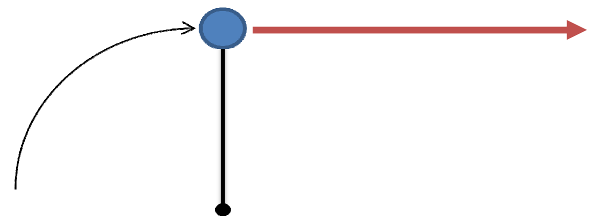
name:

group members:

PHYSICS WORKSHEET: ORBITS



Question 1: A ball of mass M swings around on a string of length r at angular velocity ω. Assume we are in outer space for this question, so there is no gravity.

a) What is the angular momentum of the system (in terms of M, r, and ω)?

$L = Mr^2\omega$

b) What is the speed of the ball (in terms of r and ω)?

$v = r\omega$

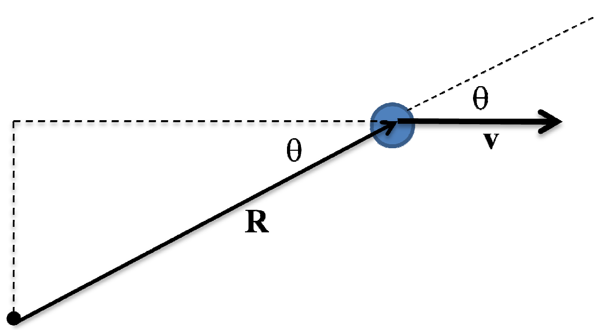
c) Express the angular momentum of the ball in terms of M, v, and r.

$L = Mvr$

This is the formula for the angular momentum of an object about a specific axis if the object's velocity is perpendicular to the vector $\vec{R}$ between the axis and the object.

c) If the string is cut just at the instant shown in the figure above, sketch the subsequent trajectory of the ball (remember, no gravity) on the figure. see above

d) After the string is cut, angular momentum is still conserved. Using this fact, express the angular momentum of the ball at a later time in terms of the quantities appearing in the diagram below.



$L = MvR\sin(\theta)$

QUESTION 2: GRAVITY AND ORBITS!

According to Newton's Law of Gravitation, the gravitational force on an object of mass m at a distance R from a spherically symmetric object of mass M is directed toward the center of this object and has magnitude

$$|\vec{F}| = \frac{G\,M\,m}{R^2}$$

where G is Newton's gravitational constant, G = 6.67×10^{-11} Nm^2/kg^2.

For this question, we will assume that M is very much larger than m, so we can ignore the influence of the smaller mass on the larger one. In this case, we can say that the smaller mass moves around in a time-independent and rotationally-symmetric environment. As a result, *mechanical energy E and angular momentum L are both conserved.* These two numbers completely characterize the different possible orbits an object can have.

In this question, we will explore these possible orbits using a simulation found here:

http://phet.colorado.edu/en/simulation/my-solar-system

In the simulation, the units for mass and distance are chosen so that G = 10000.

To start, set the accurate-fast slider bar all the way to the left (accurate). Set the masses, positions, and velocities of the 2 bodies as follows:

		Position		velocity	
	mass	x	y	x	y
body 1	10	0	0	0	0
body 2	0.01	40	0	0	0

Click the green Start button to see what happens. Click Reset when you are done.

Hopefully, you have just discovered one reason why angular momentum is important (hint: imagine body 2 = Earth without angular momentum, body 1 = Sun). Body 2 didn't have any angular momentum because we set its y velocity to 0.

a) Using your knowledge of circular motion, Newton's 2nd Law, and Newton's Law of Gravitation (with G=10000) predict what value of v_y will lead to a circular orbit. Ask for help if you are stuck getting started. When you get an answer, check it by putting this value as the v_y for body 2.

In this situation, the force due to the star is the gravitational force,

$F = G M m / r^2$.

This results in an acceleration on the planet toward the star equal to

$a = F/m = GM/ r^2$.

