SOLUTIONS TO

Physics Midterm 2

November 14, 2013

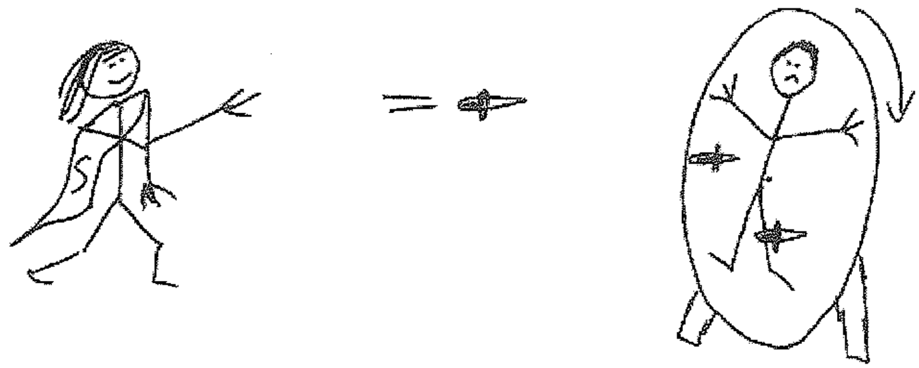
Name:
Student Number:
Bamfield Number:

Questions 1-9: Multiple Choice: 2 point each
Questions 10-12: Show your work: 19 points total

Multiple choice answers:

#1	A
#2	C
#3	D
#4	A
#5	D
#6	C
#7	E
#8	B
#9	C

Formula sheet at the back (you can remove it)



Question 1: At the circus, Sheila the Magnificent ties her assistant (Murray the Formerly-Unemployed) up to a wheel and spins it at angular velocity $\omega = 1s^{-1}$. Sheila throws five daggers which stick in the wheel, barely missing Murray. If the daggers are travelling perpendicular to the wheel when they stick, we can say that

A) the final angular velocity of the wheel will be less than $1s^{-1}$
B) the final angular velocity of the wheel will be equal to $1s^{-1}$
C) the final angular speed of the wheel will be greater than $1s^{-1}$

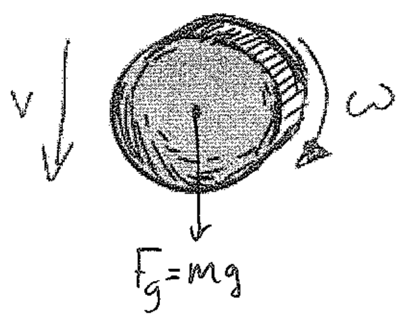
Ignore effects of friction, gravity, and air drag for this question.

L conserved
$\Rightarrow I\omega$ constant
I increases with daggers
so ω decreases.

Question 2: A falling object is spinning as shown. Ignoring effects of air drag, as the object falls, its angular velocity will:

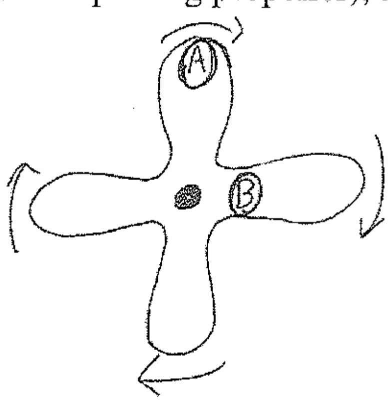
A) increase
B) decrease
C) stay the same

$\tau = 0$ since gravitational force acts right at the center of the object so no angular acceleration.



Question 3: In the picture below, what can we say about the speed v and angular velocity ω of A and B (which are stuck to the spinning propellor), assuming that the center of the wheel is fixed?

A) $v_A = v_B$, $\omega_A = \omega_B$
B) $v_A = v_B$, $\omega_A > \omega_B$
C) $v_A > v_B$, $\omega_A < \omega_B$
D) $v_A > v_B$, $\omega_A = \omega_B$
E) $v_A > v_B$, $\omega_A > \omega_B$



Same ω for all points on an object.
$V = \omega R$, so larger v for larger R.

