **changes throughout** the surface.



The flux is the sum of a bunch of little pieces.

$$\Phi_e = \sum_i E_i \; dA_i \; \cos\theta_i$$

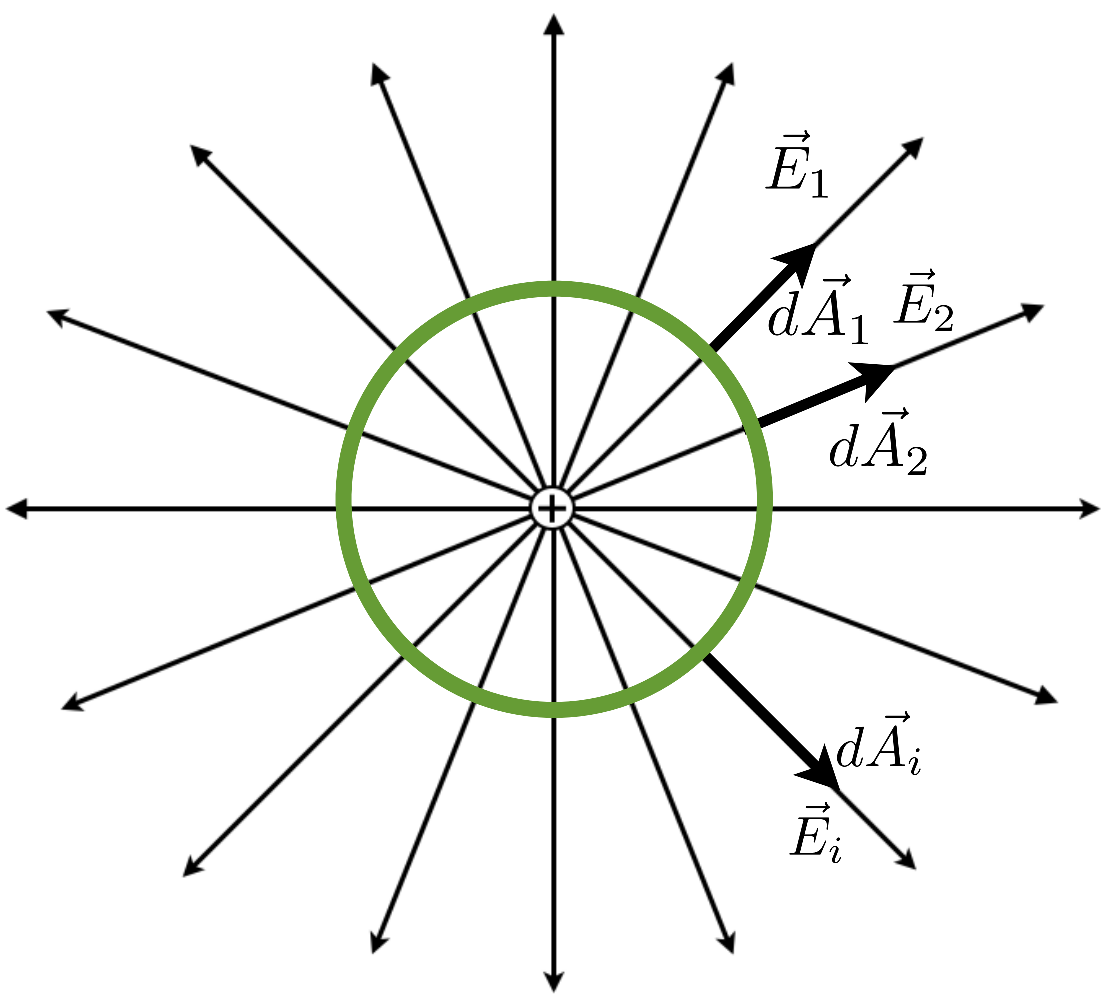
This sum is really an integral

$$\Phi_e = \int \vec{E} \cdot d\vec{A}$$

Flux Transparencies

Question: *For the electric field shown, what surface would be easy to evaluate the integral over?*

Flux and Non-Uniform Fields



For this spherical surface the electric field and area vector are always parallel and **E** has constant strength.

