Question 10: A large spacecraft with a self-sustaining population of humans and other species travels at v=0.6c from Earth to the recently discovered planet Kepler22b, 600 light years from Earth. How many years pass on the ship's clock during the voyage to planet Kepler 22b?

$t_{Earth} = \frac{600\,ly}{0.6c} = 1000$ years

A) 1250 B) 800 C) 1000 D) 600 E) 480

TIME DILATION: t_{SHIP} (proper time) $= \frac{1}{\gamma} t_{Earth} = \frac{4}{5} t_{Earth} = 800$ years

Question 11: A nucleus of mass M decays into another nucleus of mass M' by emitting an α particle. We can say that the original mass M is

A) less than $m_\alpha + M'$
B) greater than $m_\alpha + M'$
C) equal to $m_\alpha + M'$
D) any of the above are possible

BEFORE: M

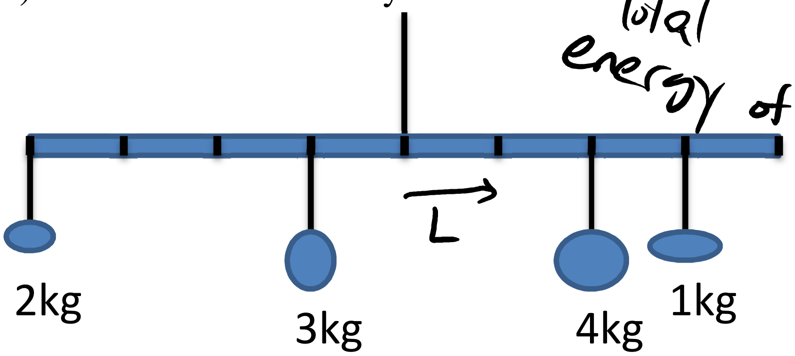
AFTER: M' m_α

Some mass energy has gone to kinetic energy

Question 12: A ball of gold cools by emitting infrared radiation. During this process,

A) the mass of the ball decreases
B) the mass of the ball increases
C) the mass of the ball stays the same

Mass = total energy of ball in its rest frame divided by c^2



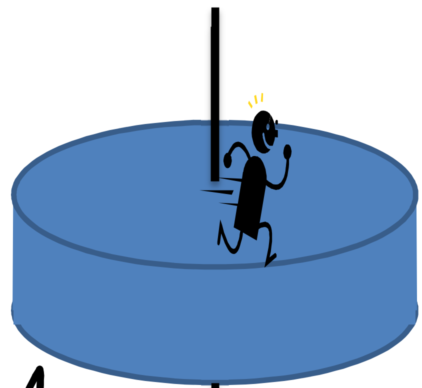
Question 13: An artist gives you the design above for a piece of art that will hang from the ceiling. He asks you whether the art work will stay horizontal. After a quick calculation, you tell him that

A) the art will tip to the right.
B) the art will tip to the left.
C) the art will stay balanced.

Net torque:

$4kg \cdot 2L + 1kg \cdot 3L - 3kg \cdot L - 2kg \cdot 4L = 0$

Question 14: A big solid disk sits on a frictionless axle. A man stands at the edge of the disk. If the man tries to run,



A) the man will stay in the same place and the disk will rotate under him.
B) the man will move counterclockwise around the axle, while the disk will rotate clockwise around the axle (viewed from the top).
C) the man and the disk will both start moving clockwise around the axis.
D) the man and the disk will both end up moving counterclockwise around the axis.
Ignore any effects associated with air resistance.