Question 4: That same old train is moving along the tracks at speed v. Observers standing near the tracks see a flash inside the train and measure the light to hit the front and back of the train at the same time *in the frame of reference of the track*. In the frame of reference of the train,

In picture above, clock at front of train is observed to read an earlier time. This means light hits the front first in the train's frame.

A) The light hits the front of the train first.
B) The light hits the back of the train first.
C) The light hits the front and the back of the train at the same time.
D) Any of A, B, or C could be correct, depending on where the observer is sitting.

Question 5: Xondar boards the space train to go from his home planet to planet Wo☺ow, 3 light hours away. He wants to take a nap during the trip and set his alarm to wake him up just as the train arrives. If the train travels at speed 3/5c, in how many hours should he set his alarm to go off?

A) 2.4 B) 3 C) 3.75 D) 4 E) 5 F) 6.25

Time for trip in frame of planets: $T = \frac{3 \text{ light hours}}{3/5\, c} = 5$ hours.

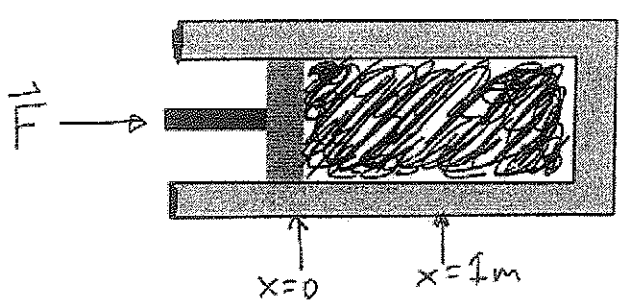
Time passed on train: $T_{proper} = \frac{1}{\gamma} \cdot T = \frac{4}{5} T = 4$ hours.

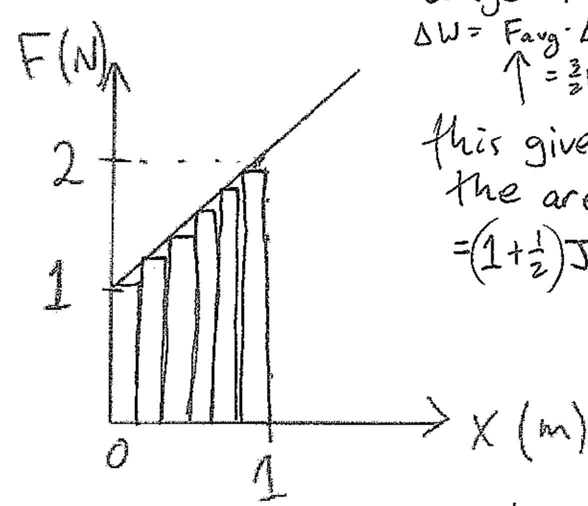
Question 6: A piston presses on some gas in a sealed container, compressing it. If the force applied to the piston as a function of position is shown in the graph below, how much work does it take to move the piston from x=0 to x=1m?

A) 0.5 J B) 1 J C) 1.5 J D) 2 J E) 3J

or just use average force:
$\Delta W = F_{avg} \cdot \Delta x = \frac{3}{2} N \cdot 1m.$

this gives the area $= (1 + \frac{1}{2}) J = 1.5 J$





$\Delta W = F \Delta x$
but F is changing w. x, so need to break interval into segments, calculate $F\Delta x$ for each & add

Question 7: Which of the following statements is true?

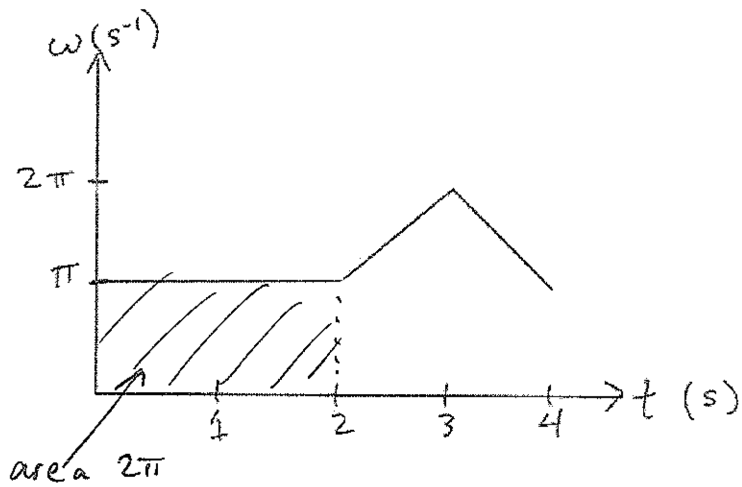
A) Since nothing can travel faster than the speed of light, there is a maximum possible momentum that an object can have. $p = \gamma m v$ γ goes to ∞ as $v \to c$

B) If an object's momentum is nonzero in one frame of reference, it is nonzero in all frames of reference. $p = 0$ in frame of object.

