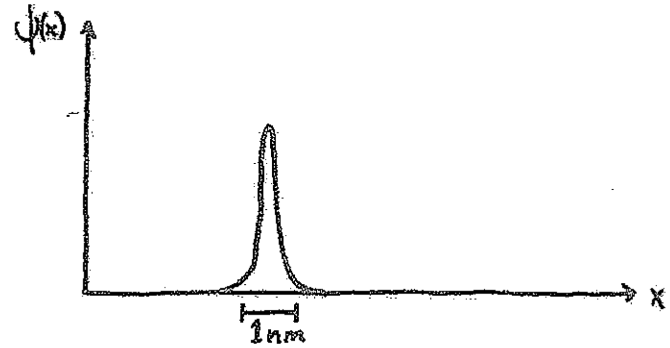
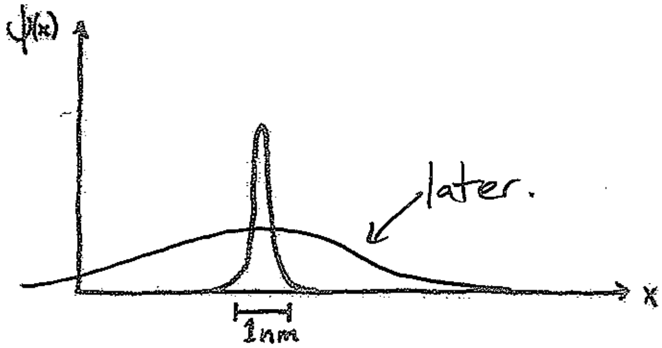


Question 26:

Shortly after a measurement of position, an electron in an infinite wire is in a state described by the wavefunction ψ(x) shown.

a) Describe what happens to the wavefunction at later times, give a qualitative explanation for your prediction. (2 points)

b) ~~Describe *quantitatively* how some feature of the wavefunction will change with time (order of magnitude estimates are fine). (2 points)~~
Briefly describe a method that could be used to determine the wavefunction at a later time (precisely)



Question 26:

Shortly after a measurement of position, an electron in an infinite wire is in a state described by the wavefunction ψ(x) shown.

a) Describe what happens to the wavefunction at later times; give a qualitative explanation for your prediction. (2 points)

Narrow wavefunction ⇒ superposition of pure waves with a big range of wavelengths

⇒ large uncertainty in velocity/momentum

∴ ~~wave peak~~ wave function spreads out quickly

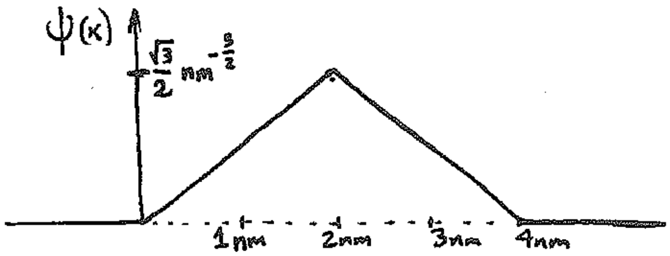
b) ~~Describe~~ ~~*quantitatively*~~ ~~how some feature of the wavefunction will change with time (order of magnitude estimates are fine).~~ ~~(2 points)~~

To find evolution:

- write $\phi(x)$ as sum of pure waves
- evolve pure waves individually (each moves at speed $\propto \frac{1}{\lambda}$ s.t frequency is $\frac{E}{h} = \frac{\frac{1}{2}mv^2}{h} = \frac{p^2}{2mh} = \frac{h}{2m\lambda^2}$)
- add up again to get ϕ at later time

OR

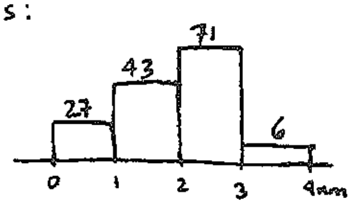
Use Schrödinger equation with initial wavefn. $\phi(x)$.

