Science One Term 1 Physics Exam

December 17, 2013

Name: SOLUTIONS
Student Number:
Bamfield Number:

Questions 1-23: Multiple Choice: 1 point each
Questions 24-31: Long answer: 27 points total
explain your work

Multiple choice answers:

#1	#2	#3	#4	#5
C	A	B	D	D

#6	#7	#8	#9	#10
D	C	B	B	B

#11	#12	#13	#14	#15
B	B	C	A	A

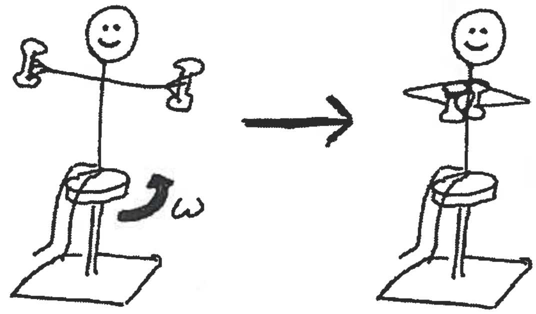
#16	#17	#18	#19	#20
C	B	C	C	C

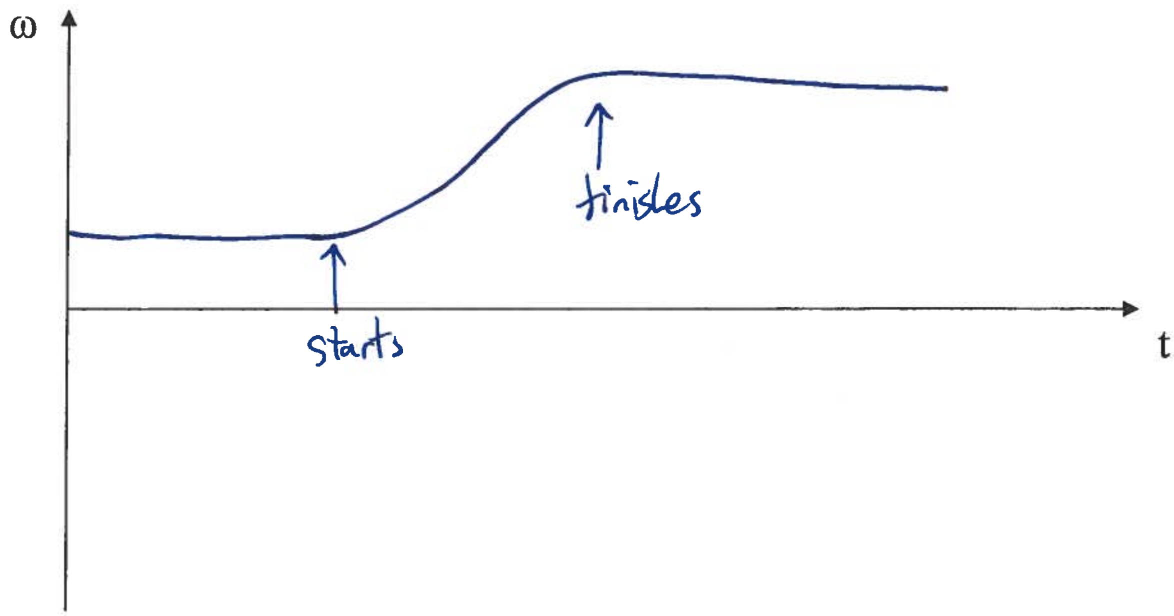
#21	#22	#23		
A	D/E	C		

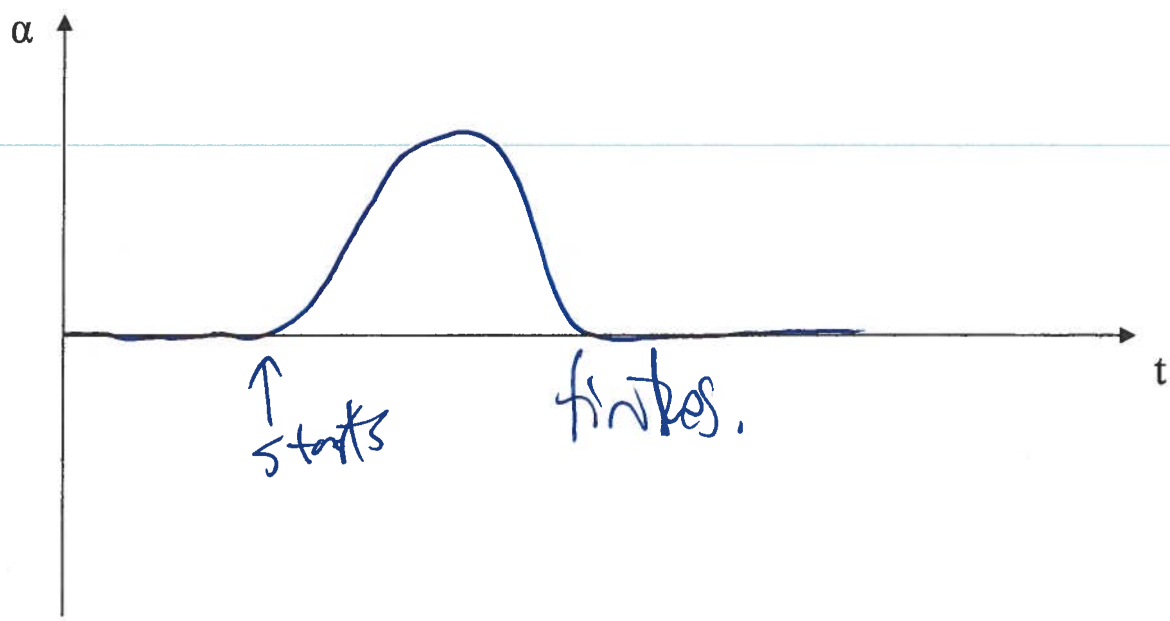
BONUS SQUARES!

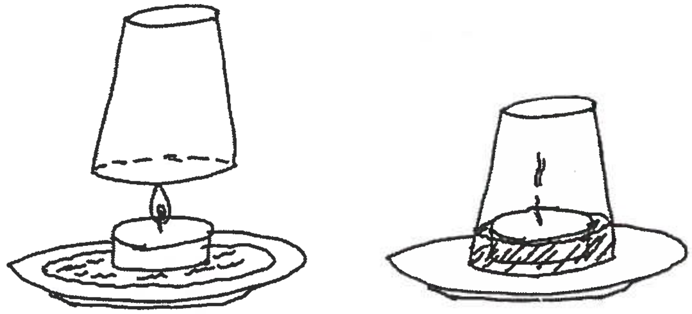
Formula sheet at the back (you can remove it...carefully!)

Question 24: Brianna is spinning around on a frictionless stool, holding weights with her arms extended. If she pulls the weights in towards her body and holds them there, sketch Brianna's angular speed and angular acceleration as a function of time on the axes below. Indicate the times when she starts and finishes moving the weights. **(2 points)**







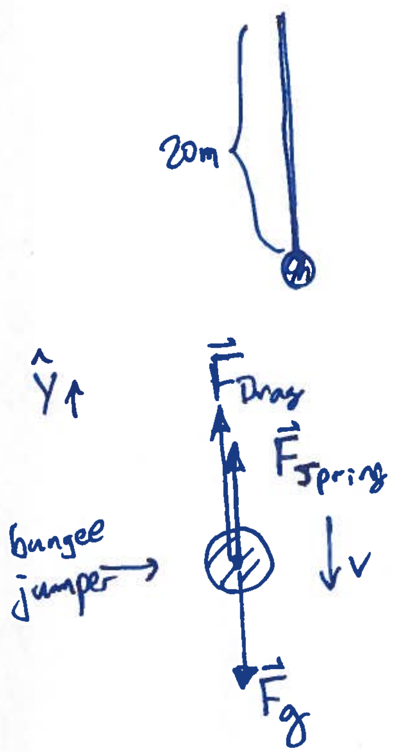


Question 25: A candle sits in a small pool of water on a dish. A glass is held above the candle, then quickly brought down over the candle so that the rim of the glass is in the water. It is observed that the candle goes out and the water is sucked up into the glass. Provide a concise explanation for this phenomenon. **(3 points)**

While the cup is above the glass, T of the air inceases, with V and P fixed (the latter since the glass is still open to the air). So by $PV=nRT$, n decreases (i.e. air flows out of the glass). After the glass is placed on the dish, the candle extinguishes after the O_2 is used up, so the air cools. If the volume remained constant, pressure would decrease, and the external pressure would be greater than the internal pressure, causing water to be pushed into the glass. Thus, the volume of air decreases (& water is sucked in) until the pressures equalize. Alternatively, we can say that the water acts as a movable piston, keeping pressure inside const & equal to the outside pressure, so as T decreases, V decreases (n is const.).