C) In a relativistic collision, momentum does not have to be conserved, since it can be converted into mass. kinetic energy can be converted to mass, but momentum must be conserved

D) All of the above are true

E) None of the above are true

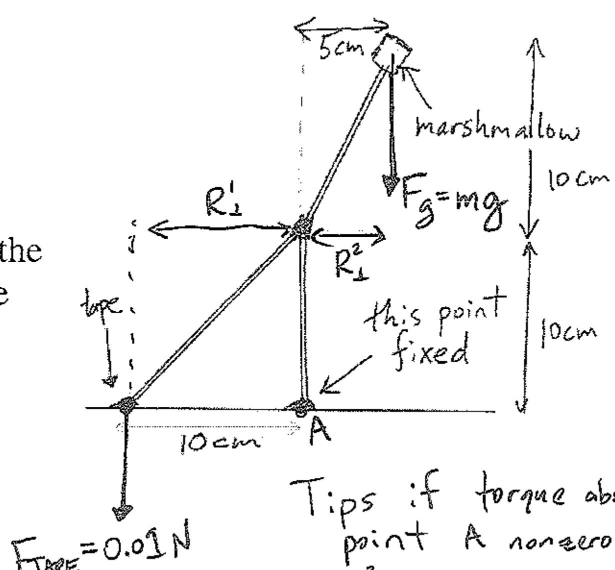


Question 8: The angular velocity ω for a wheel is plotted in the figure above. Starting from time 0, how long is it before the wheel makes one complete rotation?

A) 1s B) 2s C) 3s D) 4s

$\omega = \frac{d\theta}{dt}$ so $\Delta\theta$ = area under graph of ω from t_i to t_f

Question 9: Some students build a rigid structure out of spaghetti and then put a marshmallow on top. If the tape can exert a maximum downward force of 0.01N on the spaghetti, approximately what is mass of the heaviest marshmallow that can be placed on the top without the structure tipping over? Ignore the mass of the spaghetti.



A) 0.5g B) 1g $\boxed{\text{C) 2g}}$

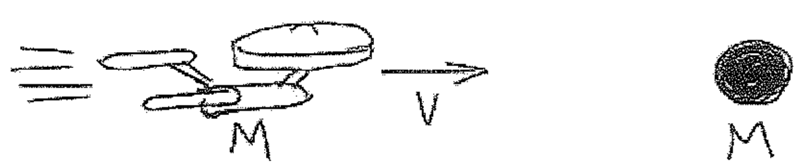
D) 3g E) 4g

$mg \cdot 5cm > 0.01N \cdot 10cm$

$\Rightarrow m > 0.002 kg$

Tips if torque about point A nonzero:

$mg \cdot R_\perp^2 > F_{TAPE} \cdot R_\perp^1$



Question 9: The Starship Enterprise (mass M) moving at speed v=3/5c collides with a small black hole (also of mass M) that is initially stationary, forming a larger black hole. Determine the mass and speed of the new black hole. (*Hint: for the purposes of this question, there is nothing special about black holes compared to other objects.)*
6 points

Let $\tilde{M}$ and $\tilde{v}$ be the mass and speed of the final black hole. Then since energy and momentum are conserved in the collision, we have:

$$E_{initial} = E_{final}$$

$$\Rightarrow \gamma M c^2 + M c^2 = \tilde{\gamma} \tilde{M} c^2$$

$$\Rightarrow \frac{5}{4} M c^2 + M c^2 = \tilde{\gamma} \tilde{M} c^2$$

$$\Rightarrow \tilde{\gamma} \tilde{M} = \frac{9}{4} M \qquad (1)$$

$$p_{initial} = p_{final}$$

$$\gamma M v + 0 = \tilde{\gamma} \tilde{M} \tilde{v}$$

$$\Rightarrow \frac{5}{4} \cdot M \cdot \frac{3}{5} c = \tilde{\gamma} \tilde{M} \cdot \tilde{v}$$

$$\Rightarrow \tilde{\gamma} \tilde{M} \tilde{v} = \frac{3}{4} c \cdot M \qquad (2)$$