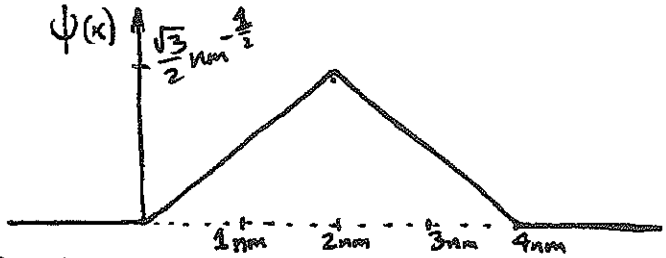


11 a) The wavefunction for an electron in a short wire is shown above. A team of physicists prepares 16,000 electrons in this state and then measures the position of each of the electrons. They make a histogram showing the number of electrons that they find in each of the four intervals [0nm,1nm], [1nm,2nm], [2nm,3nm], and [3nm,4nm]. Draw what you expect their histogram to look like and indicate the expected number in each bin. Show your calculations and explain your work.

(5 points)

your answer should look something like this:





(11) a) The wavefunction for an electron in a short wire is shown above. A team of physicists prepares 16,000 electrons in this state and then measures the position of each of the electrons. They make a histogram showing the number of electrons that they find in each of the four intervals [0nm,1nm], [1nm,2nm], [2nm,3nm], and [3nm,4nm]. Draw what you expect their histogram to look like and indicate the expected number in each bin. Show your calculations and explain your work.
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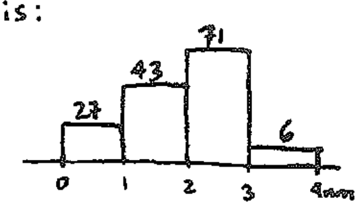


The probability density for measuring an electron at x is $|\psi(x)|^2$. For the four intervals, we have:

$$P_{[0,1]} = \int_{0nm}^{1nm} |\psi(x)|^2$$

$$P_{[1,2]} = \int_{1nm}^{2nm} |\psi(x)|^2$$

And since the wavefunction is symmetrical $P_{[2,3]} = P_{[1,2]}$, $P_{[3,4]} = P_{[0,1]}$.

~~The total probability must be 1, and we can use this to normalize the wavefunction.~~ In the region $0 \leq x \leq 2nm$, we have $\psi(x) = A \cdot x$ for some constant A, ~~be~~ where $A = \frac{\sqrt{3}}{2} nm^{-3/2} / 2nm = \frac{\sqrt{3}}{4} nm^{-3/2}$

$$P_{[0,1]} = \int_{0nm}^{1nm} A^2 x^2 = \frac{1}{3} A^2 x^3 \Big|_{0nm}^{1nm} = \frac{1}{3} A^2 (nm)^3 = \frac{1}{16}$$

$$P_{[1,2]} = \int_{1nm}^{2nm} A^2 x^2 = \frac{1}{3} A^2 x^3 \Big|_{1nm}^{2nm} = \frac{7}{3} A^2 (nm)^3 = \frac{7}{16}$$

Also $P_{[2,3]} = \frac{7}{3} A^2 (nm)^3 = \frac{7}{16}$, $P_{[3,4]} = \frac{1}{3} A^2 (nm)^3 = \frac{1}{16}$

More room for #1 a)

~~For the total probability to be 1, we need:~~

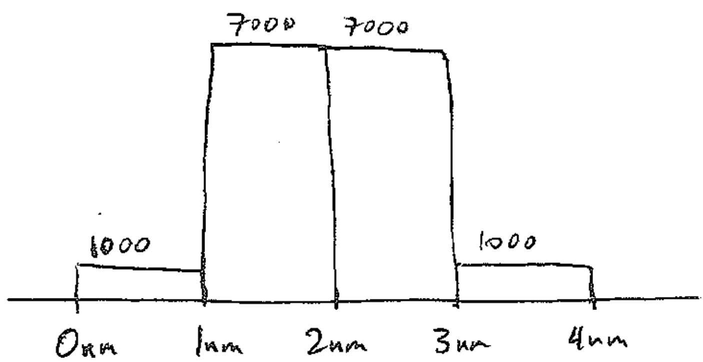
$$\cancel{P_{[0,1]} + P_{[1,2]} + P_{[2,3]} + P_{[3,4]} = 1}$$

$$\cancel{\Rightarrow \frac{1}{3}A^2(\text{nm})^3 + \frac{7}{3}A^2(\text{nm})^3 + \frac{7}{3}A^2(\text{nm})^3 + \frac{1}{3}A^2(\text{nm})^3 = 1}$$

$$\cancel{\Rightarrow A^2 = \frac{3}{16}(\text{nm})^{-3}}$$

$$\cancel{\text{So: } P_{[0,1]} = P_{[3,4]} = \frac{1}{16} \text{ and } P_{[2,3]} = P_{[1,2]} = \frac{7}{16}}$$