Question 26: A bungee jumper with mass 100kg jumps off a bridge attached to a bungee cord that can be modeled as a spring with normal length 10m and spring constant k = 100N/m. When the cord has stretched to a length of 20m (call this time t=0), the jumper's vertical velocity is -16 m/s.

If the air drag force can be modeled as $F = -C v^2$, where C = 0.3kg/m, estimate the change in the length of the cord and the change in the vertical velocity of the jumper between t=0 and t=0.1s. **(4 points)**



We have gravity, spring, and air drag forces on the jumper:

$$\vec{F}_g = -mg\,\hat{y} = -(100kg)(9.8m/s^2)\,\hat{y} = -980\,N$$

$$\vec{F}_{spring} = k(\Delta s)\,\hat{y} = (100N/m)\cdot(20m - 10m)\cdot\hat{y} = 1000N\cdot\hat{y}$$

$$\vec{F}_{drag} = +Cv^2\,\hat{y} = (0.3\,kg/m)\cdot(16m/s)^2 = 76.8N\,\hat{y}$$

Thus, $\vec{F}_{NET} = \vec{F}_g + \vec{F}_{spring} + \vec{F}_{drag} \approx 97N$

From Newton's 2nd Law,

$$\vec{a} = \frac{1}{m}\vec{F}_{NET} = \frac{1}{100kg}\cdot 97N = 0.97\,m/s^2\,\hat{y}$$

Now, we can estimate the position and velocity changes during the interval 0s to 0.1s by

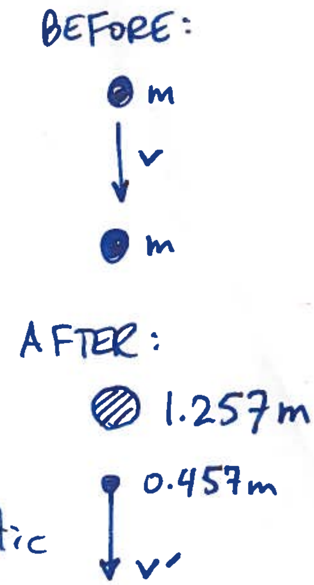
$$v_y(0.1s) - v_y(0s) \approx (0.1s)\cdot a_y(0s) \approx 0.097m/s$$

$$y(0.1s) - y(0s) \approx (0.1s)\cdot v_y(0s) \approx -1.6m.$$

extra space ↘

$\frac{5}{3}$

Question 27: A proton (mass m) from space with total energy E = 5/4 m c^2 collides with a stationary proton in the upper atmosphere. As a result of the collision, the protons annihilate, leaving a stationary Q^{++} particle (mass 1.257 m) and a meson (an unstable particle with mass 0.457 m) which travels directly towards the Earth's surface. If the collision happens 100km above the Earth's surface and the half-life of the meson is $\tau = 5.71 \times 10^{-6}$ s, what are the chances that the meson will reach the surface of the Earth before decaying? *(Note: the half life τ is defined so that for a particle at rest, the chance of that the particle hasn't decayed is 1/2 after time τ, 1/4 after time 2τ, 1/8 after time 3τ, etc…)* **(4 points)**



For the collision, total momentum and total relativistic energy are both conserved. Since there is only one unknown (v'), we only need to use one of these equations (the problem is set up to give the same answer using either one). Using energy conservation, we have:

$$E_{before} = E_{after}$$

$$mc^2 + \gamma mc^2 = 1.257mc^2 + \gamma' \cdot 0.457mc^2$$

We are given that $\gamma mc^2 = \frac{5}{4}mc^2$, so we find

$$\gamma' = \frac{mc^2 + \frac{5}{4}mc^2 - 1.257mc^2}{0.457mc^2} = 2.17$$

From this, we can calculate v' by

$$\gamma' = \sqrt{\frac{1}{1-(v')^2/c^2}} = 2.17$$

$$\Rightarrow \quad 1 - \frac{(v')^2}{c^2} = 0.212$$

$$\Rightarrow \quad v' = 0.888c$$

The half life of the meson is 5.71×10^{-6}s in its own frame, so in the frame of Earth, this corresponds to