Angular momentum conserved

BEFORE: 0 AFTER: $L_{man} + L_{disk} = 0$

So must be in opposite directions

Question 15: In roughly 5 billion years, our Sun is expected to expand into a red giant star. If its radius increases by a factor of 400, its period of rotation would

A) stay the same
B) become 20 times longer
C) become 400 times longer
D) become 160000 times longer
E) become 20 times shorter

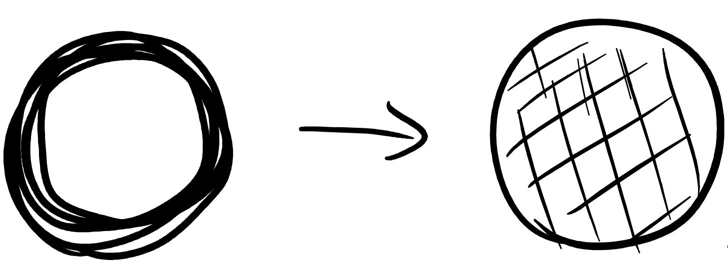
L conserved: $I_{before}\omega_{bef.} = I_{after}\omega_{after}$

$R \to 400R \Rightarrow I \to 160000 I \Rightarrow \omega \to \frac{1}{160000}\omega \Rightarrow T \to 160000\,T$

Question 16: Suppose that we replaced the wheels of a bicycle with solid disks with the same mass and radius as regular bicycle wheels. Ignoring any possible effects associated with air resistance, and assuming that the surface of the wheel is the same as a regular bicycle tire, we would expect that

A) the bicycle would be easier to pedal.
B) the bicycle would be harder to pedal.
C) the change would not affect how easy it would be to pedal the bike.

$\alpha = \tau / I$

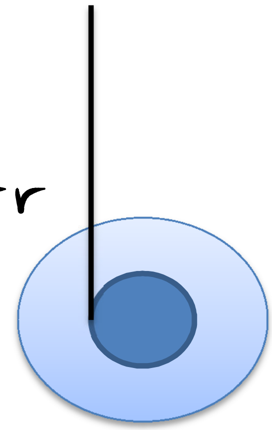


Smaller I

so: more α for same torque

easier to pedal.

Question 17: The picture at the right shows the cross-section of a yo-yo. The radius of the inner circle is r and the radius of the outer circle is R. We can say that the linear downward velocity v and the angular velocity ω of the yo-yo are related by



for $\Delta\theta = 2\pi$, $\Delta y = 2\pi r$

so $r \cdot \Delta\theta = \Delta y$

$$r \frac{\Delta\theta}{\Delta t} = \frac{\Delta y}{\Delta t}$$

A) $v = \omega R$
B) $v = \omega r$
C) $v = 2\pi\omega R$
D) $v = 2\pi\omega r$
E) none of these; v and ω are independent of each other

Question 27: The Large Hadron Collider in Geneva accelerates protons close to the speed of light, so that their total energy is 7000 times their rest energy. If the beam has 10^{10} protons per second, and we shine the beam at a 1kg block on a frictionless table so the protons reflect directly backwards at approximately the same speed, how long will it be before the block moves one meter?

For this problem, it is reasonable to ignore the change in mass of the block.

(4 points)

Since the protons have 7000 times their rest energy, we have

$$E = \gamma mc^2 = 7000\, mc^2$$

so $\gamma = 7000$.

In each collision with the block, momentum is conserved, so

$$|\Delta p_{block}| = |\Delta p_{proton}|$$

almost = c

$$= 2 m_{proton} \cdot v_{proton} \cdot \gamma$$

$$= 14000\, m_{proton}\, c$$

The collisions happen every 10^{-10} s, so the rate of change of momentum of the block is: $\dfrac{\Delta p}{\Delta t} = \dfrac{14\,000\, m_p \cdot c}{10^{-10}\, s}$