If we ~~[illegible]~~ measure 16,000 electrons in this same initial state, we therefore expect around 1000 to be measured in the first and fourth regions (each) and 7000 in each of the second and third regions. Thus, the histogram should look like:



Problem 10

A certain metal is found to emit electrons with maximum kinetic energy 5eV when it is illuminated with 200nm light. What is the maximum wavelength light that will cause electrons to be emitted? (3 points)

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A certain metal is found to emit electrons with maximum kinetic energy 5eV when it is illuminated with 200nm light. What is the maximum wavelength light that will cause electrons to be emitted? (3 points)

Suppose it takes energy W to get the electron out. We have $E^k_{max} = hf - W$ and $f = \frac{c}{\lambda}$, So:

$$E^k_{max} = \frac{hc}{\lambda} - W$$

Then: $$5eV = \frac{hc}{200nm} - W$$

$$\Rightarrow W = \frac{hc}{200nm} - 5eV$$

$$= \frac{1.24 \times 10^3 eV \cdot nm}{200nm} - 5eV = 1.20eV$$

Now the largest wavelength for which electrons are emitted occurs when the photon energy is just equal to W~~the work function~~. So:

$$E_{min} = W$$

$$\frac{hc}{\lambda_{max}} = W$$

$$\Rightarrow \lambda_{max} = \frac{hc}{W}$$

$$= \frac{1.24 \times 10^3 eV \cdot nm}{1.2eV} \quad \boxed{\approx 1.03 \times 10^3 nm.}$$

Long Answer Questions: explain your work

Problem 21

Please answer the following as concisely as possible (a couple sentences is sufficient).

a) What is wrong with the classical picture of a Hydrogen atom as an electron orbiting a proton? (2 points)

b) How does quantum mechanics resolve the problem? (2 points)

Long Answer Questions: explain your work

Problem 21

Please answer the following as concisely as possible (a couple sentences is sufficient).

a) What is wrong with the classical picture of a Hydrogen atom as an electron orbiting a proton? (2 points)

orbiting electron accelerating

accelerating charges radiate $\therefore$ lose energy & spiral into nucleus.

b) How does quantum mechanics resolve the problem? (2 points)

- there is a minimum energy state for electron.
- in this state no more loss of energy possible.

→ wavefn. is stationary for this state $\therefore$ no sense in which charge is accelerating.

Problem 23

Explain why the double-slit experiment for electrons provides evidence that general electron states do not have definite positions and can exist in quantum superpositions. Answer as concisely as possible. (4 points)

Problem 10: Describe the photoelectric effect and explain why it provides evidence for the photon picture of light.

Problem 23

Explain why the double slit experiment for electrons provides evidence that general electron states do not have definite positions and can exist in quantum superpositions. Answer as concisely as possible. (4 points)

In the double slit experiment, the pattern of hits on the screen matches the diffraction pattern for light ✓ even if we send in one electron at a time. ✓

Since the pattern for light is explained by interference of light from the two slits, the fact that we get the same result using individual electrons suggests that the single electrons ~~do~~ not just pass through one slit or the other, but "see" both slits. More evidence for this comes from the observation that the pattern changes if we cover up one slit each time and alternate which one we cover up.

For an electron to see or pass through both slits at once, it cannot exist at a definite position, so the experiment supports the idea that electrons can exist in quantum superpositions of more than one position.

(10) Describe the photoelectric effect and explain why it provides evidence for the photon picture of light. Answer as concisely as possible.

(4 points)

The photoelectric effect describes the ejection of electrons from a metal (or other material) when light with a high enough frequency is used to illuminate it. The key observation is that if the frequency is too small, no electrons are ejected regardless of how high the intensity is. On the other hand, for frequencies high enough to eject electrons, electrons are ejected for any non-zero intensity.

These observations cannot be explained by the classical picture of light, since in that picture, the flow of energy into the metal is continuous and is exactly the same for any frequency as long as the intensity is the same.