extra space ↓

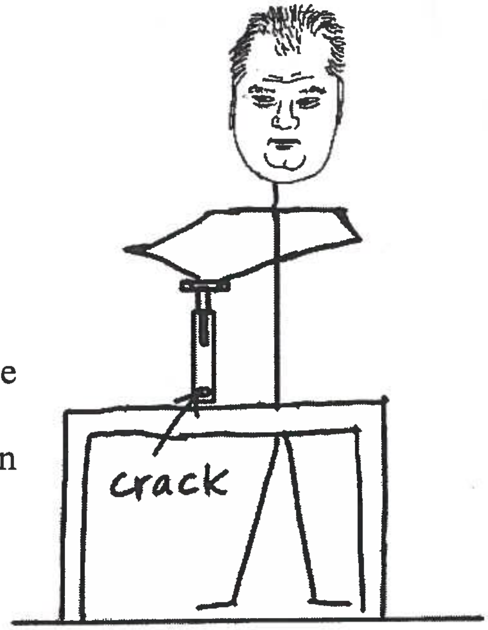
a time $\tau \cdot \gamma' = 5.71 \times 10^{-6} s \cdot (2.17) = 1.12 \times 10^{-5} s$.

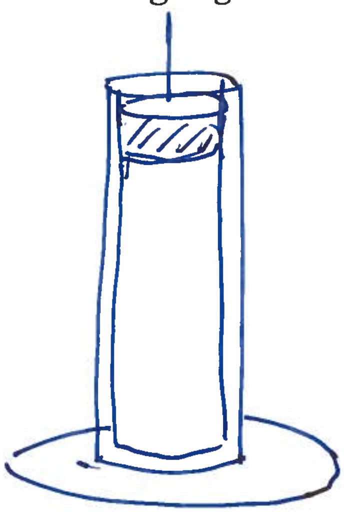
In this time, the particle travels

$$d = (1.12 \times 10^{-5} s) \cdot (0.888) \cdot (3 \times 10^{8} m/s)$$

$$\approx 3.3 \text{ km}$$

Question 28: Rob Ford (mayor of Toronto and amateur physicist) is working on a new design for a crack pipe. The idea is put the crack in an insulated cylinder of air (initially at atmospheric pressure and temperature 300K) and ignite the crack by pressing firmly down on a piston while the cylinder (crack pipe) remains in place. If the cylinder is 15 cm long and has cross sectional area 1cm^2, what constant force would ensure that the air in the tube reaches 900K (just right to ignite the crack) after the piston has been pushed 10cm down the tube? *(You may treat the air as nitrogen gas with $c_V = 5/2\ R$.)* **(4 points)**





The cylinder is insulated, so we have an adiabatic process with $Q = 0$ and $\Delta E = W$. We have $\Delta E = n \cdot c_v \Delta T$ and $W = F \Delta x$, so the required force is given by

$$\Delta x \cdot F = n c_v \Delta T$$

$$\Rightarrow F = \frac{n c_v \Delta T}{\Delta x}$$

We can calculate n using:

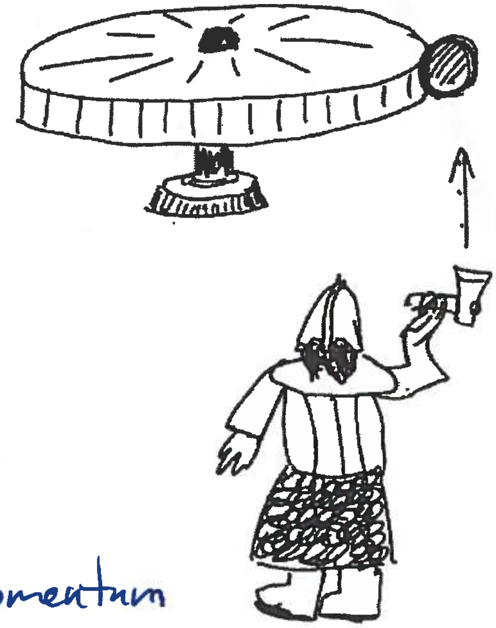
$$n = \frac{P_i V_i}{R T_i} \cancel{\neq \frac{10^5 Pa \overbrace{(0.0001 m^2)(0.15 m)}^{V = \text{Area} \times \text{length}}}{R \cdot T_i}}$$

So:

$$F = \frac{P_i \cdot A \cdot L}{R T_i} \cdot \frac{c_v \Delta T}{\Delta x} = \frac{P_i \cdot A \cdot L}{T_i} \cdot \frac{\frac{5}{2} \cdot \Delta T}{\Delta x} = 75N$$

using $A = 0.0001 m^2$, $L = 0.15 m$, $\Delta x = 0.1 m$, $\Delta T = 600K$, $T_i = 300K$, $P = 100000 Pa$.