For a circular orbit, the velocity is related to the acceleration by

$v^2/r = a$,

so we have:

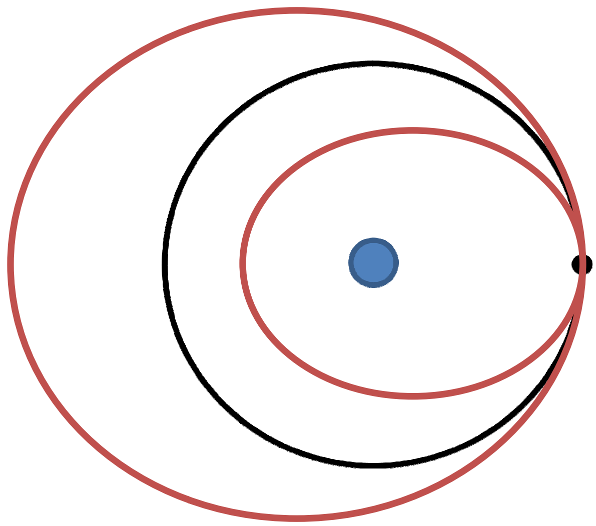
$v^2/r = GM/ r^2$, which gives

$v = (GM/r)^{1/2}$.

In our case, $G = 10000$, $M = 10$, and $r = 40$, so we get:

$v = 50$.

b) What do you think will happen if you make the initial speed a little more or a little less (from Newton's Laws, will this change the initial acceleration)? Try adding and subtracting 10 from your value for v_y from part a. Sketch these orbits below (original circular orbit shown):



c) For one of the non-circular orbits, what do you notice about the speed of body 2 as it moves around the orbit?

Object is faster when it is closer to the star and slower when it is further.

d) Explain your observation in part c using conservation of angular momentum. (You may want to have a look back at question 1c).

Angular momentum is mvr (when the object's velocity is perpendicular to the radius vector). Since angular momentum is conserved here, v must be smaller when r is larger.

e) If the initial speed is v_1, the initial distance is R_1, and the distance at the opposite side of the orbit is R_2, what is the speed v_2 at the opposite side of the orbit, in terms of v_1, R_1 , and R_2?

We must have $mv_1R_1 = mv_2R_2$, so $v_2 = v_1R_1/R_2$.

f) ESCAPE VELOCITY: If its initial speed is greater than some particular value, body 2 will keep going off to infinity. Predict this value and check your answer using the simulation (verify that it doesn't come back for speeds a little larger than your value and that it does come back for speeds a little smaller. You can move the accurate-fast slider to a faster value if things are moving too slowly for you). You may find it useful to remember that the potential energy associated with the gravitational force above is:

$$U = -\frac{G\,M\,m}{R}$$

For the object to escape to infinity, its total energy must be positive (since potential energy is zero at infinity). So we must have

$\frac{1}{2} m v^2 - GMm/R > 0$.

This gives $v > (2GM/R)^{1/2} \approx 70.7$

g) If the initial speed is less than the escape speed you derived in part f), body2 will reach some maximum distance and then start getting closer. Using conservation of energy and conservation of angular momentum, we can derive what this maximum distance is. Call the initial speed v_1, the initial distance is R_1, the distance at the opposite side of the orbit R_2, and the speed at the opposite side of the orbit v_2.

What does angular momentum conservation tell us (see part e)?

$mv_1R_1 = mv_2R_2$

What does energy conservation tell us?

$\frac{1}{2} m (v_1)^2 - GMm/R_1 = \frac{1}{2} m (v_2)^2 - GMm/R_2$

h) You now have two equations for the two unknowns R_2 and v_2. Solve for v_2 using the first equation and plug in to the second equation. After some rearranging, you should get a quadratic equation for R_2. As an example, use the simulation to find the smallest value of v_1 such that body 2 doesn't crash into body 1 (set the slider to accurate). For this value, R_2 should be the radius of body 2. Plug in this value of v_1, together with G = 10000, M = 10, and R_1 = 40 into your quadratic equation and solve for R_2 to determine the radius of body 2.

We get:

$\mathbf{(R_2)^2}[(v_1)^2 - 2GM/R_1] + 2GM\, \mathbf{R_2} - (v_1)^2 R_1 = 0$

Since $R_2 = R_1$ must be a solution, we can factor this:

$(\mathbf{R_2} - R_1)(\,[(v_1)^2 - 2GM/R_1]\, \mathbf{R_2} + (v_1)^2 R_1) = 0,$

so the solution is

$R_2 = R_1 (v_1)^2 / (2GM/R_1 - (v_1)^2).$

i) (optional) See if you can find a general solution of your quadratic equation. It looks ugly, but it actually factors, since one solution is just $R_2 = R_1$. Try to get the other solution in terms of v_1, R_1, G, and M.