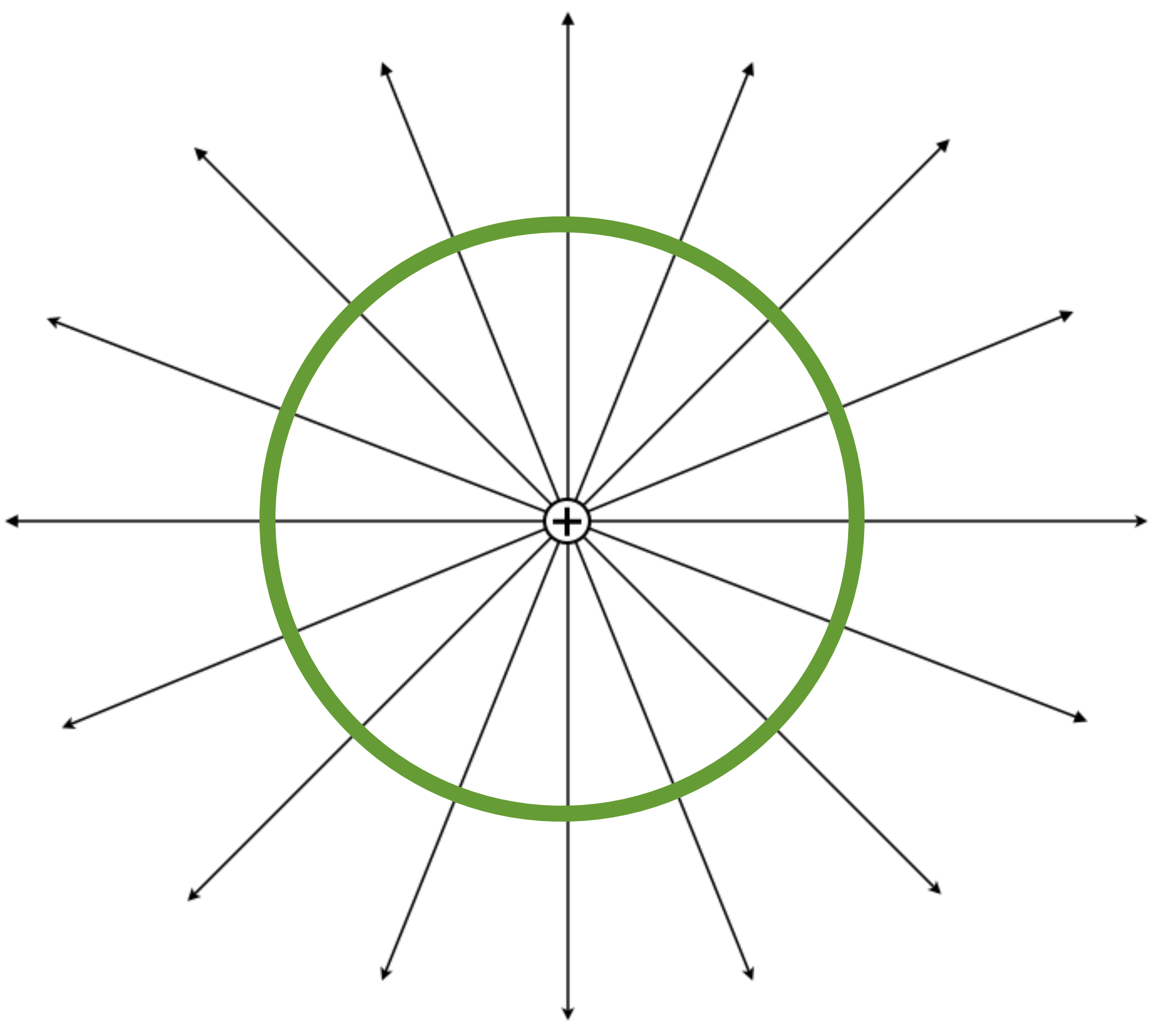
$$\begin{aligned}\Phi_e &= \sum_i E_i \; dA_i \; \cos\theta_i \\ &= \sum_i E_i dA_i \\ &= EA\end{aligned}$$

Just like the uniform field and the flat surface!

Certain electric fields have special surfaces that are easy to evaluate the flux through.

Using Gauss's Law

Let's find the electric field for a **point charge**.
We know the answer from Coulomb's Law, so this is a test of Gauss's Law.



E in same direction as **A** everywhere.
E same strength everywhere on **A** means