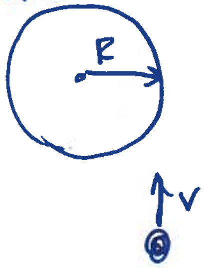
Question 29:



a) A group of dwarves are practicing axe-throwing. They have a target attached to a large wooden disk of mass 10 dwarvish stones (the standard unit of mass used by dwarves in Middle Earth, 1 ds = 15 kg) and radius 2 meters. If Glomdrin throws his axe (mass 1ds) with speed 10 m/s and it sticks in the target, determine the angular speed of the wooden disk just after the collision. *Assume that the target is also at radius 2m and that the axe hits the target perpendicularly.* **(2 points)**

In the collision, angular momentum is conserved. Before the collision,

BEFORE:



the angular momentum of the axe relative to the center of the wheel is

$$L_{BEF.} = m_{axe} v \cdot R$$

$$= (15\,kg)\cdot(10\,m/s)\cdot(2\,m) = 300\,kg\,\frac{m^2}{s}$$

AFTER:

After the collision, we have:

$$L_{AFT.} = I\omega$$

Here $I = I_{disk} + I_{axe}$

$$= \frac{1}{2} M_{disk} R^2 + M_{axe} R^2$$

$$= \frac{1}{2}(150\,kg)(2m)^2 + (15\,kg)\cdot(2m)^2$$

$$= 360\,kg\,m^2$$

So using $L_{before} = L_{after}$, we get

$$\omega = \frac{mvR}{I} = \frac{300\,kg\,\frac{m^2}{s}}{360\,kg\,m^2} = 0.833\,s^{-1}$$

b) As the disk starts spinning, some dwarf children run in and slow it down by placing their hands on it. The frictional torque due to dwarf-child hands increases with time as more and more dwarf children put their hands on the wheel. If we model this by

$$\tau = (1\,\mathrm{N\,m/s})\,t\,,$$

through what angle does the disk rotate before stopping? **(3 points)**

We have:

− sign since slowing down.

$$\alpha = \frac{d\omega}{dt} = \frac{\tau}{I} = \frac{-(1\,\mathrm{Nm/s})}{360\,\mathrm{kg\,m^2}} \cdot t = -(0.00278\ s^{-3})\cdot t$$

Thus: $\omega = -\frac{1}{2}(0.00278\,s^{-3})t^2 + C$

Since $\omega(0) = 0.833\,s^{-1}$, we get $C = 0.833\,s^{-1}$.

So $\omega = \frac{d\theta}{dt} = 0.833\,s^{-1} - \frac{1}{2}(0.00278\,s^{-3})t^2 \quad (*)$

The disk stops spinning after time

$$t = \left(\frac{2 \cdot 0.833\,s^{-1}}{0.00278\,s^{-3}}\right)^{\frac{1}{2}} = 24.5\,s$$

During this time, we have from (*) that

$$\theta(t) = (0.833\,s^{-1})\cdot t - \frac{1}{6}(0.00278\,s^{-3})\cdot t^3 + \cancel{C}^{\,0}$$

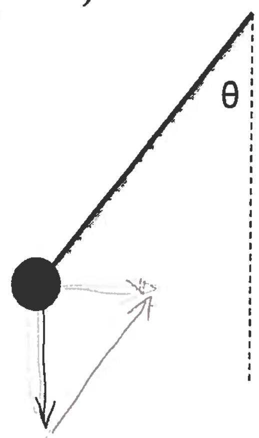
set to 0 by defining $\theta(t=0)=0$.

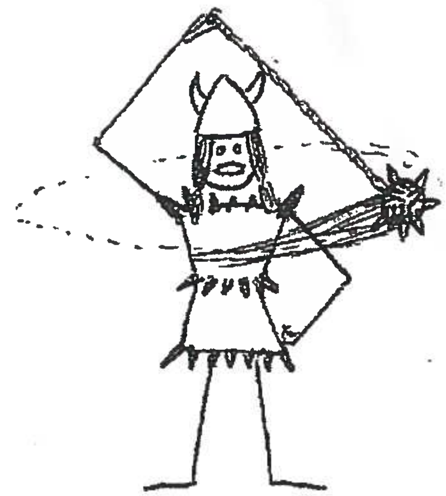
So the net change in angle is

$$\theta(24.5) = 13.6\,\mathrm{rad}.$$

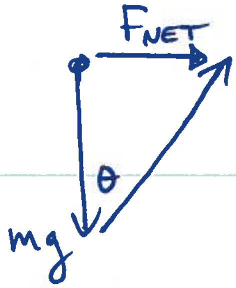
Question 30: Ethel the Invincible swings a spiky ball with mass m around on a 1m long chain. If the ball swings around once each 1.69 seconds, find the angle θ.

