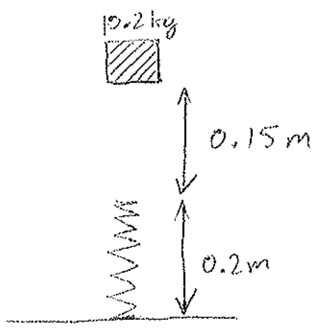
QUESTION 1

A 10.2kg block is dropped from a height of 15cm above the top of a spring with normal length 20cm. If the spring constant is 5000N/m, what is the length of the spring when it is most compressed?

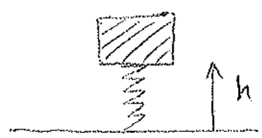
System: Block + spring.

(Useful) conserved quantity/quantities: energy.

Before diagram:



After diagram:



Conservation equations:

Energy before: $E = mgh_{BLOCK}$

$= 10.2kg \times 9.8\,m/s^2 \times 0.35m$

$= 35\,J$

Energy after: $E = \frac{1}{2}k(\Delta s)^2 + mgh$

$= \frac{1}{2}(5000N/m)(h - 0.2m)^2 + mgh$

Energy conservation:

$$2500\,N/m\,(h - 0.2m)^2 + 10.2kg \times 9.8m/s^2 \times h = 35\,J$$

Quadratic equation! $2500h^2 - 900h + 65 = 0$

2 solutions $h = \frac{900}{5000} \pm \frac{1}{5000}\sqrt{900^2 - 4\cdot65\cdot2500}$

$= 0.1m,\ 0.26m$

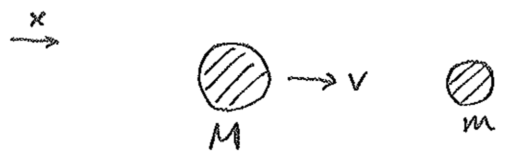
BUT: must have $h < 0.2m$ so the compressed length is $\boxed{0.1m}$.

QUESTION 2:

A billiard ball of mass m sits on a billiards table near the edge. Another ball of mass M is shot towards the first ball with speed v. Assuming that the collision is elastic and head-on, find a set of equations that determine the velocities of the two balls after the collision.

Our system is the two billiard balls. Both energy & momentum are conserved.

BEFORE:

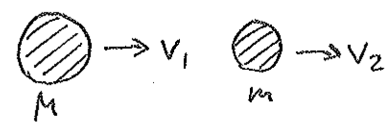


x momentum:

$$Mv$$

energy: $\frac{1}{2}Mv^2$

AFTER:



x momentum:

$$Mv_1 + mv_2$$

energy

$$\frac{1}{2}Mv_1^2 + \frac{1}{2}mv_2^2$$

(y & z momentum are not relevant.

Conservation equations:

momentum conservation gives

$$Mv = Mv_1 + mv_2$$

energy conservation gives

$$\frac{1}{2}Mv^2 = \frac{1}{2}Mv_1^2 + \frac{1}{2}mv_2^2$$

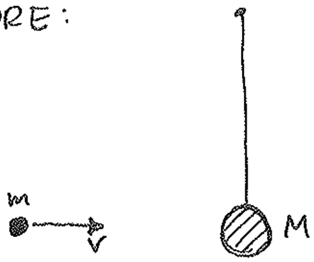
We now have 2 equations for 2 unknown quantities v_1 and v_2, so we could solve for v_1 and v_2.

Challenge question for bonus marks: if M is larger than m, the smaller ball will bounce back and forth between the larger ball and the wall a number of times before the larger ball eventually moves off with a larger speed than the smaller one. If all collisions are elastic, determine the total number of collisions (ball-ball and ball-wall) if M = m, M=100m, and M=10000m (you will probably need to use Excel). Can you guess the result for M = 100^Nm?

QUESTION 3:

A bullet with mass 10g is fired into a 2kg weight hanging from a string of length 5m. If the string swings to a 30 degree angle, how fast was the bullet travelling? (*Hint: you need to analyze this problem in two stages*)

BEFORE:



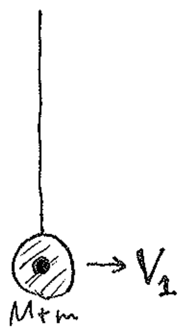
Let v be the mass of the bullet.

The original collision is inelastic, so only x momentum is conserved (there are no external horizontal forces in the collision).

The momentum before the collision is

$$P_{before} = mv$$

JUST AFTER COLLISION:



The momentum after the collision is

$$P_{after} = (M+m)V_1$$

Momentum conservation gives

$$mv = (M+m)V_1$$

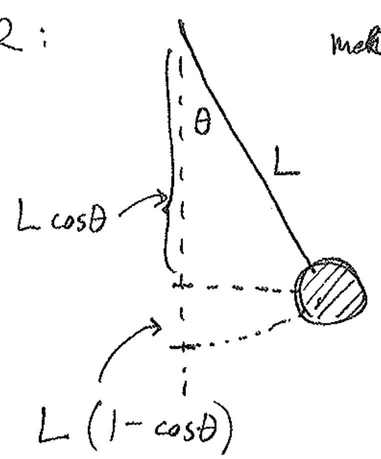
$$\Rightarrow V_1 = \frac{m}{m+M} v$$

During the swing of the weight, mechanical energy is conserved. Just after the collision, we have:

$$E = \frac{1}{2}(M+m)V_1^2$$

* we define potential energy to be 0 for the weight's lowest point.

LATER:



When the weight stops, its height is $L(1-\cos\theta)$. The mechanical energy is now just potential energy:

$$E = (M+m)gL(1-\cos\theta)$$

Energy conservation gives:

$$\frac{1}{2}(M+m)V_1^2 = (M+m)gL(1-\cos\theta)$$

$$\Rightarrow \quad V_1^2 = 2gL(1-\cos\theta) \qquad (*)$$

Now, from our momentum conservation equation we had

$$V_1 = \frac{m}{m+M} V$$

So we can use this to eliminate V_1. Equation (*) then gives

$$\left(\frac{m}{m+M}\right)^2 V^2 = 2gL(1-\cos\theta)$$

$$\Rightarrow \quad V = \left(\frac{m+M}{m}\right)\sqrt{2gL(1-\cos\theta)}$$

$$= \frac{2.01\,\mathrm{kg}}{0.01\,\mathrm{kg}}\sqrt{2 \cdot 9.8\,\mathrm{m/s^2} \cdot 5\,\mathrm{m}\,(1-\cos(30°))}$$

$$= 728\,\mathrm{m/s}$$