$$\Phi_e = EA_{\text{sphere}}$$

Gauss's law says that

$$EA_{\text{sphere}} = \frac{Q_{\text{encl}}}{\epsilon_0}$$

Solving for the electric field gives us

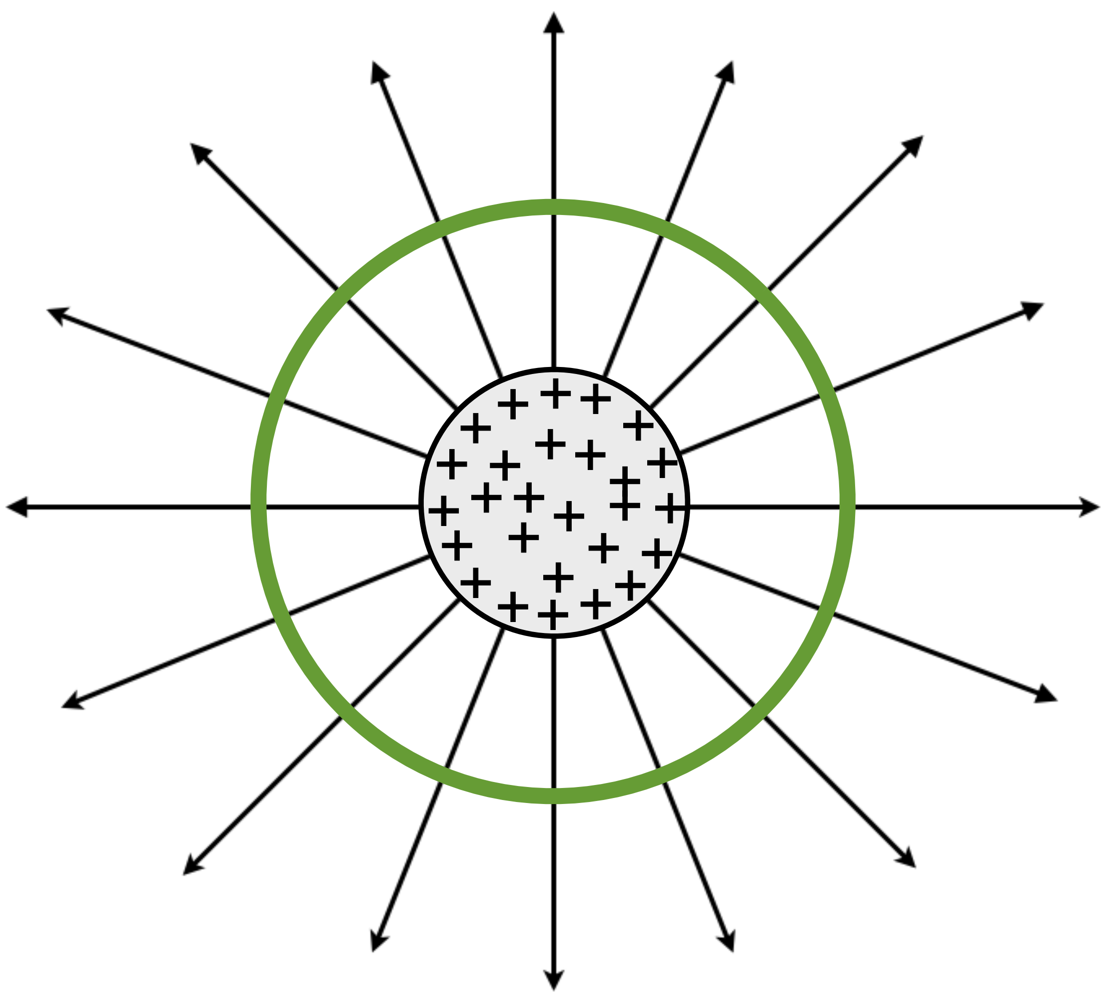
$$E = \frac{1}{A_{\text{sphere}}}\frac{Q}{\epsilon_0} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$$

Gauss's Law contains Coulomb's Law!

Using Gauss's Law

Let's find the electric field for a **uniformly charged sphere** with radius R and total charge Q.

What would you have to do to evaluate this using Coulomb's Law?