Hint: in what direction is the acceleration for an object in circular motion? **(3 points)**





The forces on the ball are gravity and the tension in the chain. Since acceleration is toward the center of the circle for circular motion, the net force must be in this direction.



From the diagram,

$$\tan\theta = \frac{F_{NET}}{mg}. \qquad (*)$$

To find F_{NET}, we use that for circular motion, the acceleration is

$|\vec{a}| = \frac{v^2}{R}$, so $F_{NET} = \frac{mv^2}{R.}$ Now

$R = L\sin\theta$ and $v = \frac{2\pi R}{t} = \frac{2\pi L \sin\theta}{t}$

so

$$F_{NET} = m \cdot \frac{4\pi^2 L \sin\theta}{t^2}$$

extra space ↴

Finally, from (*),

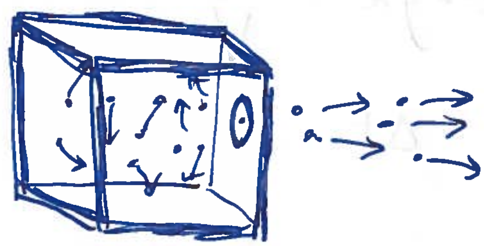
$$\tan\theta = \frac{4\pi^2 L \sin\theta}{g t^2}$$

$$\Rightarrow \cos\theta = \frac{t^2 \cdot g}{4\pi^2 L} = 0.709$$

$$\Rightarrow \theta = 0.78 \text{ rad} \approx 45^\circ$$

Question 31: A sealed container, with volume 1 liter and mass 1kg, contains 1 mole of helium gas. The container is floating in outer space, minding its own business, when a piece of space junk punctures a $1mm^2$ hole in the wall of the container. Estimate the acceleration of the container due to the escaping helium. (*It may be useful to know that the mass of a helium atom is 6.65×10^{-27} kg.*)
(2 points)

$T = 100K$



The container moves due to momentum conservation.

In a time Δt, the change in momentum of the container equals the momentum of the escaped particles

$$\Delta P_{cont.} = -\Delta P_{(particles\ escaping)}$$

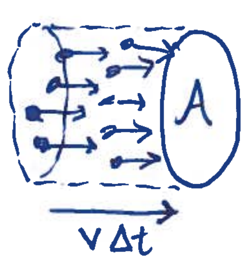
The acceleration is

$$a_{cont} = \frac{1}{m}\frac{\Delta P_{cont}}{\Delta t} = \frac{1}{m}\frac{\Delta P_{particles\ escaping}}{\Delta t}.$$

For the escaping particles, the momentum lost in time Δt is the average # of particles escaping times the average momentum per particle.

Now, $P_{avg} = m V_{avg} = \sqrt{2m\, \epsilon^{trans.kin}_{avg}}$

$$= \sqrt{2m \cdot \tfrac{3}{2} k_B \cdot T} \approx 5.2 \times 10^{-23}\ \text{kgm/s}$$



Also, in a time Δt, the number of particles that could reach the hole is about the number in the volume shown at the left (divided by 2 since only half the particles will be moving toward the wall)

This number is

$$\frac{1}{2}\cdot\left(\text{total \# molecules in container}\right)\times\frac{\text{our little volume}}{\text{container volume}}$$

$$=\frac{1}{2}\cdot\left(6.02\times10^{23}\right)\times\frac{A\cdot v_{avg}\,\Delta t}{V}$$

$$=2.38\times10^{21}\cdot\Delta t$$

Where we have used

$$v_{avg}=\frac{p_{avg}}{m}=7.9\times10^{3}\,m/s$$

So:

$$a_{cont}=\frac{1}{m}\frac{\Delta p}{\Delta t}$$

$$=\frac{1}{1kg}\cdot2.38\times10^{21}\times5.2\times10^{-23}\,kg\,m/s^{2}$$

$$\approx0.12\,m/s^{2}$$

NOTE: Many other solutions are possible.