Dividing (2) by (1), we get:

$$\tilde{v} = \frac{1}{3} c$$

We can now use (1) to find $\tilde{M}$. We have $\tilde{\gamma} = \frac{1}{\sqrt{1 - (\tilde{v}/c)^2}} = \frac{3}{2\sqrt{2}}$

So $\tilde{M} = \frac{9}{4} M \cdot \frac{1}{\tilde{\gamma}} = \frac{3\sqrt{2}}{2} M \approx 2.12 M$

Alternate solution:

The final energy and momentum of the object must be:

$$E_f = E_i = \gamma M c^2 + M c^2 = \frac{9}{4} M c^2$$

$$p_f = p_i = \gamma M v = \frac{3}{4} M c$$

So its mass must be (using $E_f^2 - p_f^2 c^2 = \tilde{M}^2 c^4$)

$$\tilde{M} = \sqrt{\frac{1}{c^4}\left(E_f^2 - p_f^2 c^2\right)}$$

$$= \sqrt{\frac{1}{c^4}\left(\frac{81}{16} M^2 c^4 - \frac{9}{16} M^2 c^4\right)}$$

$$= \frac{3\sqrt{2}}{2} M$$

Since its energy is $\frac{9}{4} M c^2$, we have

$$\tilde{\gamma} \tilde{M} c^2 = \frac{9}{4} M c^2$$

$$\Rightarrow \tilde{\gamma} \left[\frac{3\sqrt{2}}{2} M\right] c^2 = \frac{9}{4} M c^2$$

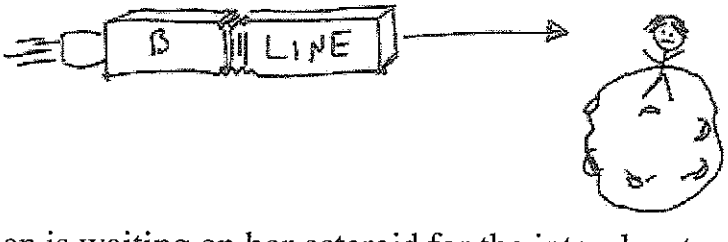
$$\Rightarrow \tilde{\gamma} = \frac{3}{2\sqrt{2}}$$

$$\Rightarrow \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{3}{2\sqrt{2}}$$

$$\Rightarrow 1 - \frac{v^2}{c^2} = \frac{8}{9}$$

$$\Rightarrow \frac{v^2}{c^2} = \frac{1}{9}$$

$$\Rightarrow v = \frac{1}{3} c.$$



Question 12: Noreen is waiting on her asteroid for the interplanetary B-line space-bus. When the bus is approaching she observes the lights on the bus to have wavelength 500nm. Unfortunately, the bus is full, so it doesn't even slow down. It takes only 10^{-7} seconds from when the front of the bus passes her to when the back of the bus passes her. When the bus is flying away, she observes the lights on the bus to have wavelength 1000nm. How long the bus (in its own frame of reference)? **5 points**

Let λ_0 be the actual wavelength of the light from the bus. When the bus is approaching, the light is blueshifted, so the observed wavelength is shorter than λ_0:

$$\lambda_{obs} = \sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}}\,\lambda_0$$

$$\Rightarrow 500\,nm = \sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}}\,\lambda_0 \qquad (1)$$

When the bus is travelling away, the light is redshifted, so

$$\lambda_{obs} = \sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}}\,\lambda_0$$

$$\Rightarrow 1000\,nm = \sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}}\,\lambda_0 \qquad (2)$$

Dividing (2) by (1), we get:

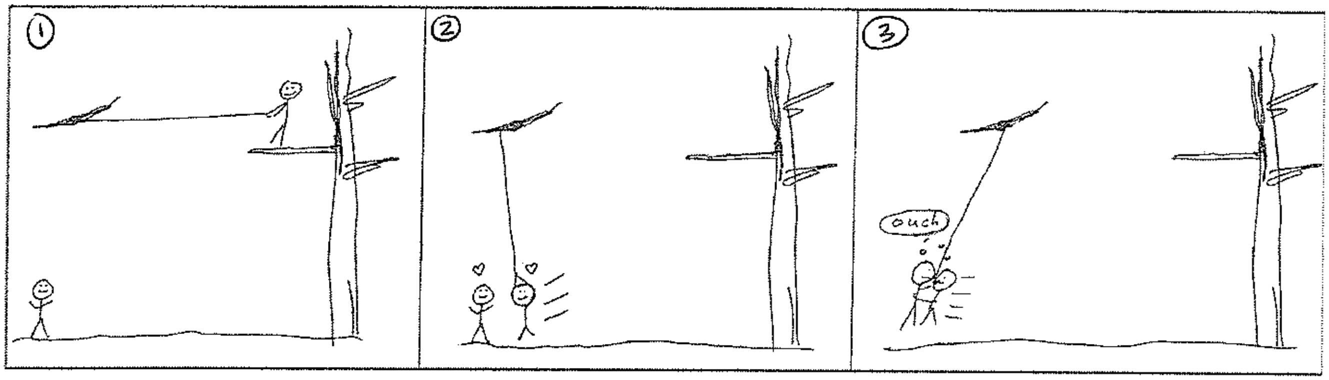
$$2 = \frac{1+\frac{v}{c}}{1-\frac{v}{c}}$$

$$\Rightarrow 2 - 2\frac{v}{c} = 1 + \frac{v}{c}$$

$$\Rightarrow \frac{v}{c} = \frac{1}{3}$$

For this v, $\gamma = \frac{3}{2\sqrt{2}}$, so the actual length of the bus is

$$L_{actual} = L_{obs}\cdot\gamma = \underbrace{(10^{-7}s)\times\left(\tfrac{1}{3}c\right)}_{\text{using length}_{obs} = \text{speed} \times (\text{time to pass})}\times\frac{3}{2\sqrt{2}} = 10.6\,m$$



Question 12:

a) Tarzan swings on a vine of length 8m that is initially horizontal. When the vine is vertical, he and Jane grab each other, and together they swing back upwards. If Tarzan and Jane have the same mass M and the mass of the vine can be ignored, to what height do they swing? **5 points**

From ① to ②, energy is conserved, so

$$E_f = E_i$$

$$\Rightarrow \frac{1}{2} I \omega^2 = Mgh$$

$$\Rightarrow \frac{1}{2}(ML^2)\omega^2 = Mgh$$

$$\Rightarrow \omega = \sqrt{\frac{2gh}{L^2}} = \sqrt{\frac{2g}{L}} \quad \text{since } h = L$$

From just before the collision to just after the collision, angular momentum is conserved, so:

$$L_f = L_i$$

$$\Rightarrow I_f \omega_f = I_i \omega_i$$

$$\Rightarrow 2ML^2 \omega_f = ML^2 \omega_i$$

$$\Rightarrow \omega_f = \frac{1}{2}\omega_i = \frac{\sqrt{2}}{2}\sqrt{\frac{g}{L}}$$

From the time just after the collision to the final height, energy is conserved, so:

$$E_f = E_i$$

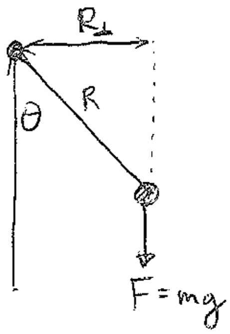
ω just after collision

$$2Mgh_f = \frac{1}{2} I_f \omega^2 = \frac{1}{2}(2ML^2)\cdot\frac{1}{2}\frac{g}{L}$$

$$\Rightarrow h_f = \frac{1}{4} L = 2m.$$

b) Estimate how long it takes Tarzan to swing down and reach Jane. **3 points**

During the swing, the angular acceleration isn't constant.