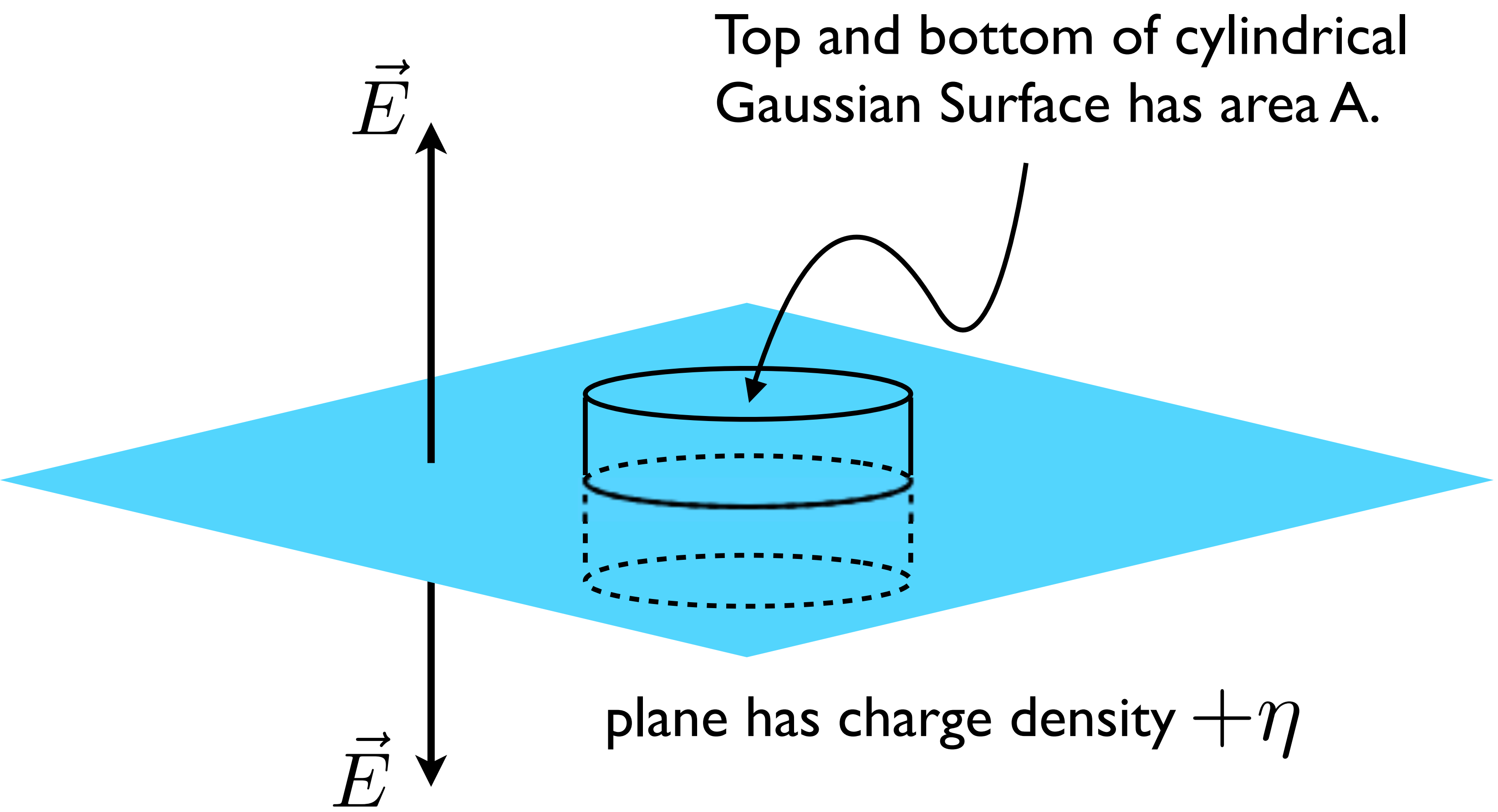
Two Cases:

First do R > 0.

Then do R < 0.

Infinite Plane of Charge

Earlier, we surmised that the field must point away from plane everywhere.



The cylinder has three surfaces. What is the net electric flux through each surface?

How much charge inside the cylinder?

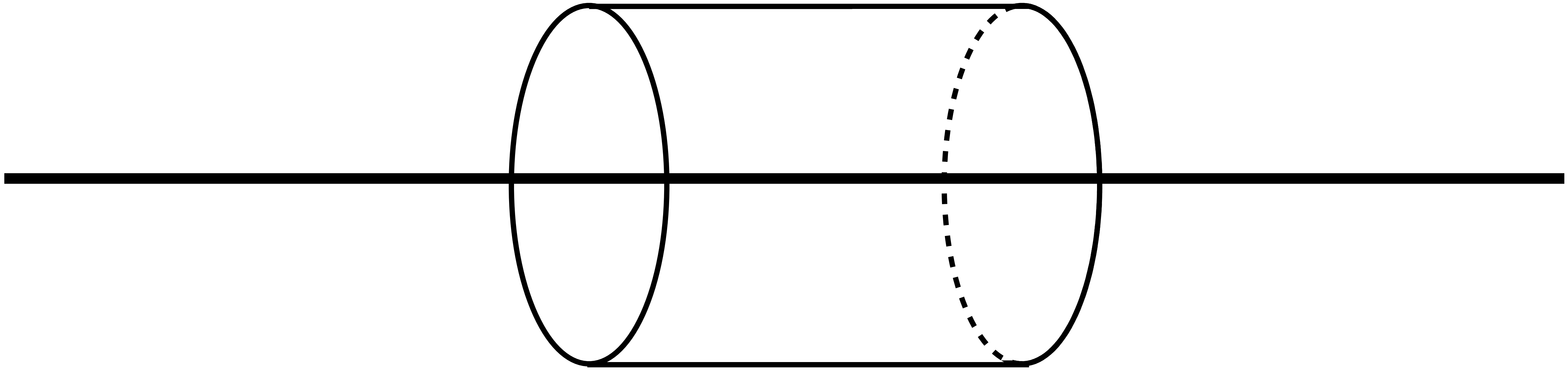
Answer those questions and we can use

$$\Phi_e = E_1 \cdot dA_1 + E_2 \cdot dA_2 + \ldots = \frac{Q_{\mathrm{encl}}}{\epsilon_0}$$

to find the electric field of the plane.

Charged Wire

Consider and infinitely charged wire with linear charge density λ.



This is the Gaussian Surface. What's the shape of the field?