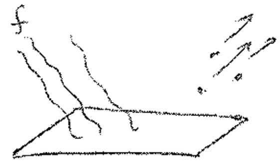
On the other hand, the photon picture says that light of frequency f comes in discrete lumps of energy $E = hf$. If we assume that there is some minimum energy W required to eject an electron from the metal and that electrons are ejected by absorbing the energy from a single photon, then we must have $hf > W$ to eject an electron. This explains the minimum frequency that is observed in the experiment. Also, the photon picture explains why the intensity doesn't matter for whether or not ~~photons~~ electrons are ejected, since it is only the individual photons that are involved.

(A shorter answer than this could have received full credit)

Problem 8

A beam of light with frequency $7.5\times 10^{14}\,s^{-1}$ is incident on a metal, and photoelectrons are observed with maximum velocity 5×10^{5} m/s. The same sample of metal is illuminated with a new light source, but this time electrons are observed with maximum velocity 10^{6} m/s. What is the frequency of the new light source? (3 points)

Problem 8



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Suppose it takes energy W to get an electron out.

Have: $E_K^{max} = hf - W$

$$\frac{1}{2}mv_1^2 = hf_1 - W \qquad v_1 = 5\times10^5\,m/s \quad f_1 = 7.5\times10^{14}\,s^{-1}$$

$$\Rightarrow W = hf_1 - \frac{1}{2}mv_1^2$$

$$\frac{1}{2}mv_2^2 = hf_2 - W \qquad v_2 = 10^6\,m/s$$

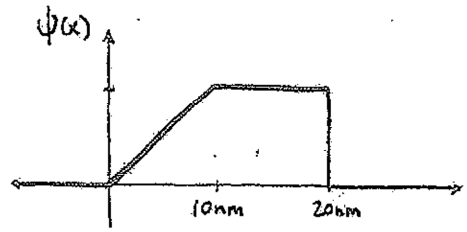
$$\Rightarrow f_2 = \frac{1}{h}\left(\frac{1}{2}mv_2^2 + W\right)$$

$$= \frac{1}{h}\left(\frac{1}{2}mv_2^2 + hf_1 - \frac{1}{2}mv_1^2\right)$$

$$= f_1 + \frac{m}{2h}\left(v_2^2 - v_1^2\right)$$

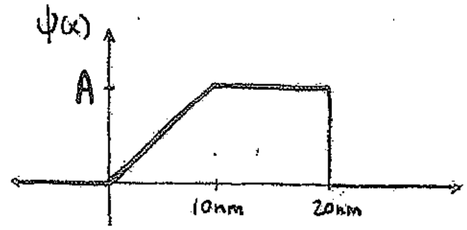
$$= 7.5\times10^{14}\,s^{-1} + \frac{9.1\times10^{-31}\,kg}{2\times6.6\times10^{-34}\,J\cdot s}\left(\left(10^6\,m/s\right)^2 - \left(5\cdot10^5\,m/s\right)^2\right)$$

$$\Rightarrow \boxed{f_2 \approx 1.3\times10^{15}\,s^{-1}}$$



Question 24: (6 points)

a) An electron in a thin wire is prepared in a state described by the wavefunction shown and its position is measured. ~~If this experiment is performed twice,~~ Calculate the probability that the electron will be found in the range between 0nm and 10nm ~~at least once~~. (4 points)



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We are not given A, so we will have to determine it.

The function shown is

line with slope $\frac{A}{10nm}$

$$\psi(x) = \begin{cases} \frac{Ax}{10nm} & 0 \leq x \leq 10nm \\ A & 10nm \leq x \leq 20nm. \end{cases}$$

The probability density is

$$|\Psi(x)|^2 = \begin{cases} \frac{A^2}{100nm^2} \cdot x^2 & 0 \leq x \leq 10nm \\ A^2 & 10nm \leq x \leq 20nm \end{cases}$$

The probability of finding the particle between 0 and 10nm is:

$$\int_0^{10nm} \Psi^2(x)\, dx = \frac{A^2}{100nm^2} \int_0^{10nm} x^2 dx = \frac{A^2}{100nm^2} \frac{(10nm)^3}{3} = A^2 \cdot \frac{10}{3} nm$$

Prob. of finding particle between 10nm and 20nm is:

$$\int_{10nm}^{20nm} \Psi^2(x)\, dx = \int_{10nm}^{20nm} A^2 dx = A^2 \cdot 10\,nm$$

Must have: $A^2 \cdot \frac{10}{3} nm + A^2 \cdot 10\,nm = 1$ so: $A^2 = \frac{3}{40}\, nm^{-1}$

Thus: $P_{[0,10nm]} = \frac{1}{4}$