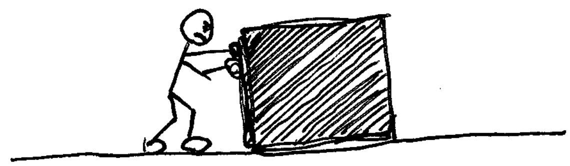
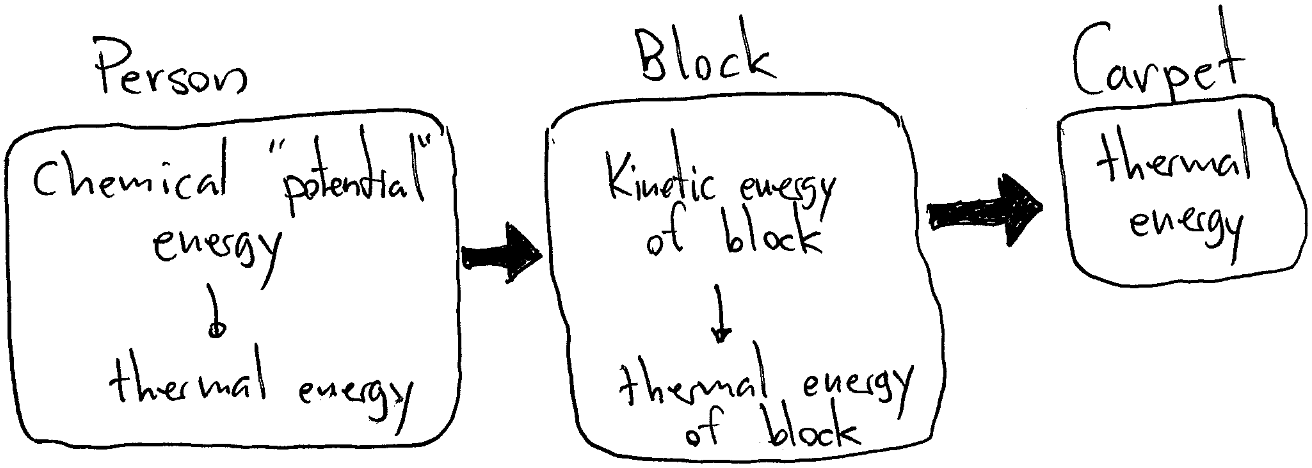
PUSHING A BLOCK WITH FRICTION:

Generally, for any system

$$\boxed{\text{total energy before} = \text{total energy after} + \text{energy in} - \text{energy out}}$$

For a block:





What's the work done on the block?

Intuitively: related to how hard and how long we push for.

SIMPLEST CASE:

A rock in outer space.

$$\begin{pmatrix} \text{work} \\ \text{done} \end{pmatrix} = \begin{pmatrix} \text{change in} \\ \text{kinetic energy.} \end{pmatrix}$$

Consider:

$$\frac{dK}{dt} = \frac{d}{dt}\left(\frac{1}{2} m v_x^2 + \frac{1}{2} m v_y^2 + \frac{1}{2} m v_z^2\right)$$

$$= m v_x \frac{dv_x}{dt} + m v_y \frac{dv_y}{dt} + m v_z \frac{dv_z}{dt}$$

$$= v_x F_x + v_y F_y + v_z F_z$$

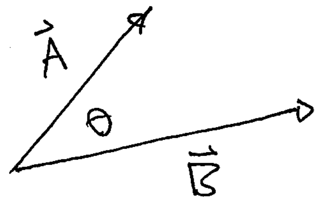
In a small time the change in kinetic energy is

$$dK = F_x v_x dt + F_y v_y dt + F_z v_z dt$$

$$\Rightarrow \quad \boxed{W = F_x dx + F_y dy + F_z dz}$$

This is actually the dot product of $\vec{F}$ and $\vec{r}$.

Dot Product:



$$\vec{A}\cdot\vec{B} = A_x B_x + A_y B_y + A_z B_z$$
$$= |\vec{A}||\vec{B}|\cos\theta$$

mathematically equal.

takes two vectors and makes a salar. A new kind of multiplication

So rewrite the work done as

$$W = \vec{F}\cdot\Delta\vec{r}$$

change in position

force applied

Conservative Forces:

$$W = \int_a^b \vec{F} \cdot d\vec{r} \qquad F_x = -\frac{dU}{dx}$$

assume potential exists.

$$= \int_a^b F_x \, dx$$

do one dimension for simplicity.

So

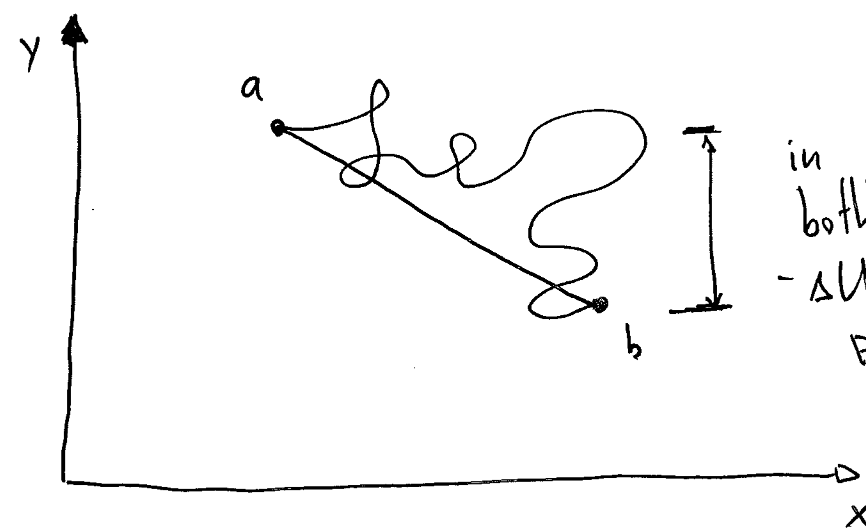
$$W = \int_a^b -\frac{dU}{dx} \, dx$$

fundamental theorem of calculus

$$= -\left[U(b) - U(a) \right] = -\Delta U$$

$$\Rightarrow \quad W = -\Delta U$$

work is always the negative of the change in potential energy.



in gravitational field both have change $-\Delta U$ regardless of path.