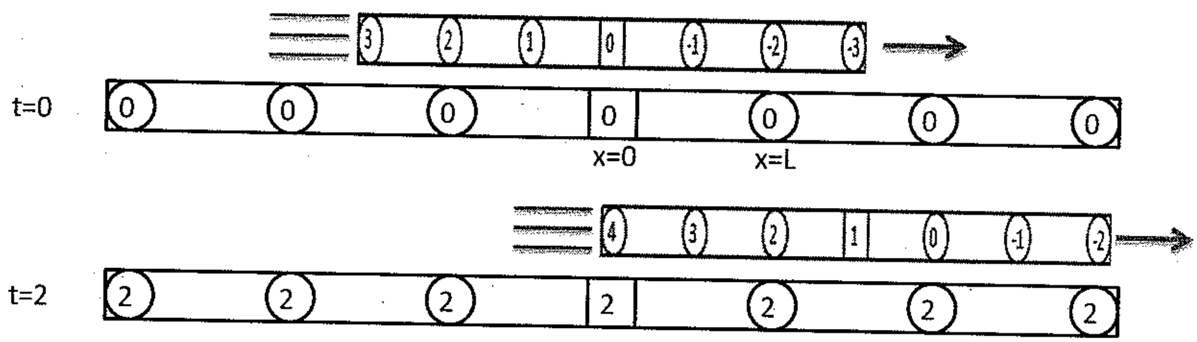
Name:

Physics Tutorial 5: Lorentz Transformations

Question 1

The pictures below show two snapshots from the video that we saw in class.



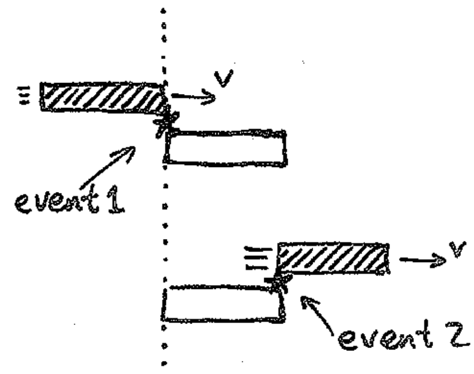
a) A firecracker explodes at position x=2L and time t=2 in the frame of the lower ruler. Where and when does this event occur in the frame of the upper (moving) ruler?

b) A cell phone rings at time t=0 and position x = -L in the frame of the lower ruler. Where and when does this event occur in the frame of the upper (moving) ruler?

Question 2

As an example of how to use the Lorentz transformation, consider the third homework problem:

A train of length 300m observes another train on a parallel track coming towards it at $v = \sqrt{3/4}\, c$. The other train appears (i.e. is measured by the first train) to have length 300m also. In the reference frame of the second train, how long does it take for the two trains to pass each other (i.e. what is the time between when the fronts align and when the backs align)



This problem is asking for the time between two events in the frame of the moving train. As a first step, what are the times and positions of these two events in the original frame?

Frame of stationary train:

we can choose our coordinates so this is true

EVENT 1: time $t_1 = 0$

position $x_1 = 0$

EVENT 2: time $t_2 =$

position $x_2 =$

Now, using the Lorentz transformation, determine the times for the two events in the frame of the moving train

Frame of moving train:

EVENT 1: time $t'_1 =$

EVENT 2: time $t'_2 =$

In the frame of the moving train, how long does it take for the two trains to pass each other?

g) Using your results for c) and f), solve for t' in terms of x,t,v,c, and γ.

t' =

(hint: it is helpful to pull out a factor of γ and write t' = γ (...). Then you can simplify what's in the brackets using $1/\gamma^2 = 1 - v^2/c^2$).

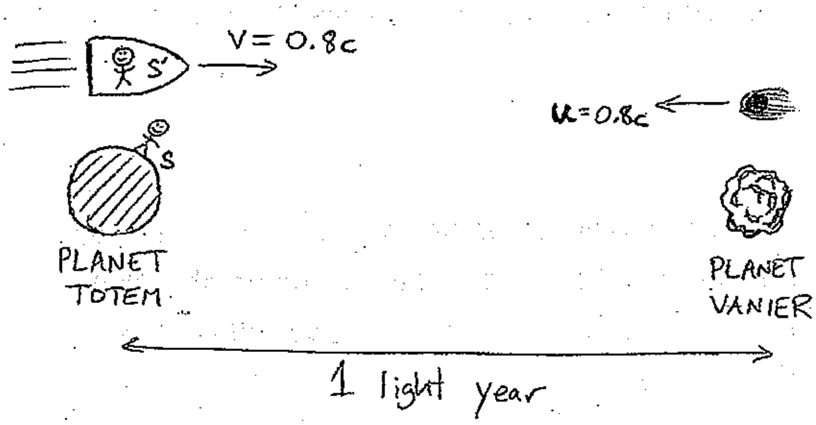
h) What are y' and z' in terms of v, c, x, y, z, and t?

y' =
z' =

Your answers to c), g), and h) give the full set of coordinates and time measured by observers in frame S' in terms of the coordinates and time measured in the frame S. You have just derived the Lorentz Transformation!

Problem 3

Many of the formulae we have derived break down if we try to use a relative velocity greater than the speed of light. I mentioned in class that nothing can actually travel faster than the speed of light. But what if we had two objects coming towards each other, both travelling at nearly the speed of light? Wouldn't their relative velocity then be greater than the speed of light? In this question, we'll use the Lorentz transformation to analyze this.



In the picture above, two planets are separated by 1 light year. At time t=0 in the frame of the planets (let's call this frame S), a spaceship passes Planet Totem at v=4/5c travelling toward Planet Vanier, and a comet passes Planet Vanier at velocity u=4/5c travelling towards planet Totem. We would like to determine what velocity observers on the ship measure for the comet. To determine the velocity, these observers measure the positions and times when the comet passes planet Vanier and Planet Totem and then plug everything in to good-old $\Delta x/\Delta t$ to determine the velocity. In order to determine the positions and times that observers in the ship's frame measure for these two events, we'll first determine the positions and times in the frame of the planets and then apply the Lorentz Transformation.

Assume that observers in frame S define the event where the ship passes Planet Totem to be at x=0 and t=0.

a) For the event where the comet passes Planet Vanier, what is the position and time as measured in the frame of the planets (frame S)?

x_V =

t_V =

b) For the event where the comet passes Planet Totem, what is the position and time as measured in the frame of the planets (frame S)?

x_T =

t_T =

c) Using the Lorentz Transformation, determine the positions and times for these events as measured in the frame of the ship. Remember that the v and γ in the Lorentz Transformation refer to the velocity of the frame S' relative to the frame S.

x'_V =

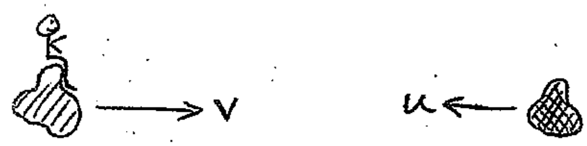
t'_V =

x'_T =

t'_T =

d) What speed do observers on the ship measure for the comet? Is this greater than c?

$u' = (x'_T - x'_V)/(t'_T - t'_V)$ =



e) Derive a general formula for the relative velocity of two objects if the objects are observed to be travelling towards each other with speed v and u respectively. What does your formula give if we take u to be c?