$= 7.0 \times 10^{-5}\, kg\, m/s^2$ ← this is the force on the block

For the block, we can use $p = mv$, so

$a = \dfrac{dv}{dt} = 7.0 \times 10^{-5}\, m/s^2$

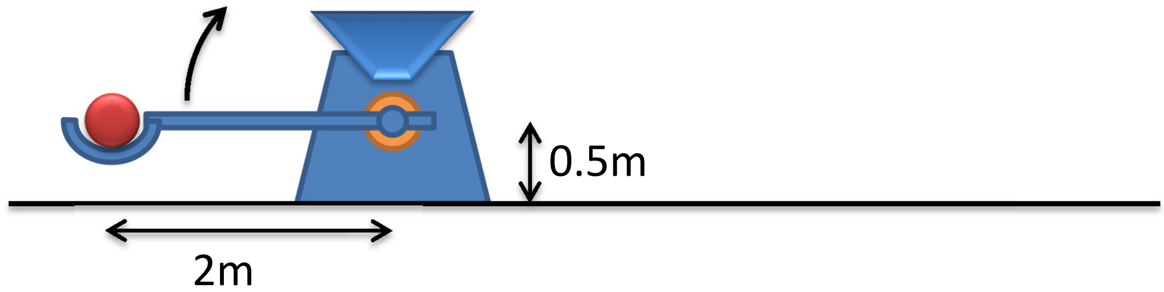
We want

$\Delta x = 1m$, so we need

$\frac{1}{2} \cdot a (\Delta t)^2 = 1m$

$\Rightarrow (\Delta t)^2 = \dfrac{2m}{7.0 \times 10^{-5}\, m/s^2}$

$\Rightarrow \Delta t \approx 169s$



Question 28: A 100kg iron ball is loaded into a catapult that starts off in the position shown. When it is fired, the catapult exerts a torque on the lever arm that increases linearly with time:

$\tau(t) = (300{,}000 \text{Nm/s})\ t$

When the catapult arm reaches an angle of 45 degrees, it hits a barrier that prevents it from moving further, leaving the ball to fly freely. Bothvar is thinking about buying this catapult to sent iron balls over the castle walls of his enemies. What are the highest castle walls over which he will be able to send balls? **(4 points)**

Assume that the mass of the lever arm can be ignored relative to the mass of the ball.

We have: $\tau = I\alpha$ with $\tau = (300000 \text{ Nm/s})\, t$
and $I = ML^2$

$\alpha = \frac{d\omega}{dt}$ so: $\frac{d\omega}{dt} = \frac{\tau}{I} = \frac{300000 \text{ Nm/s}}{100\text{kg} \cdot (2\text{m})^2} \cdot t = 750 \frac{1}{s^3} \cdot t$

$\Rightarrow \omega = 375\, s^{-3} \cdot t^2$

Also: $\omega = \frac{d\theta}{dt}$ so: $\theta(t) = \frac{1}{3} \cdot 375\, s^{-3} t^3$

The time when $\theta = 45° = \frac{\pi}{4}$ is

$\frac{\pi}{4} = \frac{1}{3} \cdot 375\, s^{-3} \cdot t^3$

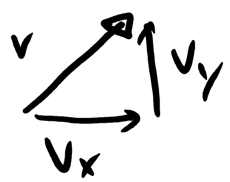
$\Rightarrow t = 0.185 s$

At this time, the angular speed is

$$\omega = 375\,s^{-3} \cdot (0.185\,s)^2$$
$$= 12.8\,s^{-1}$$

So the speed of the ball is

$$v = \omega \cdot r$$
$$= 25.6\ m/s$$



We have: $y^{o} = 0.5\,m$

$$v_y^{o} = v\,\frac{\sqrt{2}}{2} = 18.1\ m/s$$
$$a_y = -g$$

Finally: the ball reaches its highest point when $v_y = 0$ so using $\Delta v_y = a_y \Delta t$ we get: $\Delta t = \frac{-18.1\,m/s}{-9.8\,m/s^2}$

$$= 1.84\,s$$

Then $y_f = y^{o} + v_y^{o} t - \frac{1}{2} g t^2$

$$= 0.5\,m + 18.1\,m/s \cdot 1.84\,s - \frac{1}{2} \cdot 9.8\,m/s^2 \cdot (1.84\,s)^2$$
$$= 17.2\,m$$

Bothvar can send his balls over 17.2m walls.