At angle θ the torque is

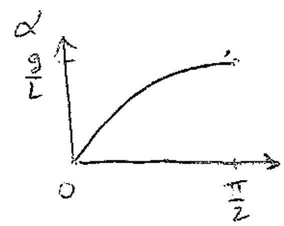
$$\tau = |\vec{F}| R_\perp$$ ← this gives the same result as $\tau = F_\perp \cdot R$ since both are $|\vec{F}| \cdot R \cdot \sin\theta$

$$= mgL\sin\theta$$

So we have: $\alpha = \frac{1}{I} \cdot \tau$

$$= \frac{mgL\sin\theta}{mL^2}$$

$$\Rightarrow \frac{d^2\theta}{dt} = \frac{g}{L}\sin\theta$$ ← this is a tricky differential equation, so we'll need to approximate.



During the swing, θ decreases from $\frac{\pi}{2}$ to 0, so α decreases from $\frac{g}{L}$ to 0.

As an approximation, we can choose an intermediate value of α and solve for the time as if we had constant acceleration.

We'll choose α to give the correct final angular speed $\sqrt{2}\sqrt{\frac{g}{L}} \approx 1.566 s^{-1}$ as calculated in the previous part.

We have: $\omega_f = \alpha \Delta t$ $\qquad$ $\theta_f = \theta_i + \frac{1}{2}\alpha(\Delta t)^2$

$1.566 s^{-1} = \alpha \Delta t$ $\qquad$ $\Rightarrow \frac{\pi}{2} = \frac{1}{2}\alpha(\Delta t)^2$

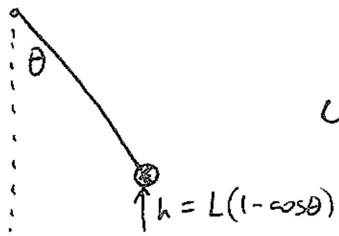
$\alpha = \frac{1.566 s^{-1}}{\Delta t}$ $\longrightarrow$ $\frac{\pi}{2} = \frac{1}{2}\left[\frac{1.566 s^{-1}}{\Delta t}\right](\Delta t)^2$

$$\Rightarrow \Delta t = \frac{\pi}{1.566 s^{-1}} \approx 2.0 s$$

Bonus question (0 points): What does the fox say?

b) Estimate how long it takes Tarzan to swing down and reach Jane. **3 points**

A SIMPLER ESTIMATE:



We have the final angular speed as

$$\omega_f = \sqrt{2}\,\sqrt{\frac{g}{L}}$$

$$= 1.566$$

During the swing, the angular speed increases from 0 to this value. So we can say that the time is at least

$$t_{min} = \frac{\frac{\pi}{2}}{\omega_f} = 1.003\,s$$

THE EXACT ANSWER:

To be more precise, we can figure out the speed at each angle θ:
Using energy conservation, we have

$$\frac{1}{2}(ML^2)\cdot\omega^2 + MgL(1-\cos\theta) = MgL$$

$$\Rightarrow \omega = \sqrt{\frac{2g}{L}\cos\theta}$$

$$\Rightarrow \frac{d\theta}{dt} = \sqrt{\frac{5}{2}}\sqrt{\cos\theta} \qquad \text{(where } t \text{ is in seconds)}$$

$$\Rightarrow \frac{1}{\sqrt{\cos\theta}}\frac{d\theta}{dt} = \sqrt{\frac{5}{2}}$$

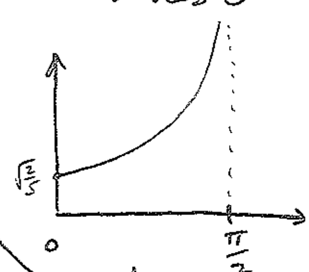
$$\Rightarrow dt = \sqrt{\frac{2}{5}}\frac{1}{\sqrt{\cos\theta}}\,d\theta$$

to find the exact time, we add this up for each $d\theta$ interval from 0 to $\frac{\pi}{2}$

So t = area under graph of $\sqrt{\frac{2}{5}}\frac{1}{\sqrt{\cos\theta}}$ from 0 to $\frac{\pi}{2}$

$$t = \int_0^{\frac{\pi}{2}} \sqrt{\frac{2}{5}}\frac{1}{\sqrt{\cos\theta}} \approx 1.658\,s$$

used a computer to find this.



Bonus question (0 points): What does the fox say?